String Phenomenology '09 – Univ. of Warsaw, June 15-19, 2009

Type-IIA flux compactifications and N=4 gauged supergravities

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arXiv:0906.0370 [hep-th]

work in collaboration with **G. Dall'Agata and F. Zwirner**

Intro:

- In many N=1 orbifolds/CY compactifications the **closed string sector** can be described by a truncation of the **underlying *N*=4 supergravity in 4d**.

- N=4 4d supergravity is highly constrained:

  "***the EFT is fixed just by the gauging***"

- Extrema of the N=4 potential give all the extrema of the truncations
  - useful for dS/cosmo no-go's *(see talk by Zagermann)*
  - to generate solution (AdS_4/CFT_3, susy-*Mink*, etc...)

- *Aim:* relation between:

  Type IIA flux compactifications on (twisted) tori with O6-planes

  and

  N=4 4d gauged supergravities

  10d fluxes + geometry ↔ structure constants of the 4d gauging

  10d global constraints ↔ 4d generalized Jacobi Id.

Outline

- **The 10d Type-IIA setup**;
- **Mini-review** of N=4 gauged supergravities
- **From 10d to 4d:** "fluxes $\leftrightarrow$ gaugings" correspondence
  - SO(1,1)-twists and ξ-parameters
- **An application:** N=4 uplift of N=1 AdS_4 vacua
  - **gaugings** and properties
  - ***N*=8 gauged supergravity** uplift
    - M-theory uplift and massive IIA parameter
  - **relation** with other non-N=4 AdS_4 solutions
  - **scales and EFT**
  - **comments on AdS/CFT**
- **Outlook**

The 10d setup

Type IIA D6/O6 on twisted tori, with *N*=4 preserving sources:

[D6/O6 along (μ=$0..3$, i=$1..3$) orthogonal to (a=$1..3$)]

- *The metric:* $ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu + g_{ab}\, \eta^a \eta^b + g_{ij} (\eta^i + V^i_\mu dx^\mu)(\eta^j + V^j_\nu dx^\nu)$

$$d\eta^k = \frac{1}{2}\, \omega_{ij}{}^k\, \eta^i\, \eta^j + \frac{1}{2}\, \omega_{ab}{}^k\, \eta^a\, \eta^b\,,$$

$$d\eta^c = \omega_{ib}{}^c\, \eta^i\, \eta^b\,,$$

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- *Fluxes:* NSNS (H) and RR ($G^{(0)}$,$G^{(2)}$,$G^{(4)}$,$G^{(6)}$)

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- *Fluxes:* NSNS (H) and RR ($G^{(0)}$,$G^{(2)}$,$G^{(4)}$,$G^{(6)}$)

- *Global constraints:*

<u>NSNS:</u> $\omega\,\omega = -\omega_{[mn}{}^q\,\omega_{p]q}{}^r = 0$ $\qquad \omega_{mn}{}^n = 0 \quad \Rightarrow \quad \omega_{ik}{}^k + \omega_{ic}{}^c = 0$

$$dH = 0 \qquad \omega\,\overline{H} = 0$$

<u>RR:</u> $dG^{(p)} + H\,G^{(p-2)} = 0$ $\qquad \omega\,\overline{G}^{(p)} + \overline{H}\,\overline{G}^{(p-2)} = 0$

$$\omega\,\overline{G}^{(2)} + \overline{H}\,\overline{G}^{(0)} = Q(\pi_6)$$

<u>Localized:</u> $\overline{H}\,[\pi_6] = 0\,,$ $\quad \omega\,[\pi_6] = 0$ $\qquad \omega_{ik}{}^k = 0\,,$ $\quad \omega_{ic}{}^c = 0$

N=4 gauged supergravities

- *Scalar potential:* $\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SO}(6,n)}{\mathrm{SO}(6) \times \mathrm{SO}(n)}$. *the only deformation of the theory is a gauging*

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The most general gauging is defined by:

de Roo, Wagemans '85
Schon, Weidner '06

1) the gauge group
2) the de Roo-Wagemans phases, → $f_{\alpha\, MN}{}^{R}$
3) the Schon-Weidner parameters → $\xi_{\alpha\, M}$

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- *Quadratic constraints (generalized Jacobi Id.)* →

$$\xi_\alpha^M \xi_{\beta M} = 0$$
$$\xi^P_{(\alpha} f_{\beta)PMN} = 0$$
$$3\, f_{\alpha R[MN} f_{\beta PQ]}{}^R + 2\,\xi_{(\alpha[M} f_{\beta)NPQ]} = 0$$
$$\epsilon^{\alpha\beta}\left(\xi^P_\alpha f_{\beta PMN} + \xi_{\alpha M}\xi_{\beta N}\right) = 0$$
$$\epsilon^{\alpha\beta}\left(f_{\alpha MNR} f_{\beta PQ}{}^R - \xi^R_\alpha f_{\beta R[M[P}\eta_{Q]N]} - \xi_{\alpha[M} f_{N][PQ]\beta} + \xi_{\alpha[P} f_{Q][MN]\beta}\right) = 0$$

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<u>*All the action is then fixed*</u>

- *e.g. fermion variations* $$\delta\psi^I_\mu = 2D_\mu\epsilon^I - \frac{2}{3}\, A_1^{IJ}\,\Gamma_\mu\epsilon_J + \dots\,, \quad \delta\chi^I = \frac{4}{3}\, i\, A_2^{IJ}\epsilon_J + \dots\,, \quad \delta\lambda^I_A = 2\,i\,(A_{2A})_J{}^I\,\epsilon^J + \dots$$

$$A_1^{IJ} = \epsilon^{\alpha\beta}\,\mathcal{V}^\star_\alpha\,\mathcal{V}^M_{KL}\mathcal{V}^{N\,IK}\mathcal{V}^{P\,JL} f_{\beta MNP}\,,$$
$$A_2^{IJ} = \epsilon^{\alpha\beta}\mathcal{V}_\alpha\mathcal{V}^M_{KL}\mathcal{V}^{N\,IK}\mathcal{V}^{P\,JL} f_{\beta MNP} + \frac{3}{2}\,\epsilon^{\alpha\beta}\,\mathcal{V}_\alpha\,\mathcal{V}^{IJ}_M\,\xi^M_\beta\,,$$
$$(\overline{A}_{2A})^I{}_J = -\epsilon^{\alpha\beta}\mathcal{V}^\star_\alpha\mathcal{V}^M_A\mathcal{V}^{N\,IK}\mathcal{V}^P_{JK}\, f_{\beta MNP} - \frac{1}{4}\,\epsilon^{\alpha\beta}\,\mathcal{V}^\star_\alpha\,\mathcal{V}^M_A\,\delta^I_J\,\xi_{\beta M}$$

- *and scalar potential*

$$\frac{1}{3}\,A_1^{IK}\,\overline{A}_{1\,JK} - \frac{1}{9}\,A_2^{IK}\,\overline{A}_{2\,JK} - \frac{1}{2}\,A_{2\,AJ}{}^{K}\,\overline{A}_{2\,A}{}^{I}{}_{K} = -\frac{1}{4}\,\delta^I_J\,V$$

From 10d to 4d - I

- *vector fields:*

$$A_\mu^{\bar{\imath}\,-} = \widetilde{V}_{\mu\,i}\,, \qquad A_\mu^{i\,-} = \epsilon^{ijk} C^{(3)}_{\mu jk}\,, \qquad A_\mu^{\bar{a}\,-} = \frac{1}{6}\epsilon^{ijk} C^{(5)}_{\mu aijk}\,, \qquad A_\mu^{a\,-} = \frac{1}{6}\epsilon^{ijk}\epsilon^{abc} B^{(6)}_{\mu ijkbc}\,,$$

$$A_\mu^{i\,+} = V_\mu^i\,, \qquad A_\mu^{\bar{\imath}\,+} = \frac{1}{6}\epsilon^{abc} C^{(5)}_{\mu abci}\,, \qquad A_\mu^{a\,+} = \frac{1}{2}\epsilon^{abc} C^{(3)}_{\mu bc}\,, \qquad A_\mu^{\bar{a}\,+} = B_{\mu a}\,,$$

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- *the SU(1,1) axion* ⇒ No SW parameters ($\xi_{\alpha M} = 0$)

$$D_\mu C^{(3)}_{ijk} = \partial_\mu C^{(3)}_{ijk} - \omega_{[il}{}^{l}\, C^{(3)}_{\mu jk]} + V_\mu^h \omega_{hl}{}^{l}\, C^{(3)}_{ijk}$$

$$D_\mu \tau = \partial_\mu \tau + A_\mu^{M-}\,\xi_{+M} + \left(A_\mu^{M+}\,\xi_{+M} - A_\mu^{M-}\,\xi_{-M}\right)\tau - A_\mu^{M+}\,\xi_{-M}\,\tau^2$$

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- *other scalars*: (em vectors and dRW phases)

	$A_\mu^{i\,-}$	$A_\mu^{a\,+}$	$A_\mu^{\bar{i}\,+}$	$A_\mu^{\bar{a}\,-}$
$C^{(3)}_{iab}$	$\omega_{ab}{}^k$	$\omega_{ia}{}^b$	0	0
$C^{(5)}_{ijabc}$	$\overline{H}_{abc}$	$\overline{H}_{ijc}$	$\omega_{ij}{}^k$	$\omega_{ab}{}^k$

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- *A rule-of-thumb for dRW phases*

Analogously for non-geo./S-dual fluxes
see also: Aldazabal, Camara, Rosabal '08

> *The NSNS fluxes leading to non-trivial dRW phases are those and only those with lower indices orthogonal to the O-planes and upper indices parallel to the O-planes.*

From 10d to 4d - II

- *from field strengths to structure constants*

$$V^i_{\mu\nu} = 2\,\partial_{[\mu} V^i_{\nu]} - \omega_{ij}{}^k\, V^i_\mu\, V^j_\nu$$

$$V^i_{\mu\nu} = 2\,\partial_{[\mu} A^{+\,i}_{\nu]} - \omega_{ij}{}^k\, A^{+\,i}_\mu A^{+\,j}_\nu$$

$$\mathcal{H}^{M\,+}_{\mu\nu} = 2\,\partial_{[\mu} A_{\nu]}{}^{M\,+} - \widehat{f}^{\,M}_{\alpha N P}\, A^{N\alpha}_{[\mu}\, A^{P\,+}_{\nu]} + \ldots$$

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$$\begin{aligned}
f_{-\,ijk} &= -\frac{1}{6}\overline{H}_{abc}\,\epsilon^{abc}\,\epsilon_{ijk}\,,\\
f_{-\,ij}{}^c &= -\frac{1}{2}\omega_{ab}{}^k\,\epsilon^{abc}\,\epsilon_{ijk}\,,\\
f_+{}^{abc} &= \overline{G}^{(0)}\,\epsilon^{abc}\,,\\
f_{+\,i}{}^{bc} &= -\overline{G}^{(2)}_{ia}\,\epsilon^{abc}\,,\\
f_{+\,ij}{}^c &= -\frac{1}{2}\overline{G}^{(4)}_{ijab}\,\epsilon^{abc}\,,\\
f_{+\,ijk} &= \frac{1}{6}\overline{G}^{(6)}_{ijkabc}\,\epsilon^{abc}\,,\\
f_{+\,ij}{}^k &= \omega_{ij}{}^k\,,\\
f_{+\,ija} &= -\overline{H}_{ija}\,,\\
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- *10d BI ⇔ 4d generalized Jacobi*

$$f_{\alpha R[MN} f_{\beta PQ]}{}^{R} = 0\,, \qquad \epsilon^{\alpha\beta} f_{\alpha MNR} f_{\beta PQ}{}^{R} = 0$$

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$$f_{\alpha R[MN} f_{\beta PQ]}{}^R = 0\,, \qquad \epsilon^{\alpha\beta} f_{\alpha MNR} f_{\beta PQ}{}^R = 0$$

$$\left(\omega\overline{G}^{(2)} + \overline{H}\overline{G}^{(0)}\right)_{ijc} = 0$$

$$\left(\omega\overline{G}^{(4)} + \overline{H}\overline{G}^{(2)}\right)_{ijkab} = 0$$

$$(\omega\omega)_{ija}{}^b = 0$$

$$(\omega\overline{H})_{ijka} = 0$$

$$(\omega\overline{H})_{iabc} = 0$$

$$(\omega\omega)_{abi}{}^k = 0$$

$\xi_{\alpha M}$ and SO(1,1)-twist

along the lines of:
GV, Zwirner '04
Derendinger, Petropoulos, Prezas '07

- *twisting the SO(1,1)*

$$g \to e^{\lambda/2}\, g\,, \quad B \to e^{\lambda/2}\, B\,, \quad \Phi \to \Phi + \lambda\,, \quad C^{(p)} \to e^{\left(\frac{p}{4}-1\right)\lambda}\, C^{(p)}$$

the metric is not invariant ⇒ induces a ω–flux: $\quad \mathrm{tr}\ \omega \neq 0$

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- *implementation*

$$d\Phi = d_4\Phi + \overline{\Delta}$$

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- *constraints* $\quad \mathcal{D}^2 = 0\,, \qquad \mathcal{D}G = Q_{RR}\,.$

$$\boxed{\omega_{ij}{}^{j} = \frac{3}{4}\overline{\Delta}_i\,, \qquad \omega_{aj}{}^{j} = -\frac{3}{4}\overline{\Delta}_i} \qquad \boxed{\overline{\Delta}_i \overline{G}^{(0)} = 0}$$

$$\begin{aligned} G^{(4)}_{\mu ijk} &= \partial_\mu C^{(3)}_{ijk} - (\omega_{ij}{}^{l} C^{(3)}_{lk\mu} + 2\mathrm{Perm}_{ijk}) - \frac{1}{2}(\overline{\Delta}_i C^{(3)}_{jk\mu} + 2\mathrm{Perm}_{ijk}) \\ &= \partial_\mu C^{(3)}_{ijk} + \overline{\Delta}_i C^{(3)}_{\mu jk} + 2\mathrm{Perm}_{ijk}\,, \end{aligned}$$

$\Rightarrow$ $\boxed{\xi_{+i} = \overline{\Delta}_i}$

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⇒ $\boxed{\xi_{+i} = \overline{\Delta}_i}$

...but SO(1,1) is exact only at 2-derivative level...

...no uplift at the full string theory level?

An example: AdS_4 vacua – I

uplift to *N*=4 of *N*=1 AdS_4 vacua from $T^6 / Z_2 \times Z_2$ compactifications
(analogous for other orbifold vacua)

GV, Zwirner '05
see also:
Camara, Font, Ibanez 05
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- *the families of vacua*

$$\frac{1}{9}\overline{G}^{(6)} = -t_0^2\,\overline{G}^{(2)} = \frac{t_0\,u_0}{6}\omega_1 = \frac{s_0\,t_0}{2}\omega_2 = \frac{t_0\,u_0}{6}\omega_3\,,$$

$$\frac{t_0}{3}\overline{G}^{(4)} = \frac{t_0^3}{5}\overline{G}^{(0)} = -\frac{s_0}{2}\overline{H}_0 = \frac{u_0}{2}\overline{H}_1\,,$$

condition for *N*=4 sources (no net D6 at angles): $$5\,u_0^2\,\overline{H}_1^2 = 3\,s_0^2\,t_0^2\,\omega_2^2\,.$$

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- *uplift to N=4*

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  fluxes spontaneously break susy N=4→1

An example: AdS_4 vacua – I

uplift to N=4 of N=1 AdS_4 vacua from $T^6 / Z_2 \times Z_2$ compactifications
(analogous for other orbifold vacua)

GV, Zwirner '05
see also:
Camara, Font, Ibanez 05
Aldazabal, Font '07

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- *gauging* $$G = \mathrm{SU}(2) \ltimes N_{9,3}\,.$$

$$[X_i, X_j] = \epsilon_{ijk}X_k, \qquad [X_i, A_j^I] = \epsilon_{ijk}A_k^I,$$

$$[A_i^1, A_j^1] = \epsilon_{ijk}A_k^2, \qquad [A_i^1, A_j^2] = \epsilon_{ijk}A_k^3.$$

An example: AdS_4 vacua – II

- *Net D6/O6 charge cancel* on the AdS vacua, $\omega\,\overline{G}^{(2)} + \overline{H}\,\overline{G}^{(0)} = Q(\pi_6) = 0$
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Hull '98

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Hull '98

An example: AdS_4 vacua – III

- *the geometry of the vacuum:*

on the lines of
Aldazabal, Font '07

$$d\eta^\Lambda = \omega_1^\Lambda\, \eta^\Sigma \eta^\Gamma + \omega_2^\Lambda\, \widetilde{\eta}^\Sigma \widetilde{\eta}^\Gamma\,, \qquad d\widetilde{\eta}^\Lambda = \omega_{3\Sigma\Gamma}\, \eta^\Sigma \widetilde{\eta}^\Gamma + \omega_{3\Gamma\Sigma}\, \widetilde{\eta}^\Sigma \eta^\Gamma$$

$$\Rightarrow \quad \xi^\Lambda \equiv \sqrt{\omega_1^\Sigma \omega_1^\Gamma}\left(\eta^\Lambda + \frac{\widehat{u}_\Lambda}{\sqrt{3}}\,\widetilde{\eta}^\Lambda\right), \qquad \widetilde{\xi}^\Lambda \equiv \sqrt{\omega_1^\Sigma \omega_1^\Gamma}\left(\eta^\Lambda - \frac{\widehat{u}_\Lambda}{\sqrt{3}}\,\widetilde{\eta}^\Lambda\right)$$

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$$\overline{G}^{(0)} = g_0\,, \qquad \overline{G}^{(6)} = g_6\, \xi^1 \widetilde{\xi}^1 \xi^2 \widetilde{\xi}^2 \xi^3 \widetilde{\xi}^3$$

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String coupling: $e^{-2\Phi} = \frac{2^{4/3} \cdot 3}{5^{5/6}} \left(g_0^5\, g_6\right)^{1/3}$

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- *equivalent to SUSY conditions:*

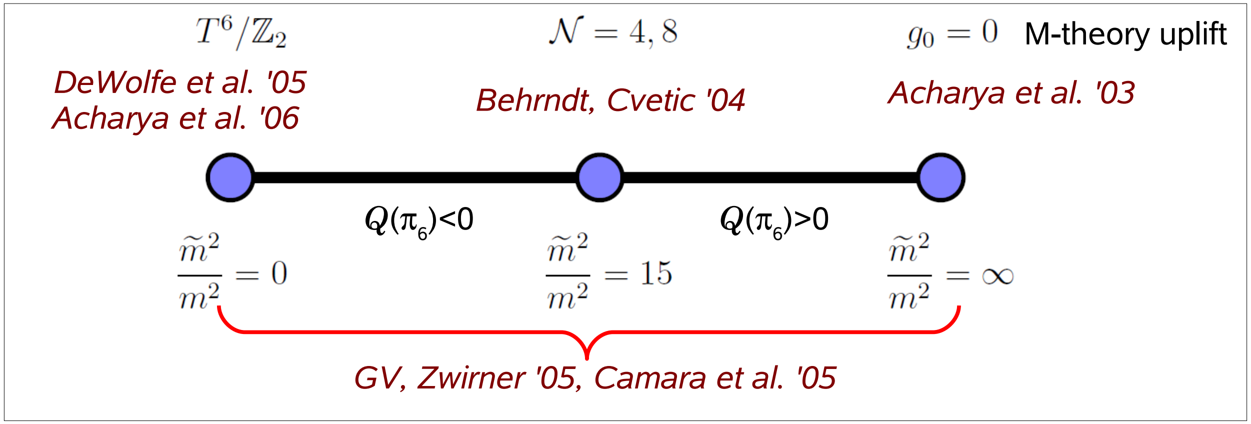
Behrndt, Cvetic '04

$$dJ = 2\widetilde{m} \mathrm{Re}\Omega\,, \qquad d\Omega = i \left(W_2^- J - \frac{4}{3} \widetilde{m} J^2 \right), \qquad H = -2m \mathrm{Re}\Omega;$$

$$G^{(0)} = 5m e^{-\Phi}, \quad \mathrm{e}^{\Phi} G^{(2)} = -W_2^- + \frac{1}{3} \widetilde{m} J, \quad G^{(4)} = \frac{3}{2} m e^{-\Phi} J^2, \quad G^{(6)} = -\frac{1}{2} \widetilde{m} \mathrm{e}^{-\Phi} J^3$$

An example: *AdS*$_4$ vacua – IV

- *other solutions:* $$dG^{(2)} + HG^{(0)} = \frac{2}{3}\mathrm{e}^{-\Phi}\left(\widetilde{m}^2 - 15m^2\right)\mathrm{Re}\Omega = Q(\pi_6)$$



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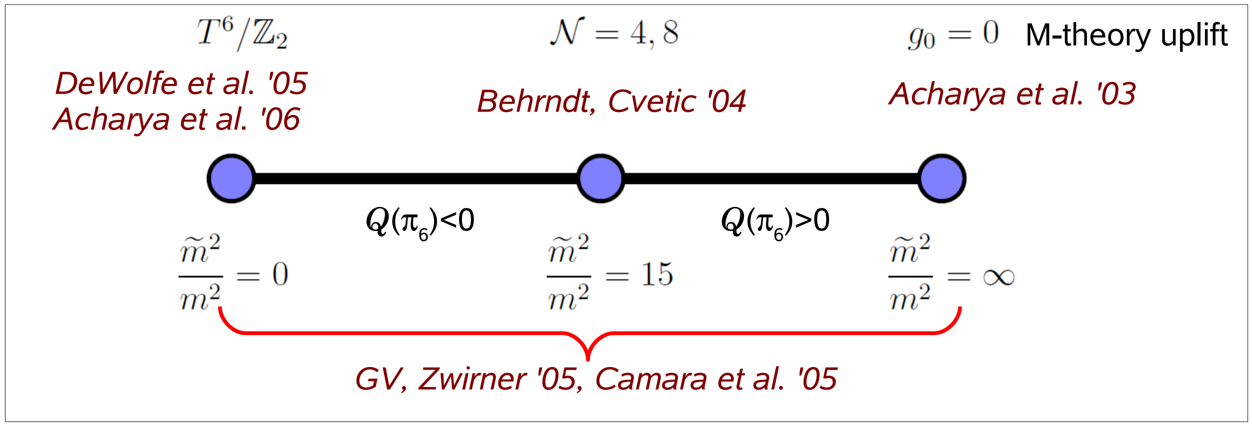

- *scales and EFT:* $\rho^2 \sim \left(\frac{g_6}{g_0} \right)^{1/3}, \qquad e^{2\Phi} \sim \frac{1}{g_0^{5/3} g_6^{1/3}} \sim \frac{1}{g_0^2 \rho^2}$

$S^3 \times S^3$: $\ell_{AdS}^{-2} = \frac{V_0}{M_P^2} \sim \frac{g_0^2 \sqrt{g}}{e^{-2\Phi} \sqrt{g}} \sim \rho^{-2}$

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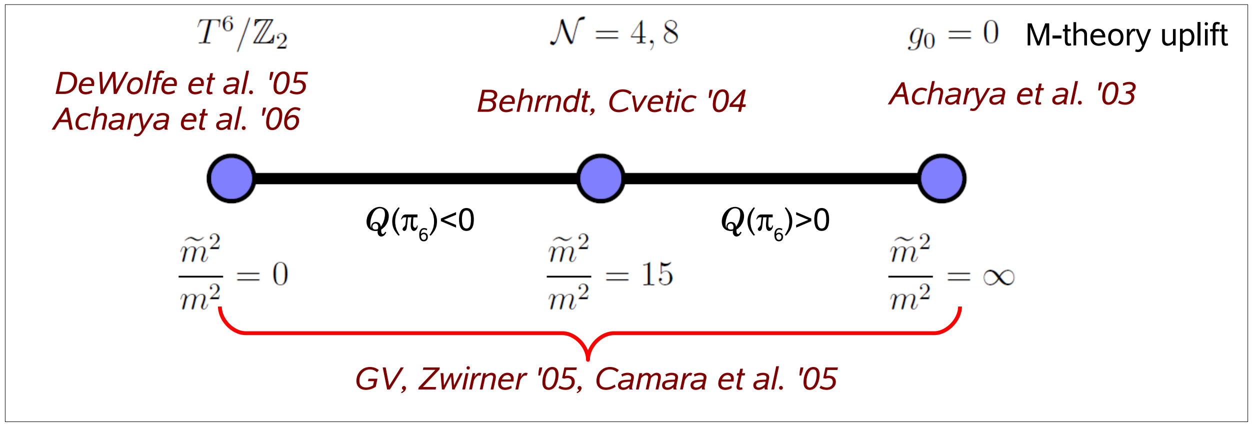

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- *AdS$_4$/ CFT$_3$* ABJM-like scaling: $N \sim g_6 \sim \frac{\rho^5}{e^{\Phi}}$, $\frac{N}{k} \sim \frac{g_6}{g_2} \sim \rho^4$

non-trivial 3-cycles → axions $C^{(3)} = a(\xi^1\xi^2\xi^3 + \tilde{\xi}^1\tilde{\xi}^2\tilde{\xi}^3)$

E2-instantons $Ae^{-\int_{E2}(e^{-\Phi}\mathrm{Re}\,\Omega + iC^{(3)})} \sim Ae^{-\frac{\mathrm{vol}(S^3)}{g_s} + ia}$ ⇒ $e^{-\mathrm{const}\sqrt{kN}}$

Conclusions

- IIA O6 twisted tori compactifications $\leftrightarrow$ 4d *N*=4 gauged supergravities
  - *(Mapping of 4d Jacobi w/ 10d global constraints)*
  - *Identification of dRW phases*
- SO(1,1)-twist and ξ –parameters
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& Outlook

- Extension to non-geo./S-dual fluxes
- ξ –parameters in string theory?
  - consequences for moduli stab./ *dS* vacua
- dS vacua? stabilized susy-Mink vacua?
- CFT dual to AdS vacua?
  - 3-cycles, axions, chiral fermions...