

# Ultrafast nonlinear spectroscopy and control of semiconductor nanostructures: From ensembles to individual quantum states

**Wolfgang Langbein**

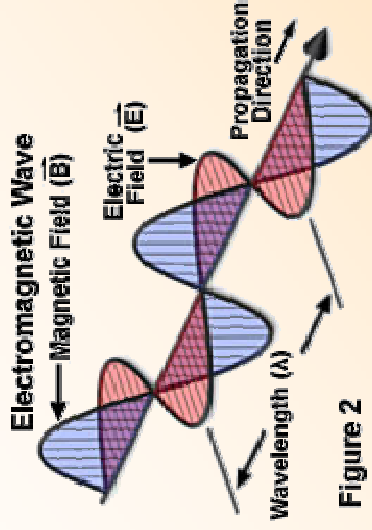
*School of Physics and Astronomy, Cardiff University*

- Linear and non-linear optical spectroscopy
- III-V semiconductors and nanostructures, excitonic properties, disorder
- Transient coherent spectroscopy & four-wave mixing, photon echo
- Excitons, Biexcitons and dephasing in quantum wells, wires and dots
- Rabi-oscillations in quantum dot ensembles
- Four-wave mixing of individual quantum states:
  - Photon echo formation
  - Rabi oscillations & coherent control
  - Two-dimensional four-wave mixing
  - Coherent coupling between states

# Optical spectroscopy

We use **electromagnetic radiation** to investigate semiconductor structures

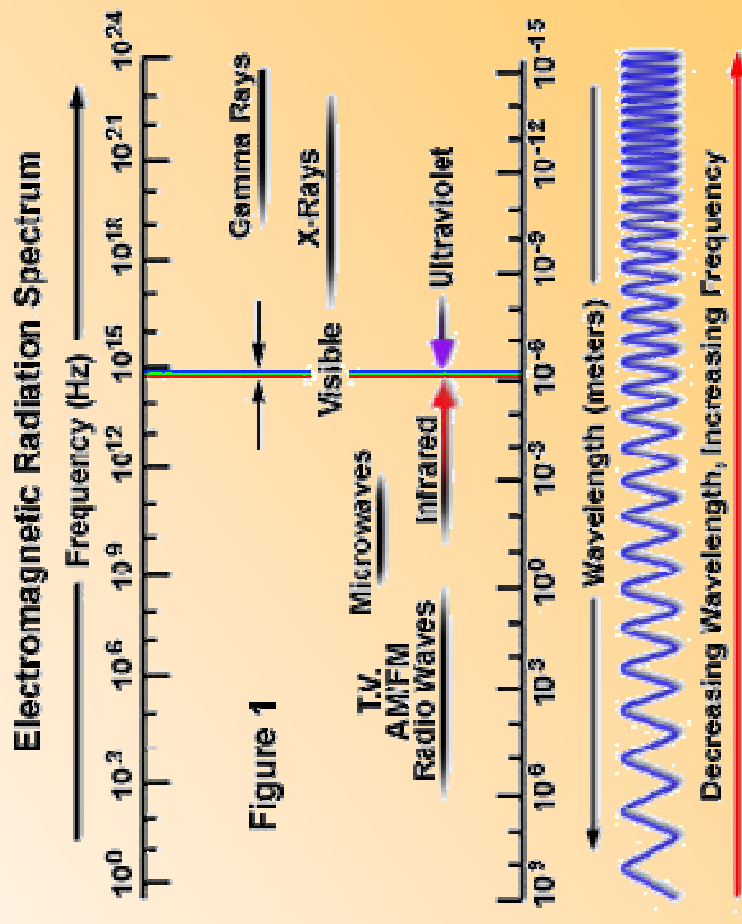
classical field of a light wave



amplitude  $E_0$ ,  
wavelength  $\lambda$ , frequency  $\nu$   
direction  $\mathbf{k}$

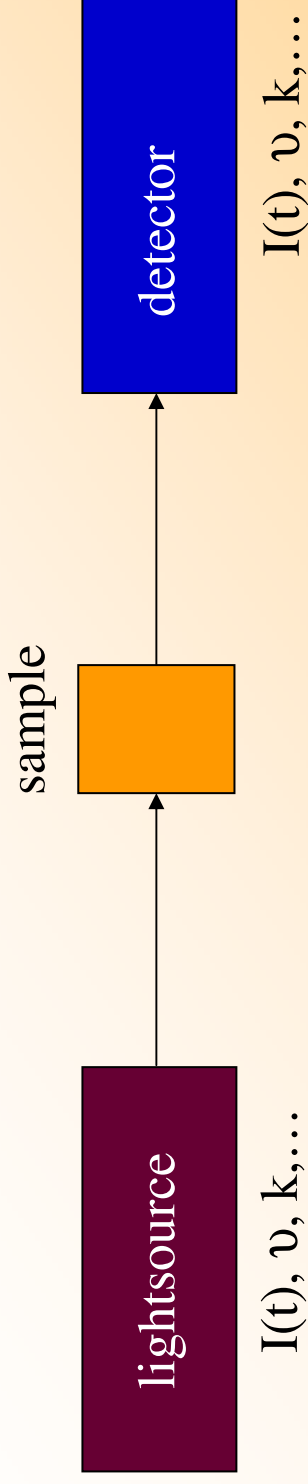
refractive index  $n$        $\lambda = \frac{c}{n\nu}$   
light velocity  $c$

<http://micro.magnet.fsu.edu/primer/lightandcolor/electromagintro.html>



thermal energy  $kT$   
a room temperature

# Linear and non-linear spectroscopy



**Linear Optics:** detected field is proportional to the exciting field,  
i.e the material polarization is

$$\vec{P} = \tilde{\chi} \vec{E}$$

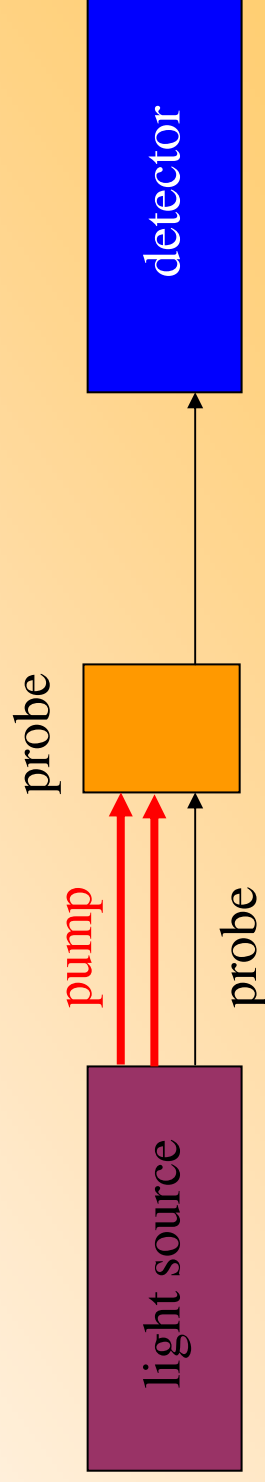
**Non-linear Optics:** detected field is **not** proportional to the exciting field.  
The material polarization can be developed in orders of the exciting field  $E$

$$\vec{P} = \tilde{\chi} \vec{E} + \tilde{\chi}^{(2)} \vec{E}^2 + \tilde{\chi}^{(3)} \vec{E}^3 + \dots$$

# Linear and non-linear optical spectroscopy

**Non-linear optics: the excitation field is changing the material properties**

- ⇒ material processing (laser-welding, laser-evaporation)
- ⇒ usage of light not only to measure, but also to modify the sample: very specific preparation possible
- ⇒ micrometer spatial resolution, femtoseconds temporal, Hz frequency (the best time standard uses nonlinear optical spectroscopy)
- ⇒ **typical experimental configuration: pump-probe technique with one or more pump beams and one probe beam**



# III-V Semiconductors

		d-shell (10)										p-shell (6)							
Group	Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1	H																	2 He
2	2	Li	Be																10 Ne
3	3	Na	Mg																18 Ar
4	4	K	Ca																36 Kr
5	5	Rb	Sr																54 Xe
6	6	Cs	Ba	*															86 Rn
7	7	Fr	Ra	**															118 Uuo

		III					V				
5	B	6	C	7	N	8	O	9	F	10	Ne
13	Al	14	Si	15	P	16	S	17	Cl	18	Ar
31	Ga	32	Ge	33	As	34	Se	35	Br	36	Kr
49	In	50	Sn	51	Sb	52	Te	53	I	54	Xe
81	Tl	82	Pb	83	Bi	84	Po	85	At	86	Rn
113	Uut	114	Uuq	115	Uup	116	Uuh	117	Uus	118	Uuo



III - V Semiconductor (3+5=8, filling of a shell)

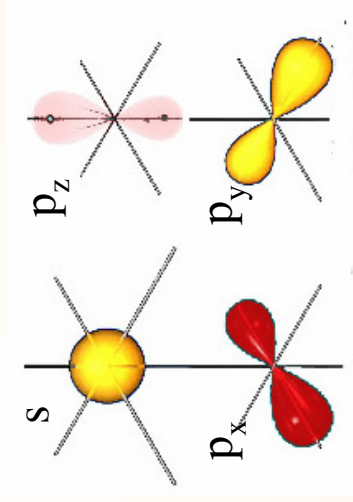
others: In, Ga, Al - Sb, As, P, N

Elements of Group III and V  
i.e. 3 or 5 Electrons in the lowest  
not fully occupied s p shell

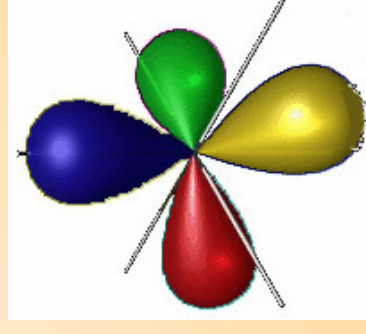
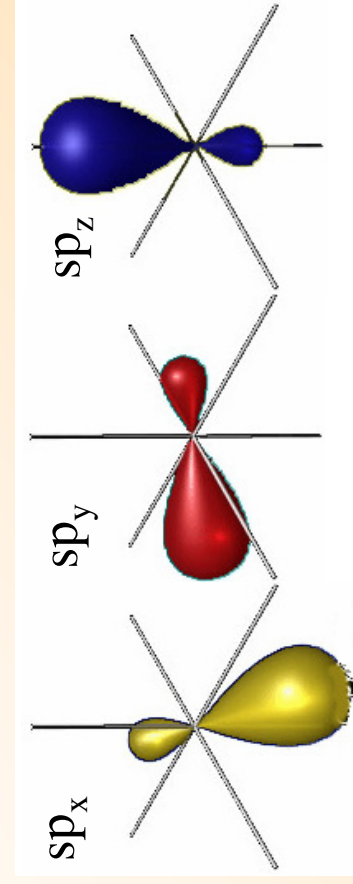


# Zincblende type crystals: chemical binding

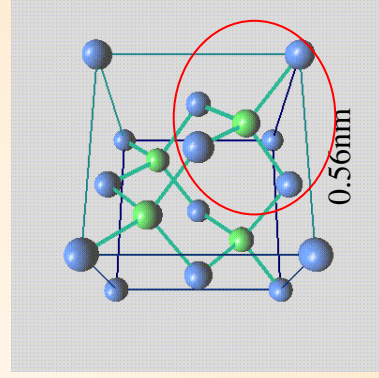
Topmost atomic orbitals



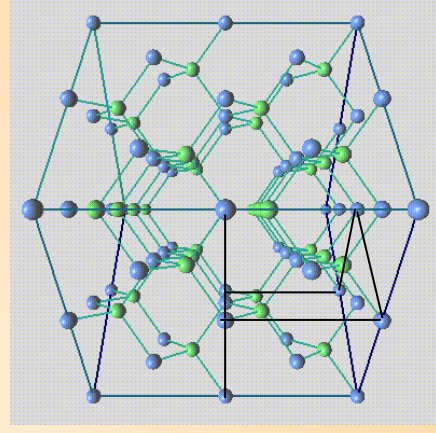
sp-3 Hybridization gives binding geometry: tetraedric



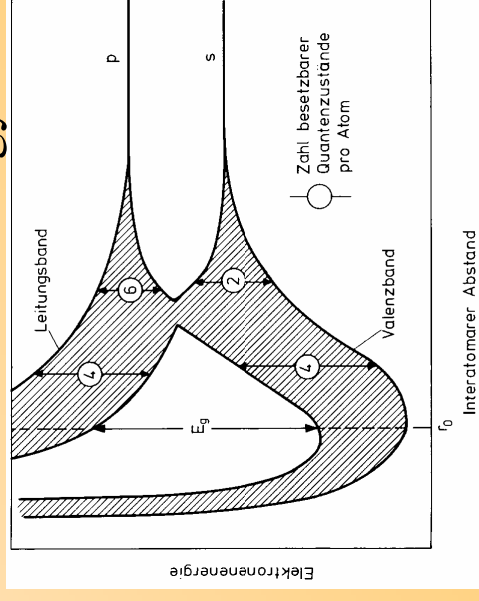
periodic arrangement of atoms Gallium :  $s^2p^1$  Arsen :  $s^2p^3$



unit cell of GaAs



atomic levels -> energy bands





# Semiconductor band structure

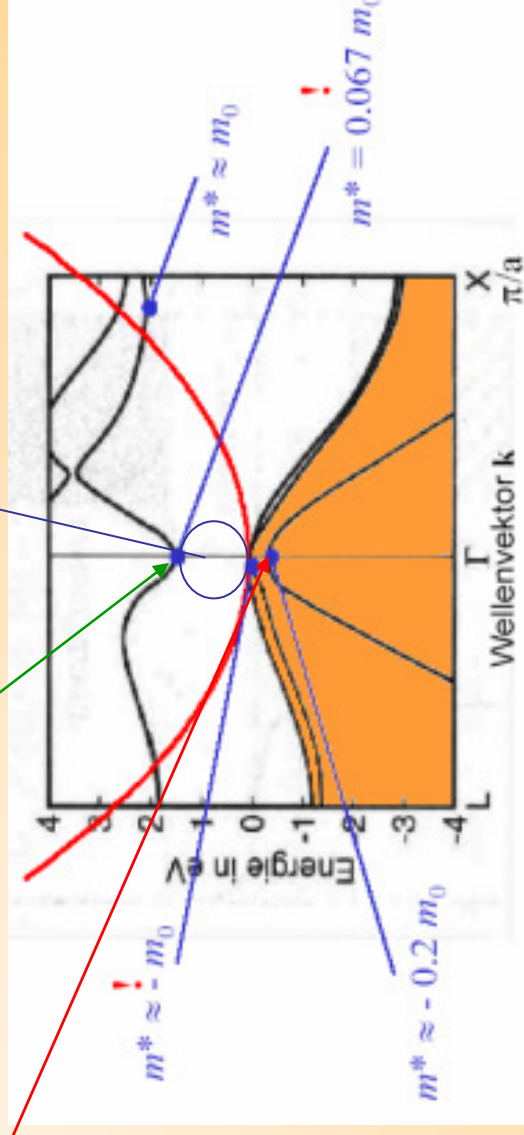
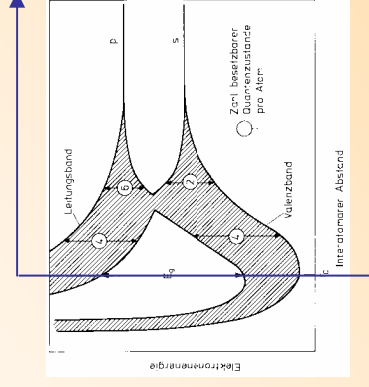
At **maximum** / **minimum** of energy bands:

Dispersion is parabolic, like for a particle with mass

**Band gap** between filled and empty states

**electrons** in the (otherwise empty) conduction band: positive mass  
 no electronic excitations with small energy are possible  
 No electric conductivity

**missing electrons (holes)** in the (otherwise full) valence band: positive mass

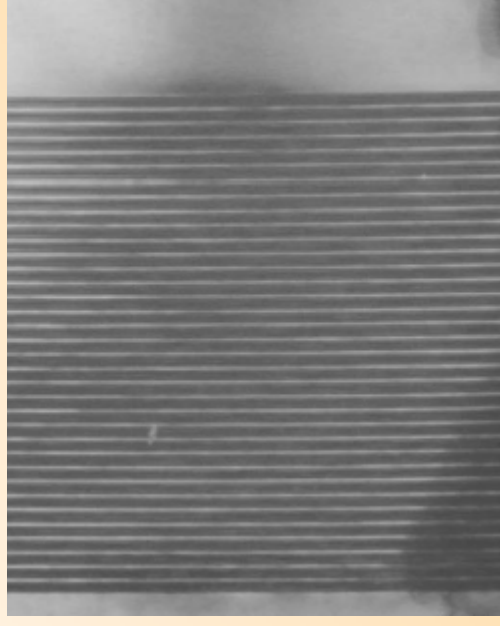
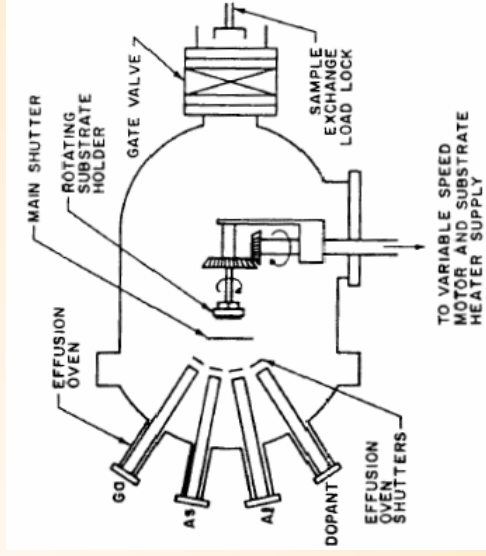


# MBE growth of thin semiconductor layers

## Molecular beam epitaxy:

Evaporation of element sources onto a heated substrate in ultrahigh vacuum (pressure  $<10^{-13}$  atmospheres, the atom mean free path is more than a meter)

Control of the layer thickness to the lattice constant, (0.3nm)

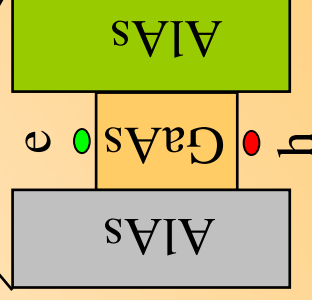


Transmission  
electron  
microscopy

Conduction band edge

Band gap

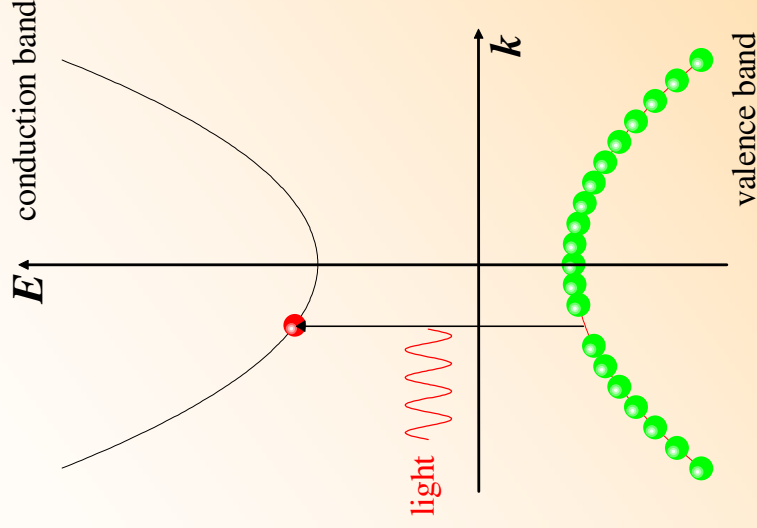
Valence band edge



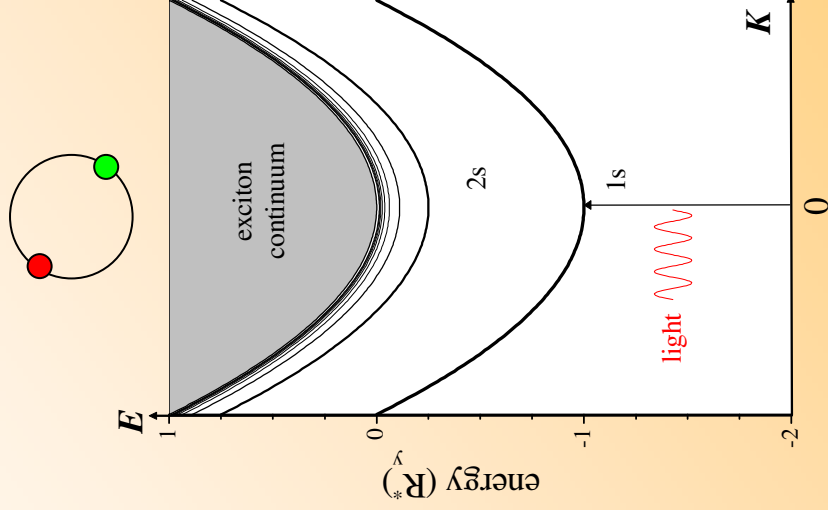


# Interband transitions and excitons

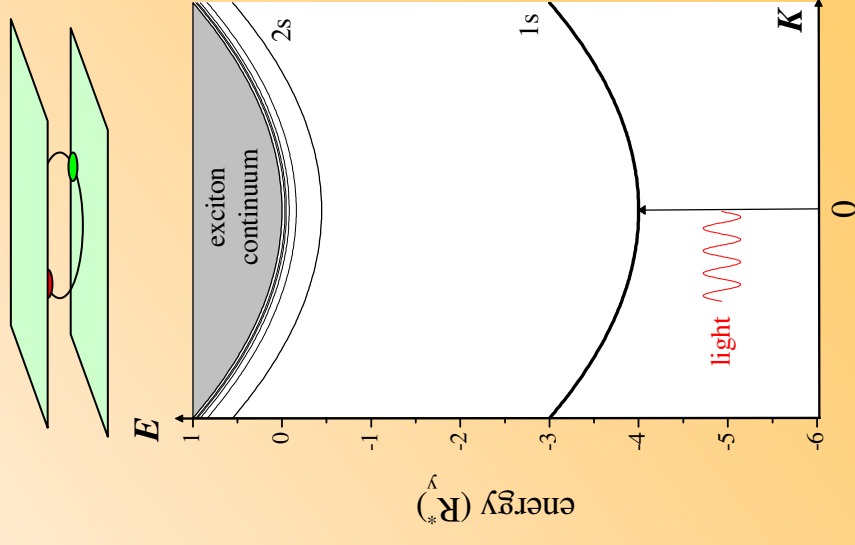
Coulomb attraction between electron and hole:  
Formation of a bound state (like Hydrogen atom)



three-dimensional  
exciton



two-dimensional  
exciton



# Exciton-polaritons in bulk semiconductors

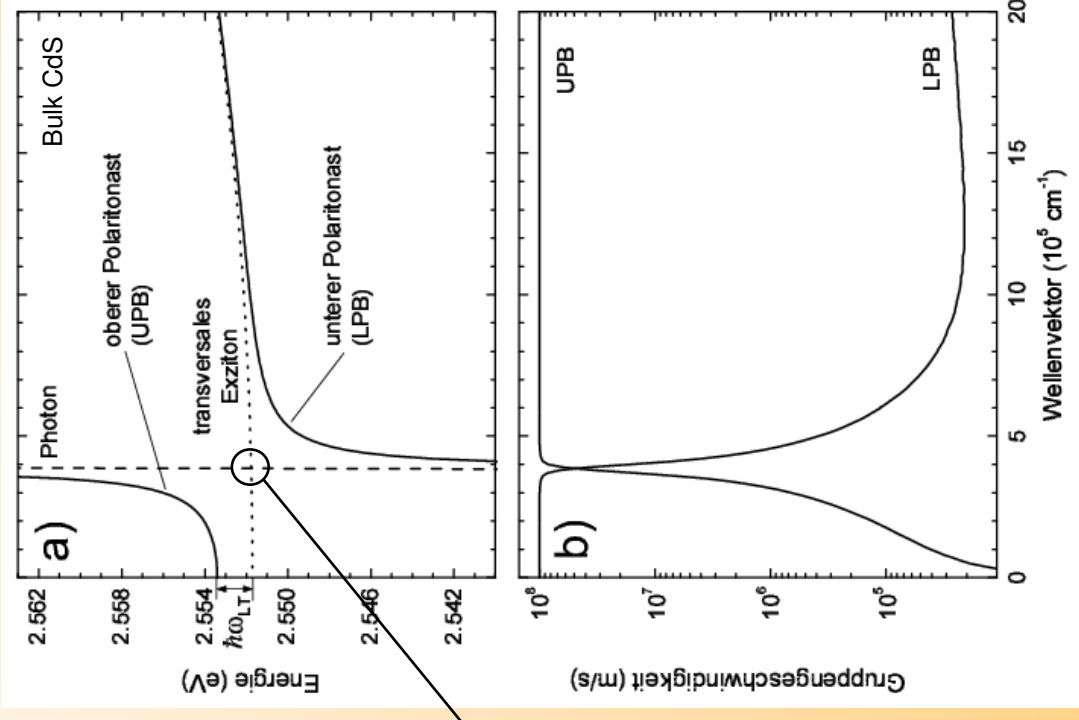
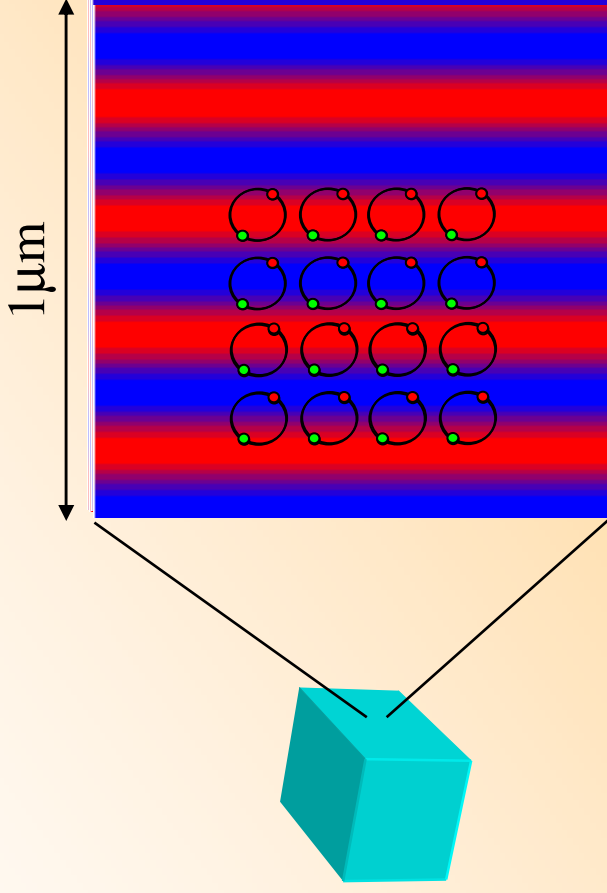
Exciton and photon have **3D translational symmetry**:  
coupled exciton-photon states: exciton polaritons  
with defined wavevector

Exciton: heavy ( $\sim$ electron mass), slow

Photon: massless, fast ( $c/n$ )

Exciton-polaritons:

widely variable velocity and mass



# Exciton-polaritons in quantum wells

QW Exciton:

2D translational symmetry

In-plane wavevector **inside light** coupling to a continuum of 3D p radiative decay

**Outside light cone** ( $|\mathbf{k}| > \omega c^{-1}$ ):

Formation of stable interface polaritons **inside light cone**

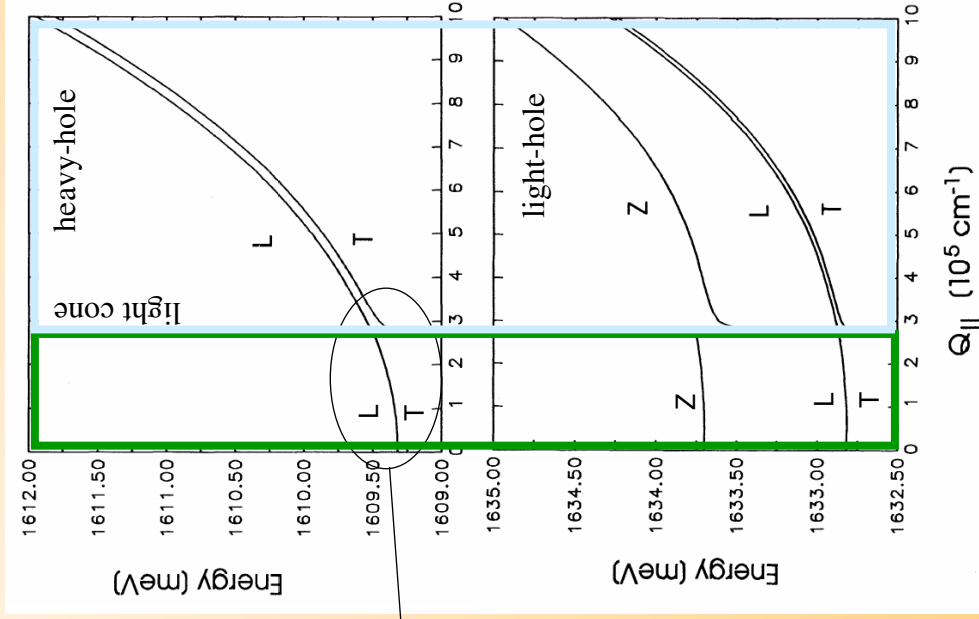
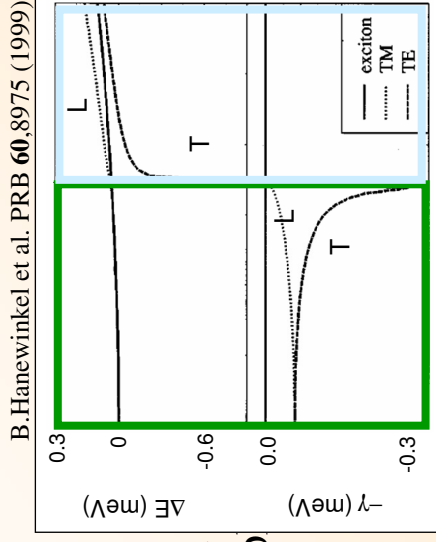
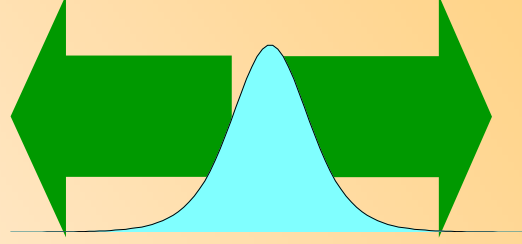
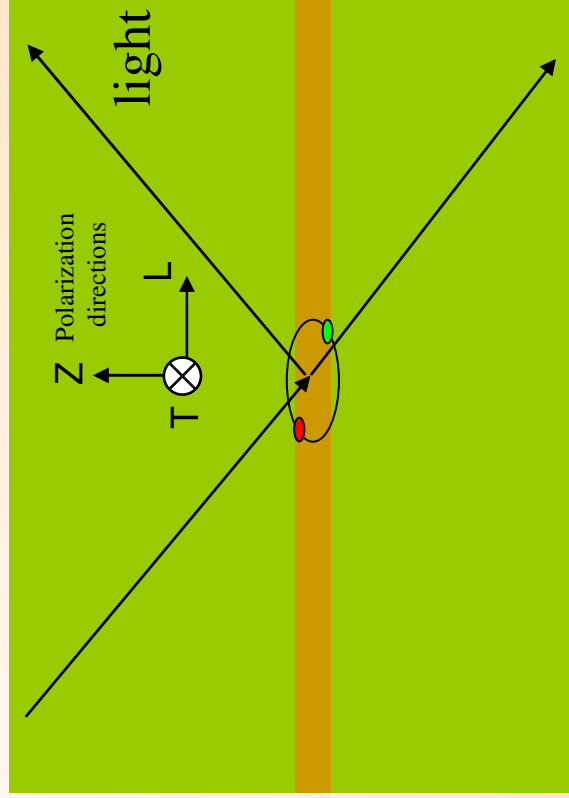
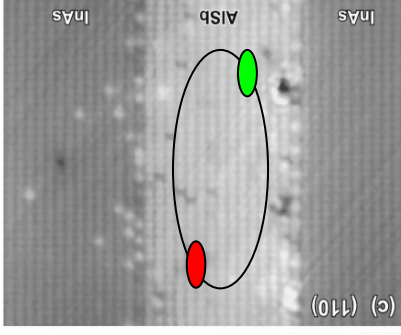


FIG. 5. LH polariton dispersion for a 50-Å QW. Resonant and localized polaritons are separated by the dashed line, which is given by  $\hbar\omega = \hbar cQ_{||}/\sqrt{\epsilon_{\infty}}$ .

S. Jorda et al. PRB **48**, 1669 (1993)



# Disorder in quantum wells



## Real quantum wells:

Deviation from in-plane translational symmetry due to interface roughness, segregation, alloy disorder:

- Formation of an effective two-dimensional disorder potential  $V(\mathbf{R})$  for the motion of the 1s exciton in the QW plane
- $V$  is typically exceeding the coupling to the photon field, so that they are spread in k-space wider than the radiative cone

Excitons in quantum wells:

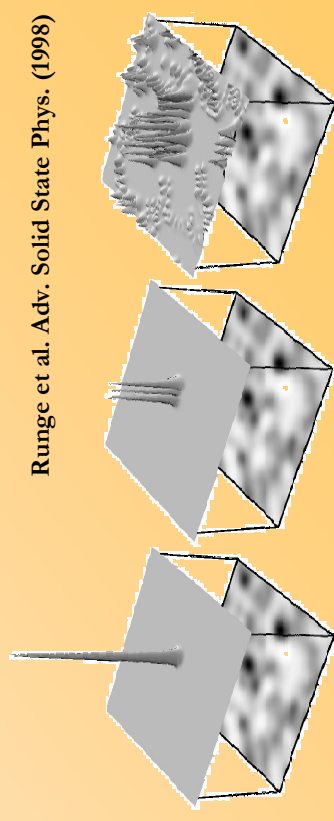
Model for a quantum-mechanical particle moving in a two-dimensional potential landscape

Advantage: excitons can be created and detected by light

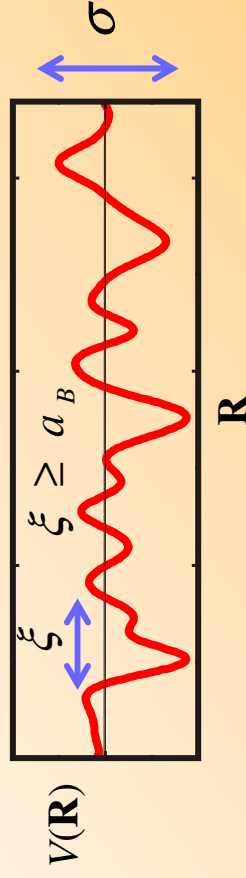
Schrödinger equation for center-of-mass (COM) motion

$$H_R \psi_n(\mathbf{R}) = \left( -\frac{\hbar^2}{2M} \nabla_{\mathbf{R}}^2 + V(\mathbf{R}) \right) \psi_n(\mathbf{R}) = E_n \psi_n(\mathbf{R})$$

Localised exciton states  $\psi_\alpha(\mathbf{R})$   
with eigenenergies  $\hbar\omega_\alpha$

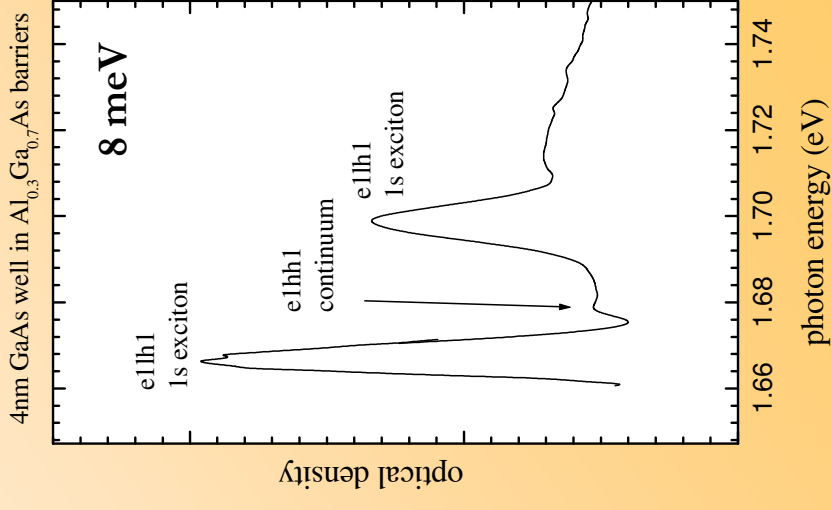
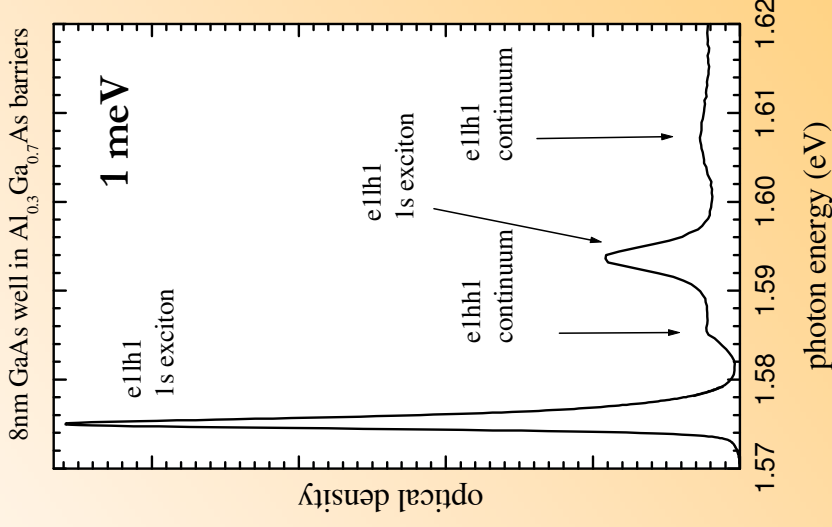
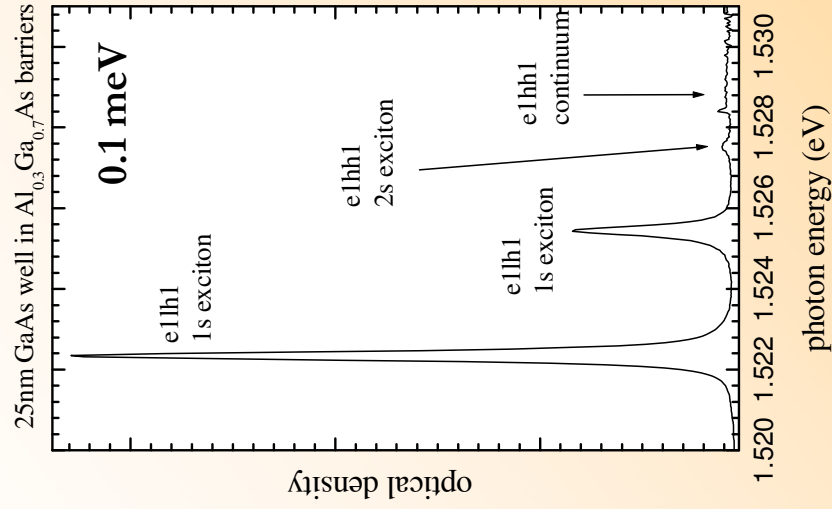
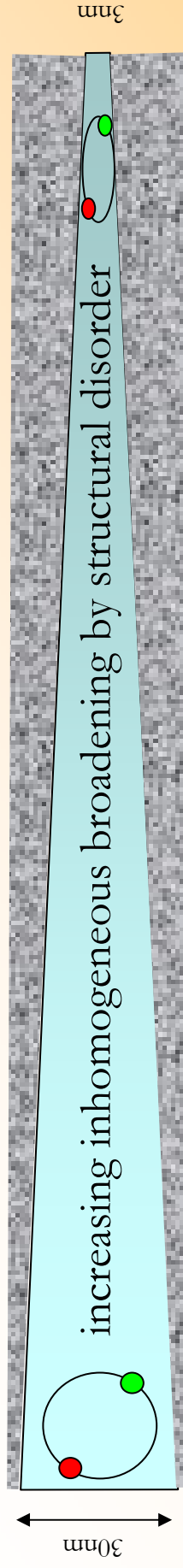


Runge et al. Adv. Solid State Phys. (1998)



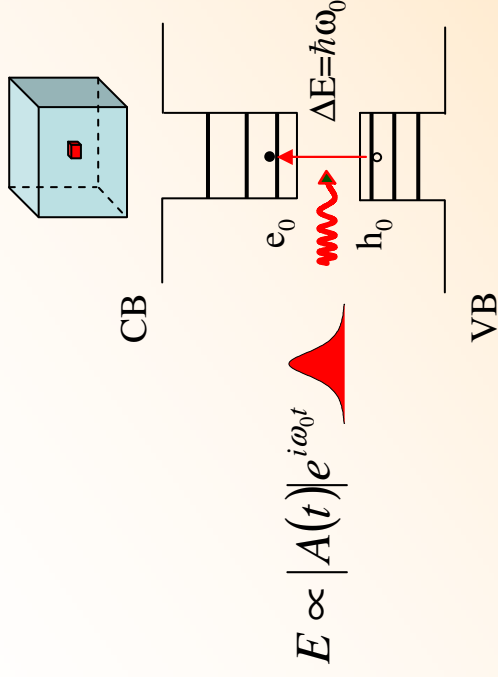
# Excitonic absorption in GaAs/AlGaAs quantum wells

with decreasing well width:



# Transient coherent spectroscopy

QD



A short light pulse induces a coherent *polarization P*.  
The average over many repeats and/or many systems is measured (even in single dot experiments)

$$\vec{P} = \sum_k P_k \langle \vec{p} \rangle_k \quad \text{„macroscopic“ polarization}$$

$P_k$ : probability for the state  $\Psi_k$      $\langle \vec{p} \rangle_k = \langle \Psi_k | \vec{p} | \Psi_k \rangle$

Dynamics of  $P$  is given by

- *amplitude decay* of the individual polarizations
- *mutual phase coherence* in the ensemble average.

Relaxation time approximation for ensembles of equivalent systems:

$$P \propto \exp(-t/T_2)$$

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_2'}$$

↑ population relaxation      ↑ pure dephasing

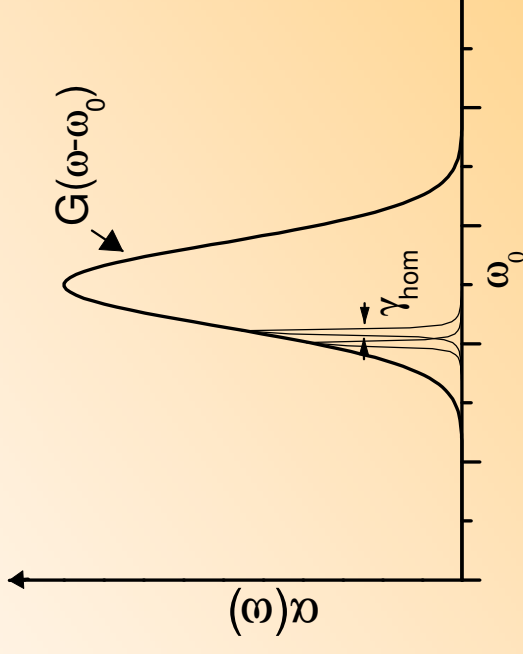


# Inhomogeneous broadening

Non-equivalent systems: Distribution of transition frequencies

$$G(\omega_i) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left\{ -\frac{(\omega_i - \omega_0)^2}{2\sigma^2} \right\}$$

$$P \propto \exp\left(-t/T_2 - \frac{1}{2}t^2\sigma^2\right)$$



**Fast decay of macroscopic polarization** due to destructive interference:  
 $\Rightarrow$  microscopic dephasing  $T_2$  not measurable

The way out: Consider the third-order nonlinear polarization

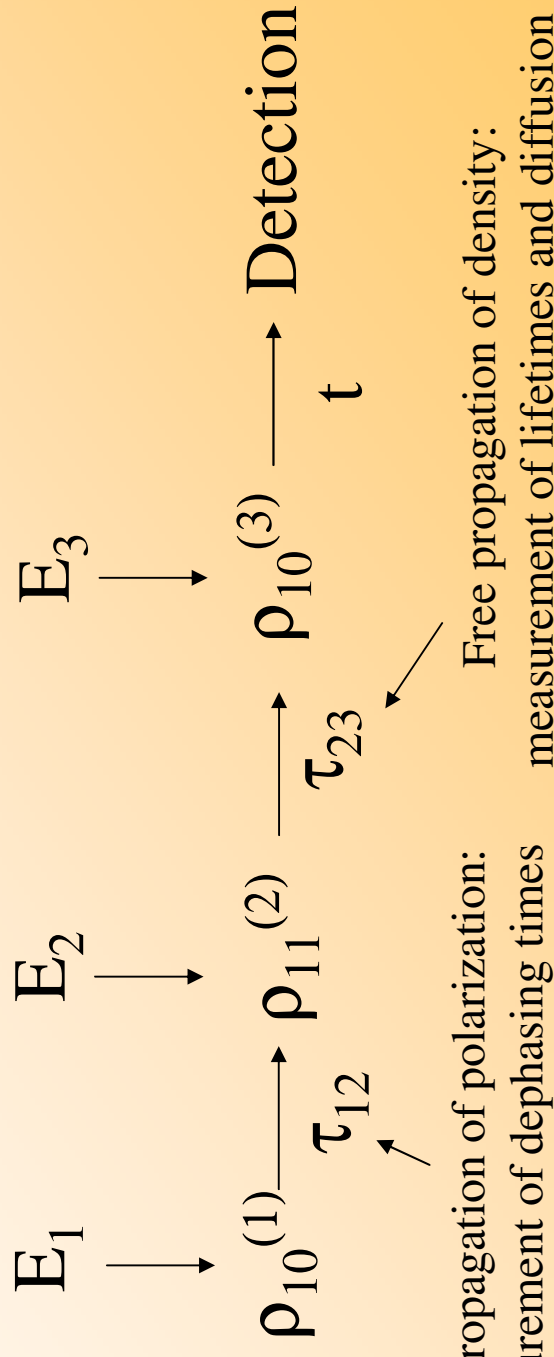
# Four-wave mixing non-linearity

3 excitation fields  $E_i$ , 1 emitted field by third-order polarization  $P^{(3)}$

$$\vec{P}^{(3)} \propto \chi^{(3)} \vec{E}_3 \vec{E}_2 \vec{E}_1^*$$

Using excitation pulses (transient FWM) the time-ordering of the excitation fields is defined

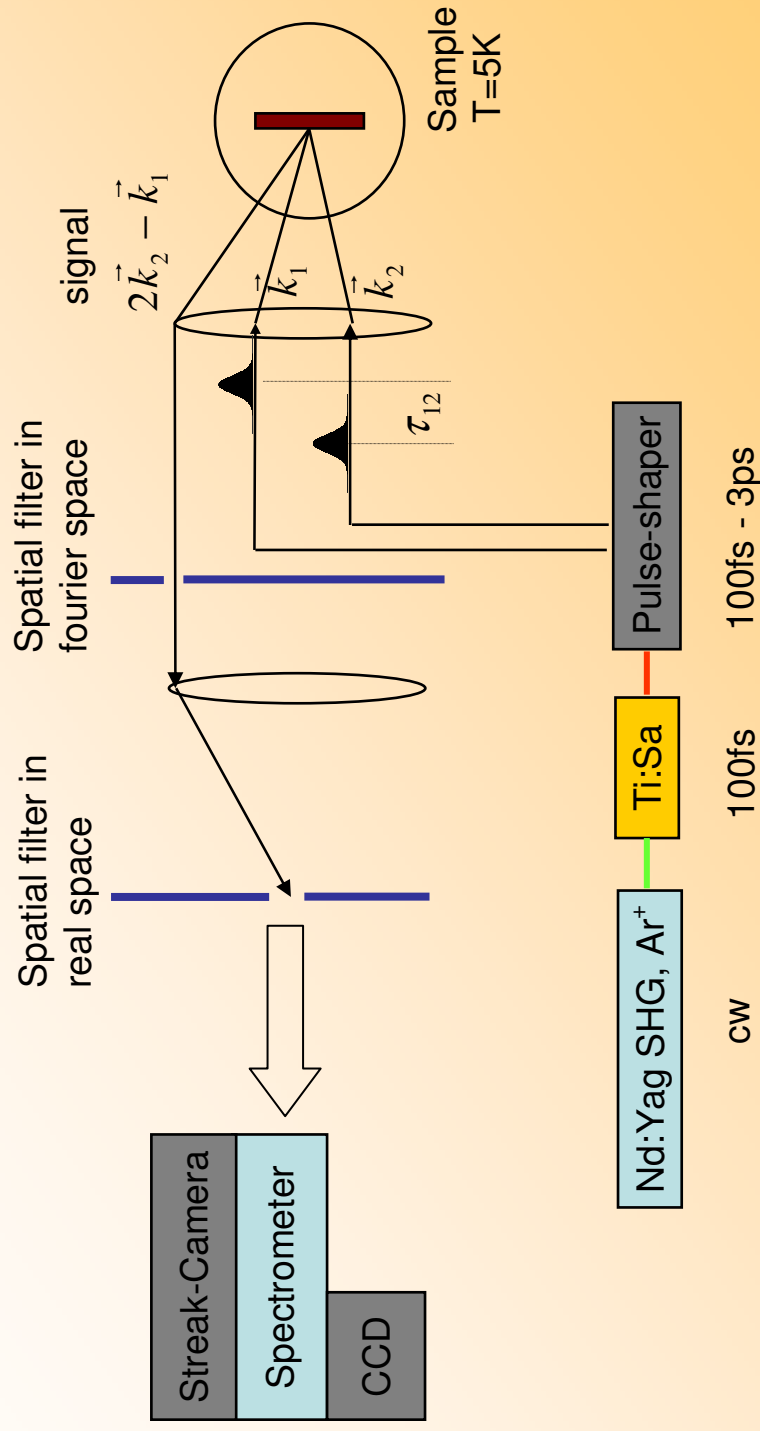
**Simplest case: 2 level system, 2x2 density matrix  $\rho_{ij}$**





# Directional selection setup: reflection geometry

For SQWs: thickness much less than wavelength, only in-plane wavevector conserved:  
Reflection geometry possible (good for absorptive substrates, like GaAs for GaAs QWs)



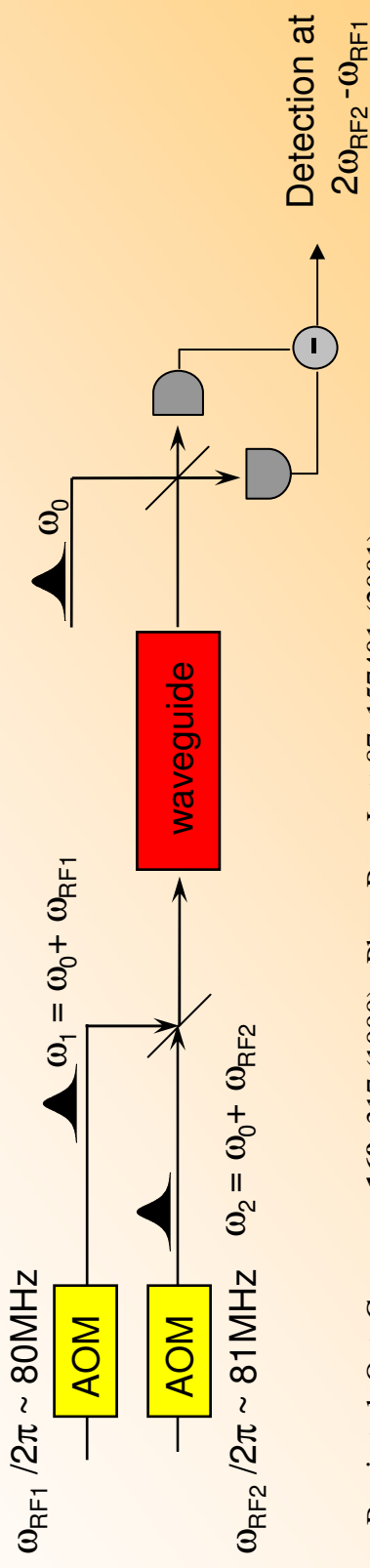
# Detection of FWM

$$\vec{P}^{(3)} \propto \tilde{\chi}^{(3)} \vec{E}_2 \vec{E}_1 \vec{E}_1^*$$

$$\vec{E} \propto \exp(i\omega t - i\vec{k}\vec{r})$$

**Frequency selection:**  $\vec{P}^{(3)} \propto \exp(i(2\omega_2 - \omega_1)t)$

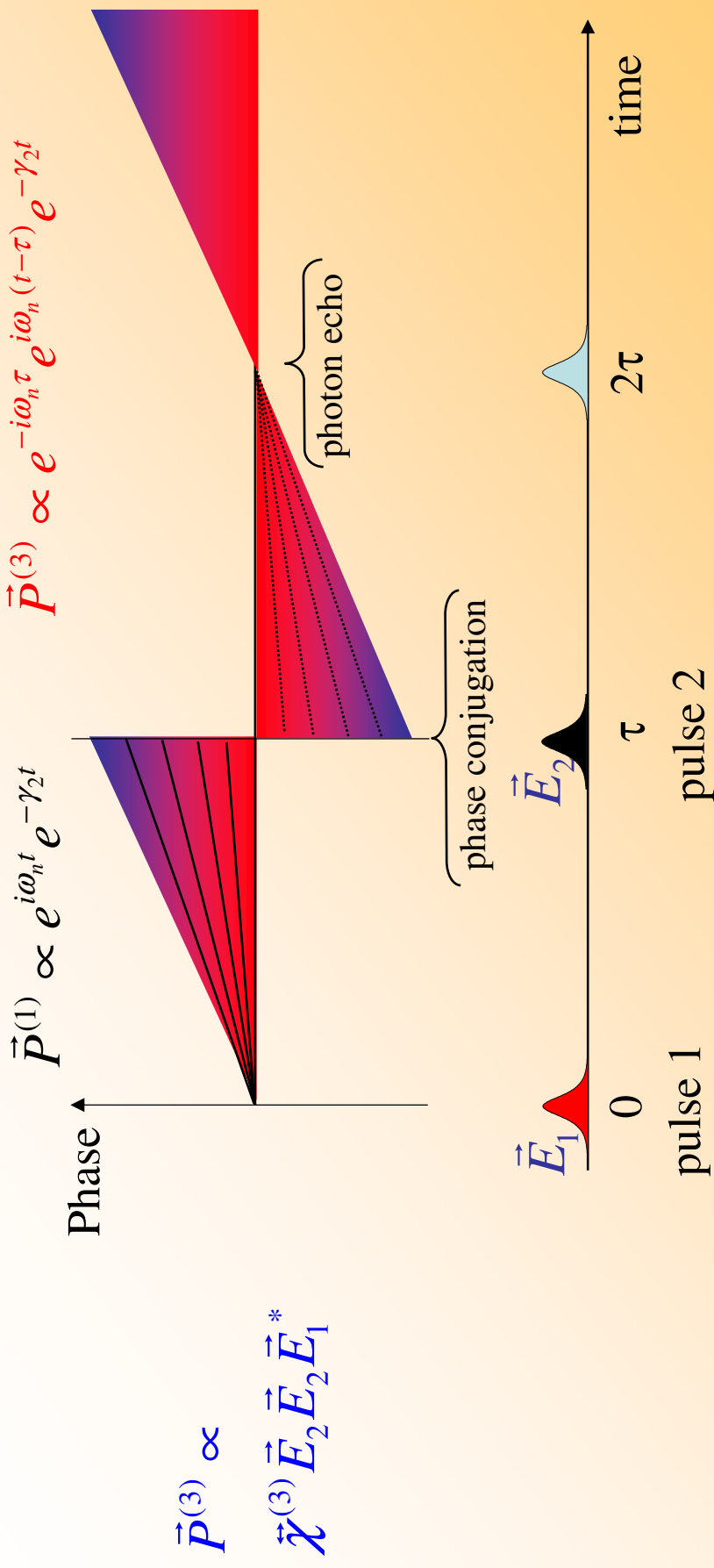
K. Hall *et al.*, Optics Lett. **17**, 874 (1992); A. Mecozzi *et al.* Optics Lett. **21**, 1017 (1996)



Borri *et al.* Opt. Commun. **169**, 317 (1999), Phys. Rev. Lett. **87**, 157401 (2001)

- ☺ Uses time-invariance, which applies also to disordered samples
- ☺ Can be used in colinear geometry with perfect phase matching (waveguides)
- ☹ Colinear geometry: excitation pulses create classical noise & detector saturation
- ☺ Balanced detection reduces classical noise due non-FWM intensities
- ☺ Coherent (heterodyne) detection rejects incoherent background (luminescence)
- ☹ Single channel detection inefficient for complex FWM signals needing spectral or time-resolution

# Photon echo by phase conjugation

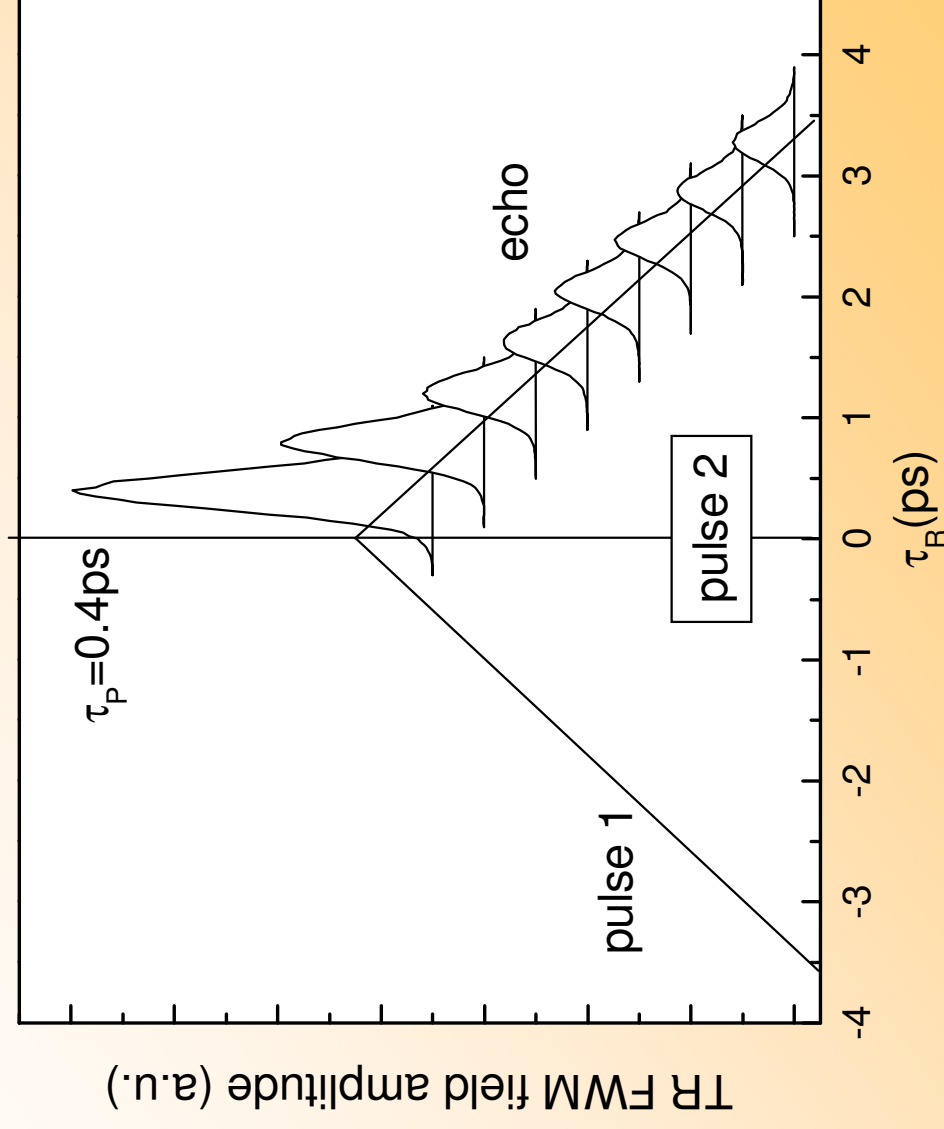


Rephasing of all polarizations at  $t=2\tau$  so that the macroscopic polarization is

- independent of inhomogeneous broadening
- **only sensitive to microscopic dephasing**
- **insensitive to slow spectral diffusion**



# Measured FWM signal from an ensemble of InGaAs quantum dots in a waveguide

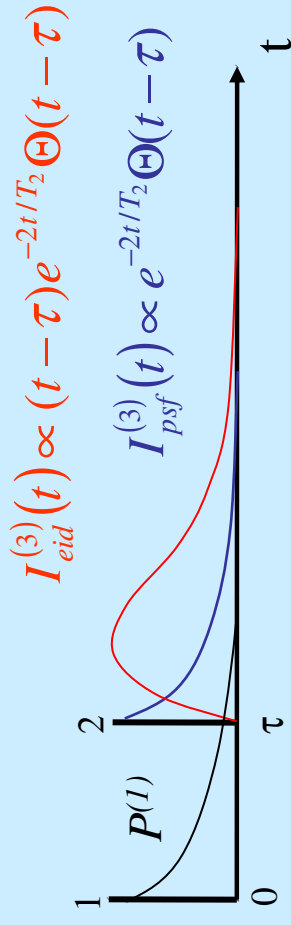


# Free polarization decay and photon echo

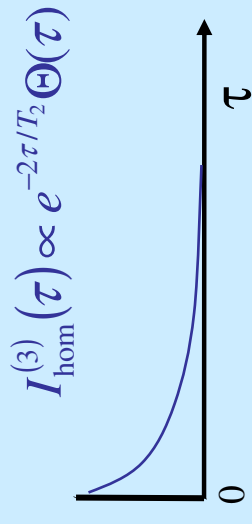
Ensemble of two-level systems with variance  $\sigma$  of inhomogeneous broadening, polarization decay time  $T_2$ , homogeneous broadening  $\gamma = \frac{2\hbar}{T_2}$

Time-resolved

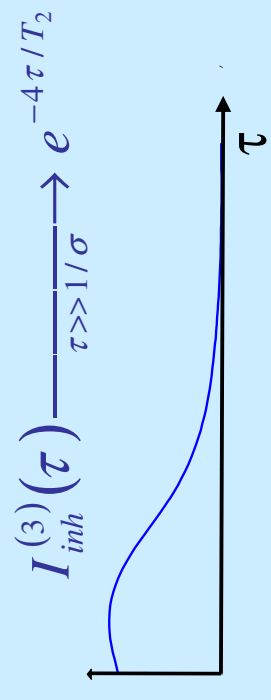
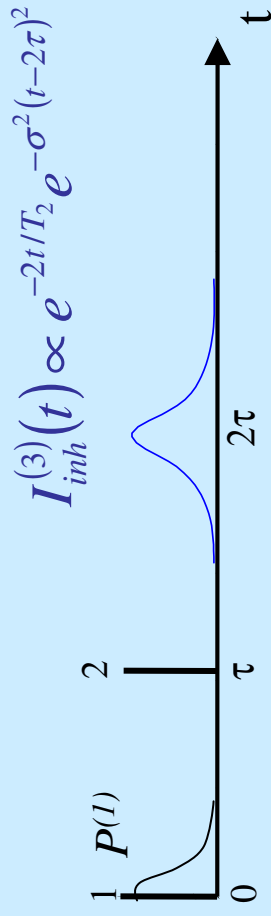
Homogeneous case:  $\sigma=0$ : Free Polarization Decay



Time-integrated



Inhomogeneous case:  $\sigma \gg \gamma$ : Photon Echo



# FWM versus inhomogeneous broadening

Borri et al., Phys. Rev. B **63**, 035307 (2001)

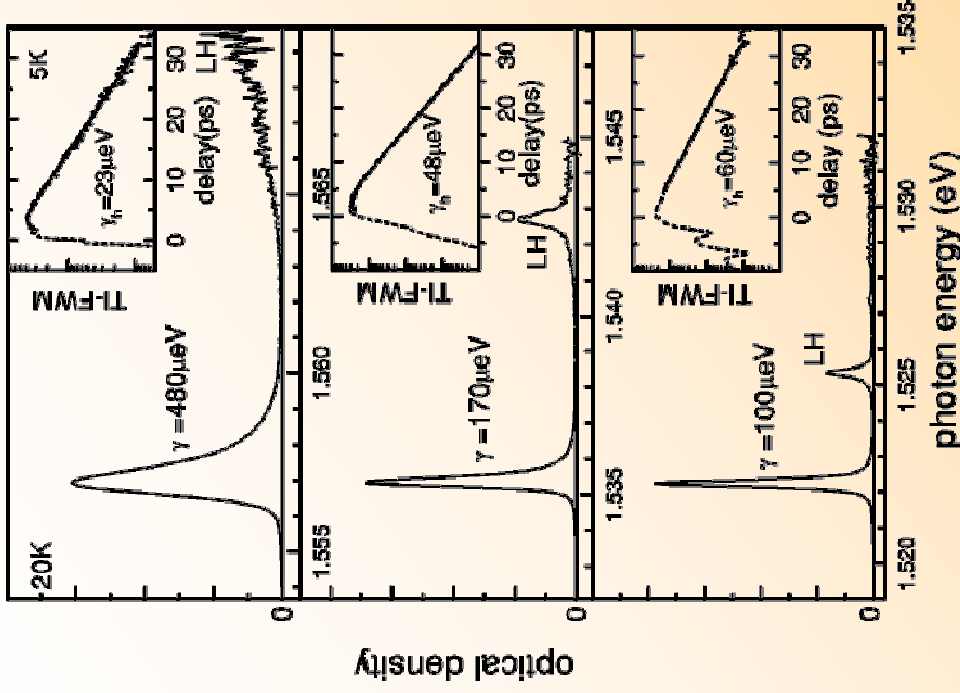


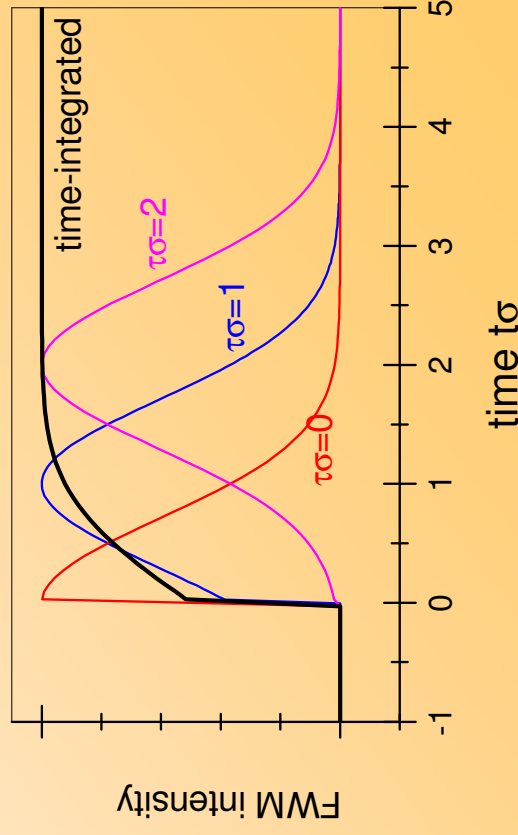
FIG. 1. Absorption profile, as deduced from the photoluminescence spectrum at 20 K corrected by a Boltzmann population factor, for a 25 nm (bottom), a 15 nm (middle), and a 10 nm (top) GaAs/Al<sub>0.3</sub>Ga<sub>0.7</sub>As single quantum well. In the insets, four-wave mixing (FWM) traces at 5 K are given with the corresponding fits of the homogeneous broadening.

For a Gaussian broadening of FWHM  $\Gamma = \sigma\sqrt{8\ln(2)}$

$$I_{\text{TI-FWM}}(\omega, \tau_{12}) \propto \left| \exp\left(-\gamma\tau_{12} - 4\ln 2 \frac{(\omega_x - \omega)^2 - (\gamma/2)^2}{\Gamma^2}\right) \times \text{erfc}\left[2\sqrt{\ln 2}\left(\frac{\gamma}{2\Gamma} - \frac{\Gamma\tau_{12} + i\frac{\omega_x - \omega}{\Gamma}}{8\ln 2}\right)\right] \right|^2,$$

Erland et al., Phys. Rev. B **50**, 15047 (1994), Borri et al., Phys. Rev. B **59**, 2215 (1999)

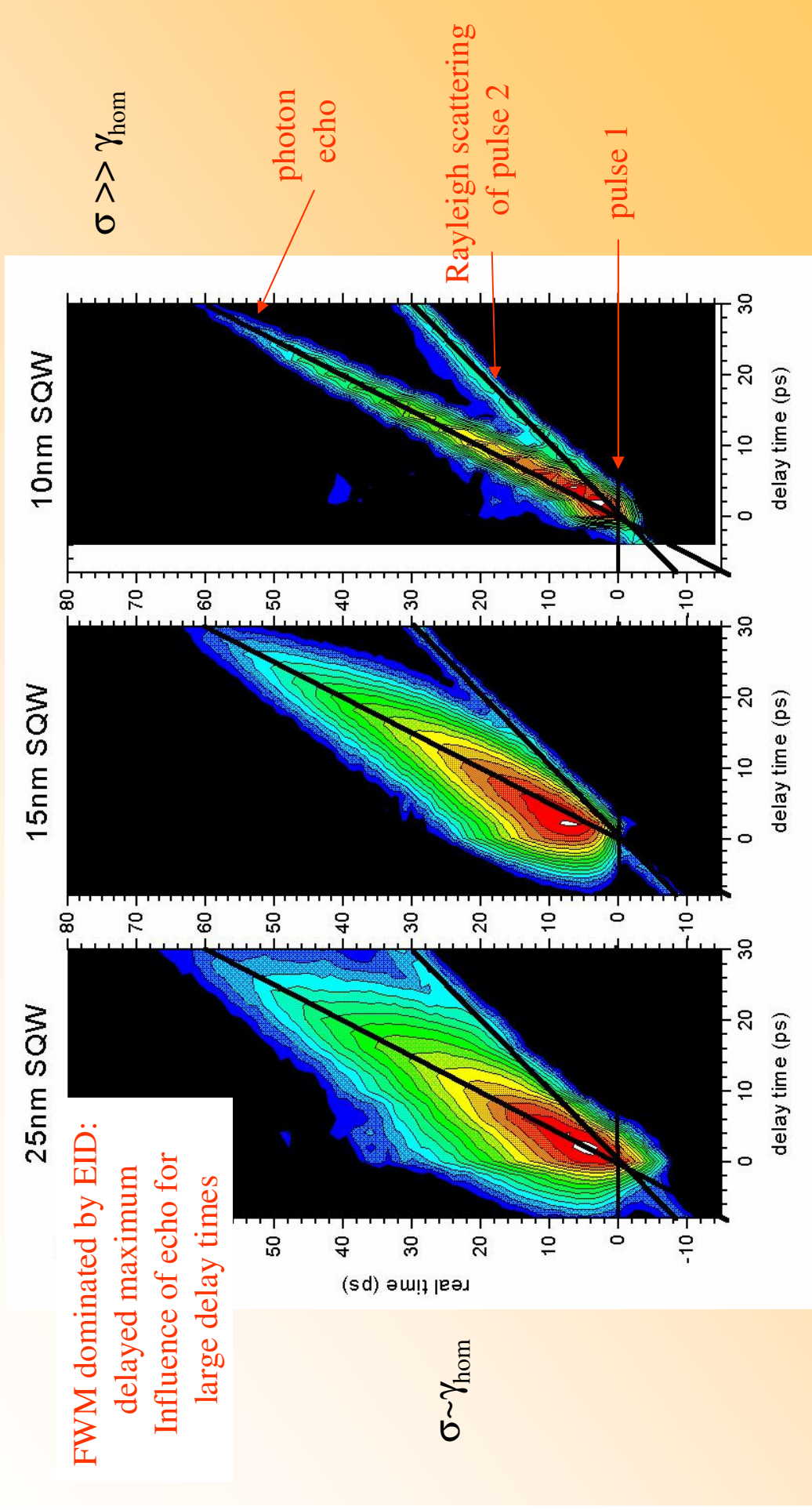
- Spectrally resolved FWM has a width of  $\Gamma/\sqrt{2}$
- Initial intensity rise with delay time until echo is fully developed



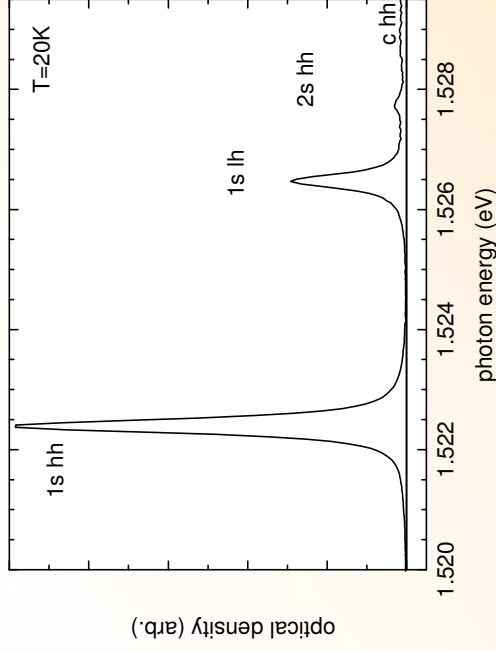
W. Langbein, Ultrafast nonlinear spectroscopy and control of semiconductor nanostructures

# Time-resolved FWM intensity versus inhomogeneous broadening

Circularly polarized excitation & detection polarization to avoid signal from bound biexcitons  
 Streak camera detection, logarithmic color scale over 2 orders of magnitude



# FWM on the exciton-biexciton system



Sample:  
25nm GaAs SQW

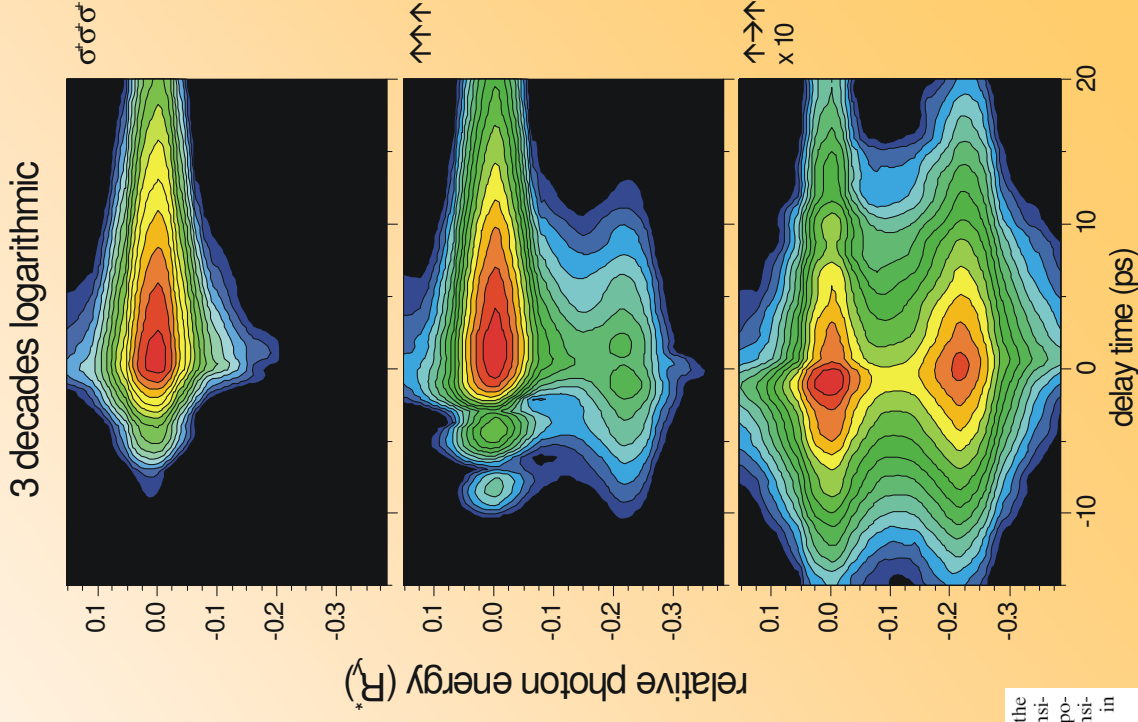
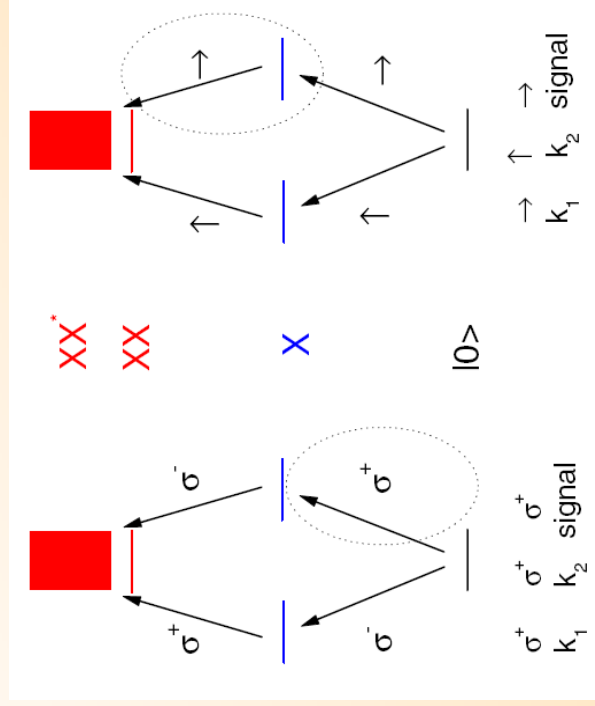
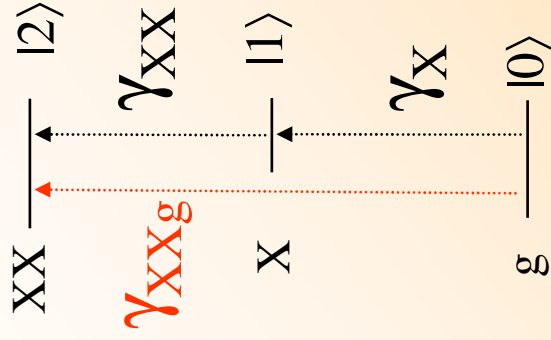


Fig. 1. Schematic representation of the five-level system and the optical transitions for a circular (left) and linear polarized (right) exciton basis. The transitions emitting the FWM signal  $p^{(3)}$  in  $2k_2 - k_1$  direction are encircled

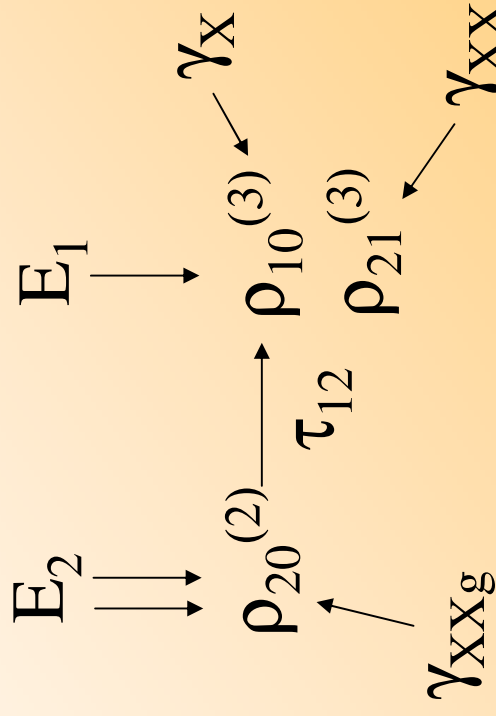


## 5-level model

# Polarization decay in the exciton-biexciton system



negative delay time,  $E_2$  before  $E_1$



- $\gamma_{XXg}$ : measured in decay of FWM intensity for negative delay times
- $\gamma_X$ : measured in decay of FWM intensity for positive delay times
- $\gamma_{XX}$ : measured in FWM spectrum



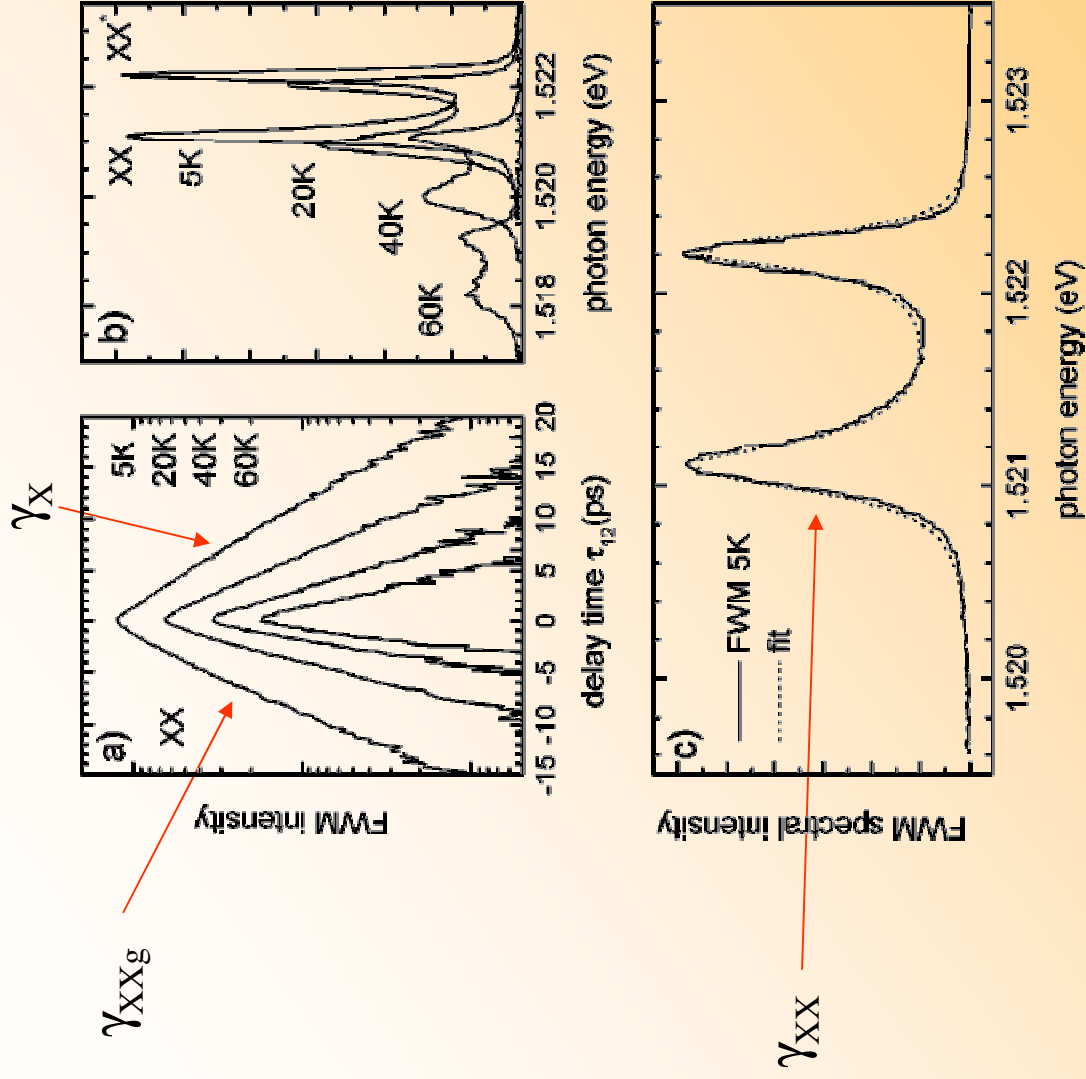
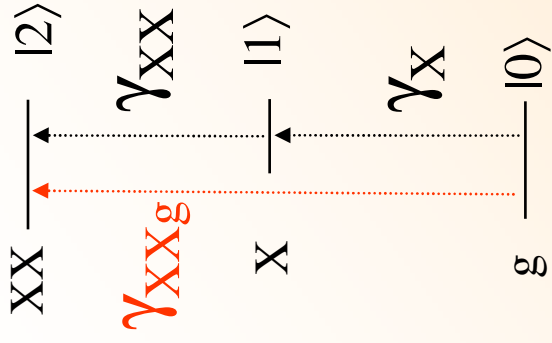


FIG. 2. (a),(b) Temperature dependency of the four-wave mixing signal for ( $\rightarrow$ ) configuration. (a) Delay-time traces detected at the exciton-biexciton transition. (b) Spectra for  $\tau_{12}=1$  ps. (c) Four-wave mixing spectrum at  $\tau_{12}=1$  ps, and 5 K lattice temperature, together with a fit using Eq. (1).

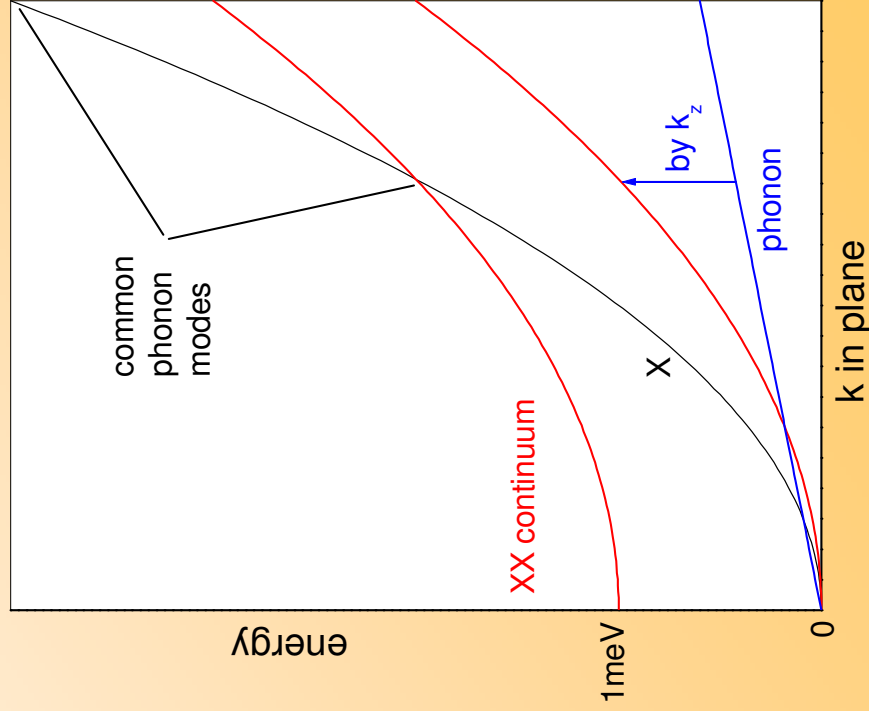
Relation between decay rates gives information about the correlation  $R$  between the scattering processes of excitons and biexcitons



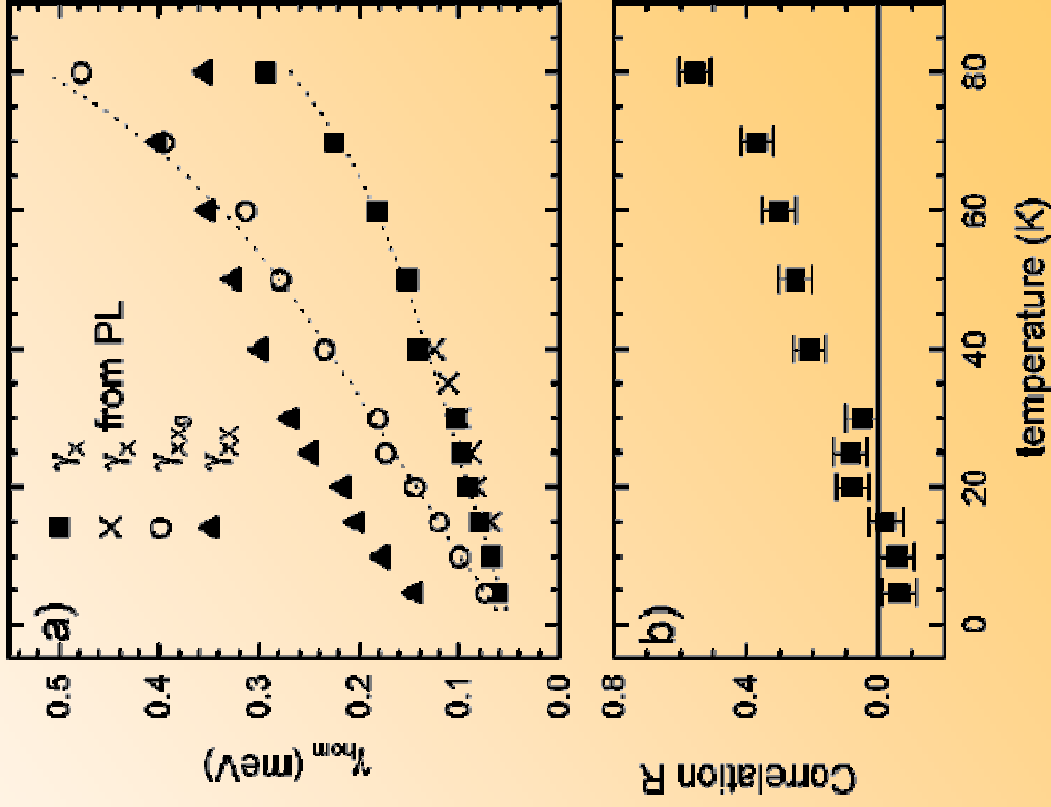
$$\gamma_{XX} = \gamma_X + \gamma_{XXg} - 2R\sqrt{\gamma_X\gamma_{XXg}}, \quad |R| \leq 1.$$

Decay rates are expected to be due to

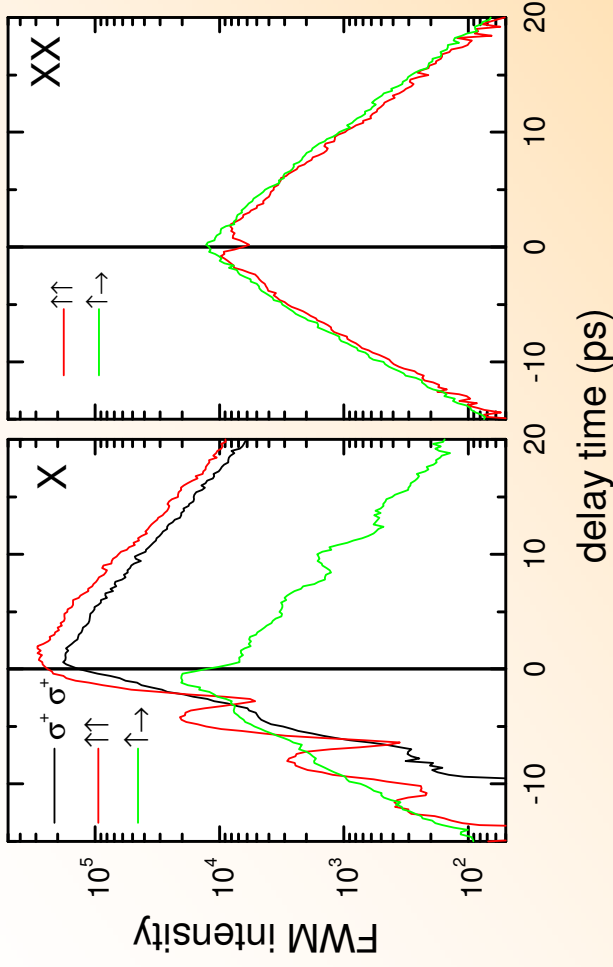
- **radiative decay:** should be uncorrelated since emitted photons have different energies
- **phonon absorption:** uncorrelated since exciton and biexciton dispersions are different, but scattering into the biexciton continuum allows equal modes to couple



- At low temperature scattering is uncorrelated: expected since dominated by radiative decay
- with increasing temperature scattering gets partially correlated ( $kT \gg$  biexciton binding), as expected from the scattering into the XX continuum



# Exciton-exciton interaction effects in FWM



## Excitation-induced dephasing (EID):

$(\uparrow\uparrow), (\sigma^+\sigma^+)$ : X signal-enhancement

$(\uparrow\uparrow), t < 0$ : beats with BIF at X

## Biexciton-formation (BIF):

$(\uparrow\uparrow), (\uparrow\rightarrow)$ : signal at XX

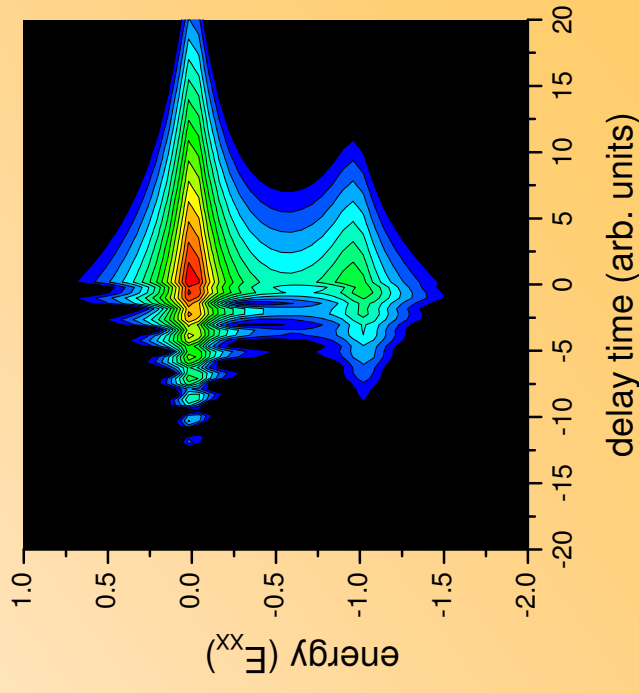
$(\uparrow\uparrow), t < 0$ : beats with EID and LFC at X

$(\uparrow\rightarrow), t < 0$ : beats with LFC at X

## Local Field Correction (LFC):

X signal-enhancement

$(\uparrow\uparrow), (\uparrow\rightarrow), t < 0$ : beats with BIF at X

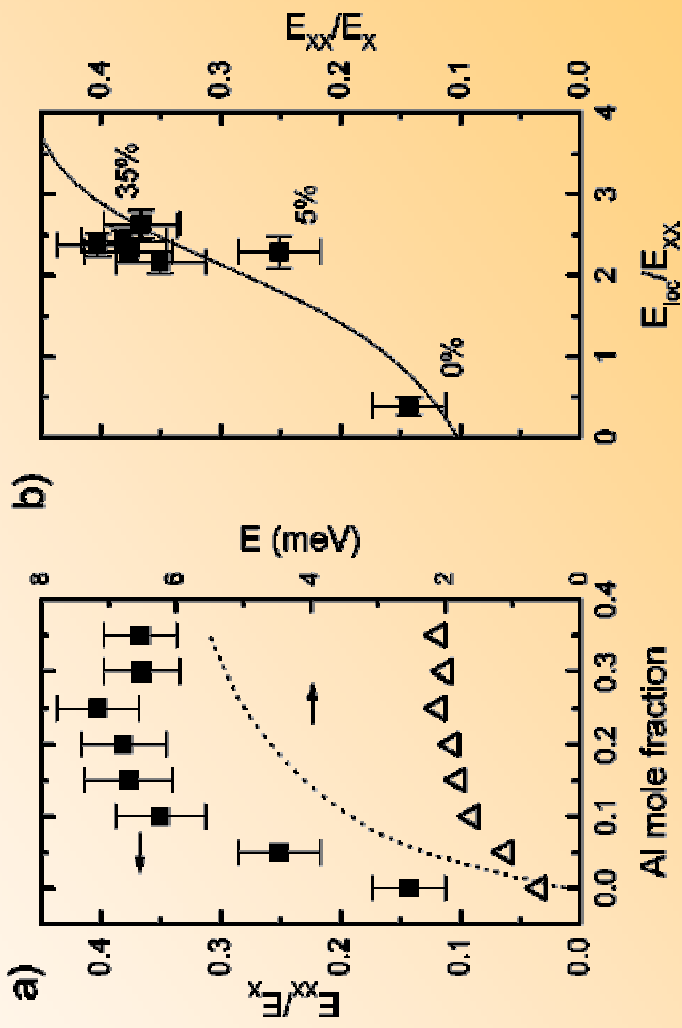
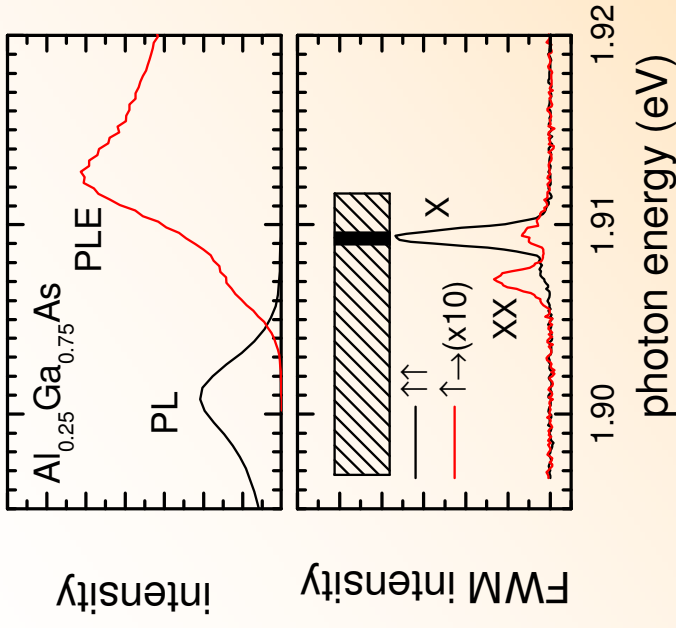


Theoretical Modeling:

5-level system + EID + LFC

# Localization enhanced biexciton binding in $\text{Al}_x\text{Ga}_{1-x}\text{As}$

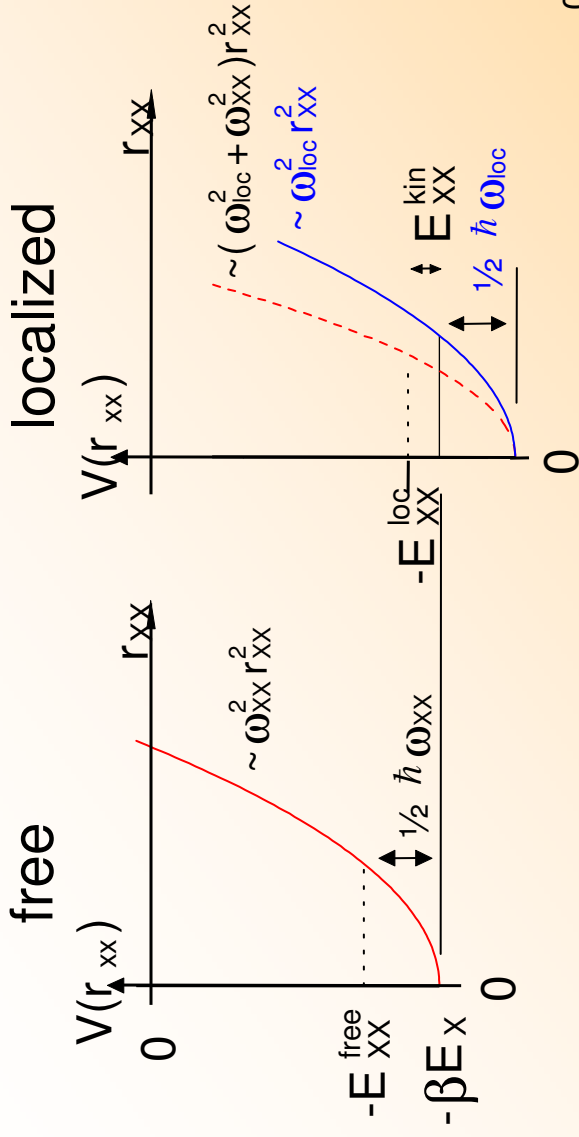
Langbein & Hvam, Phys. Rev. B **59**, 15405 (1999)



**Biexciton binding energy  $E_{XX}$  increases from 0.5 meV to 2 meV with increasing disorder.** Binding energy ratio between biexciton and exciton  $E_{XX}/E_X$  increases from 0.13 to 0.37 when the inhomogeneous broadening  $E_{1oc}$  gets larger than  $E_{XX}$

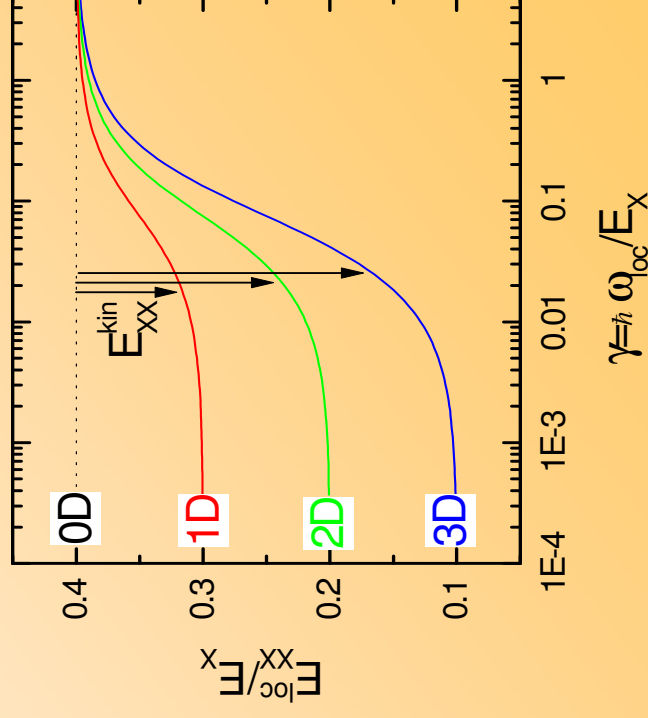
FIG. 1. (a) Biexciton binding energy  $E_{XX}$  (triangles), photoluminescence linewidth  $E_{1oc}$  (dotted line), and ratio  $E_{XX}/E_X$  (squares) for the  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  mixed crystal samples as a function of the Al mole fraction  $x$ . (b)  $E_{XX}/E_X$  as a function of the localization ratio  $E_{1oc}/E_{XX}$ . Line: calculated dependence [Eq. (5)] for  $n=3$ ,  $E_{1oc} = 4\hbar\omega_{1oc}$ ,  $\alpha=0.312$ ,  $\beta=2.4$ , and  $\sigma=0.2$ .

# Binding energy of localized biexcitons



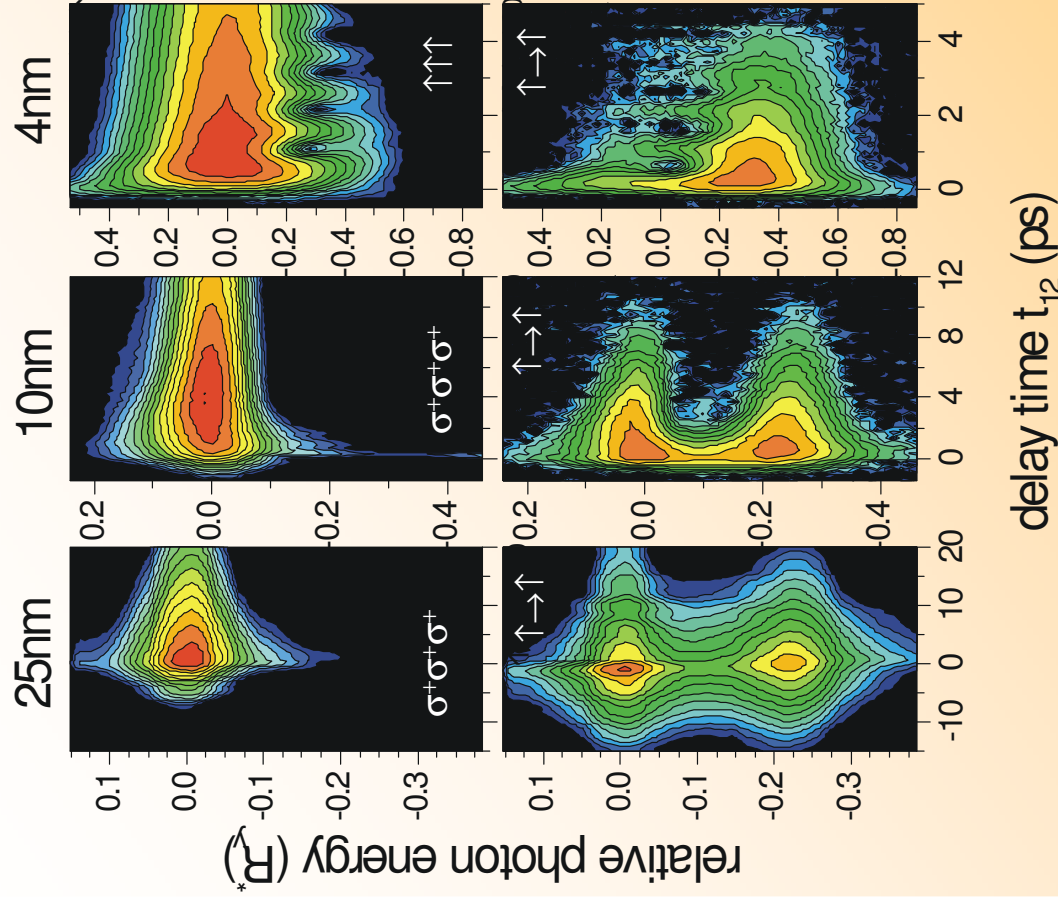
Enhancement of the binding energy  
by the quenching  
of the binding-induced kinetic energy

$$E_{xx}^{kin}$$



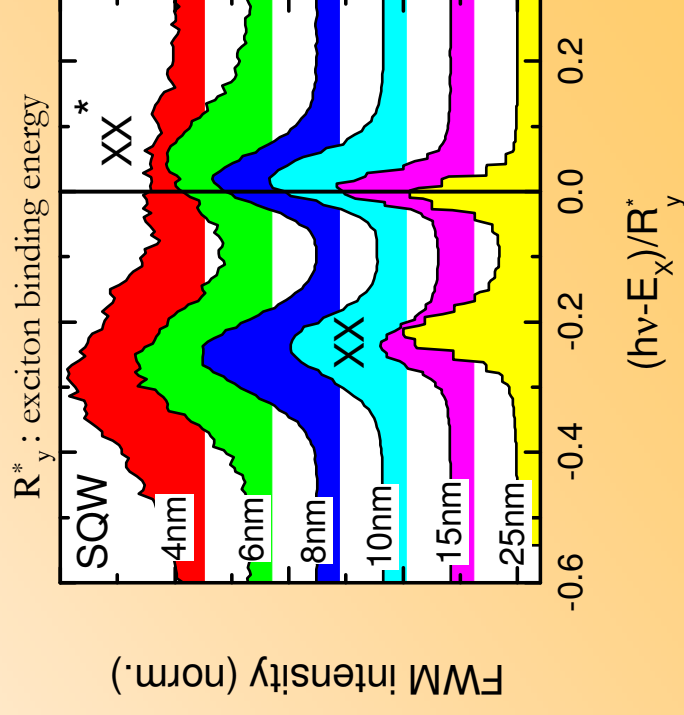


# Localization enhanced biexciton binding in GaAs QW



The **biexciton binding energy  $-E_{XX}$**  increases with increasing localization due to a quenching of the binding-induced kinetic energy.

The **biexcitonic continuum edge  $(XX^*)$**  is quantized to higher energies and **reduced in oscillator strength** by the localization.



# Localization enhanced biexciton binding in GaAs QWs

Binding energy ratio between biexciton and exciton  $E_{XX}/E_X$  increases from 0.2 to 0.32 when the inhomogeneous broadening  $E_{loc}$  gets larger than the biexciton binding  $E_{XX}$

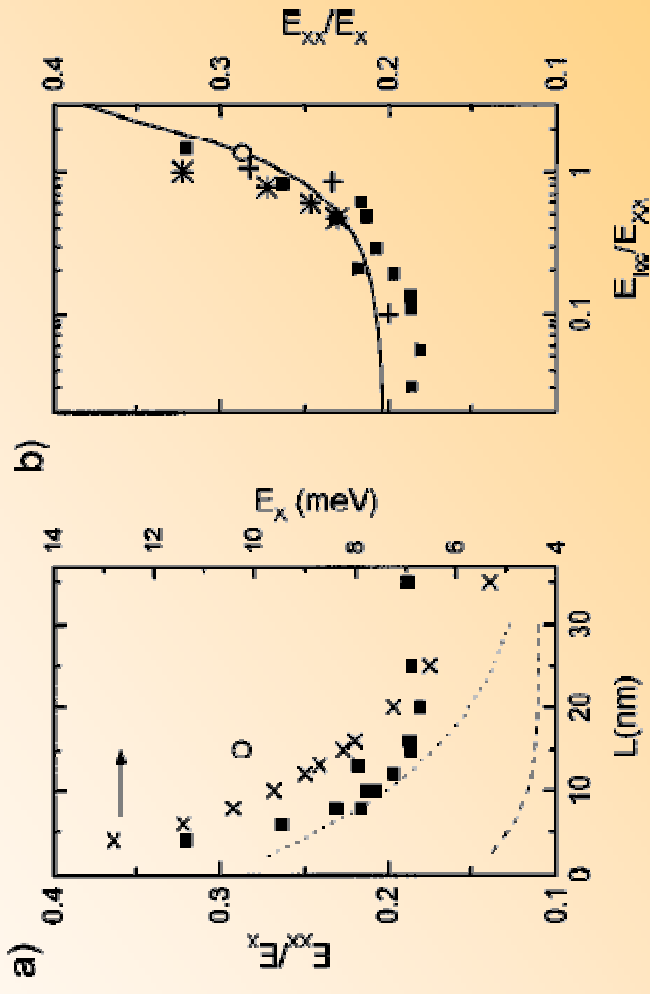
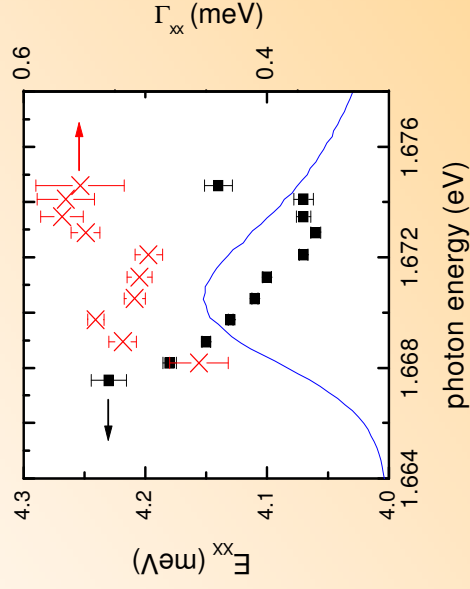
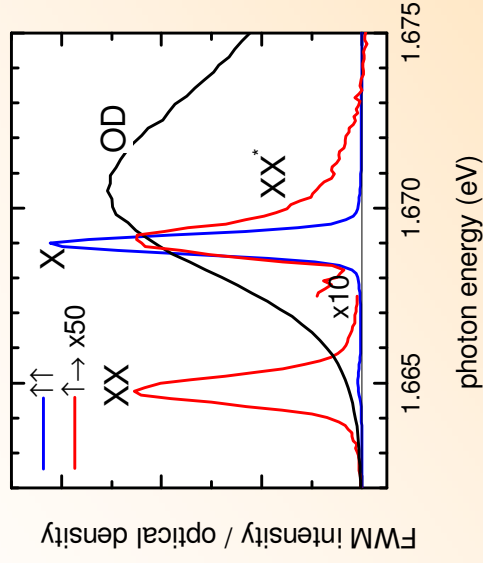


FIG. 3. Experimental and calculated data for quasi-2D samples. (a) Symbols: experimental data for GaAs/Al<sub>0.3</sub>Ga<sub>0.7</sub>As QWs as a function of the well thickness  $L$ . Crosses: Exciton binding energy; squares: experimental ratio  $E_{XX}/E_X$ . Circle:  $E_{XX}/E_X$  for an Al<sub>0.1</sub>Ga<sub>0.9</sub>As/Al<sub>0.3</sub>Ga<sub>0.7</sub>As QW. Lines are calculations (Refs. 30, 35 and 36). (b)  $E_{XX}/E_X$  versus localization strength  $E_{loc}/E_{XX}$  for different quasi-2D material systems: Open diamonds: this work; filled squares: GaAs/Al <sub>$x$</sub> Ga <sub>$1-x$</sub> As QWs; crosses: ZnSe/Zn <sub>$x$</sub> Mg <sub>$1-x$</sub> Se <sub>$y$</sub> Se <sub>$1-y$</sub>  (Refs. 25, 37, and 38); stars: In <sub>$1-x$</sub> Ga <sub>$x$</sub> As/GaAs (Ref. 39). Line: calculated with Eq. (5) for  $n=2$ , other parameters as in Fig. 1.

# Inhomogeneous broadening of biexciton binding

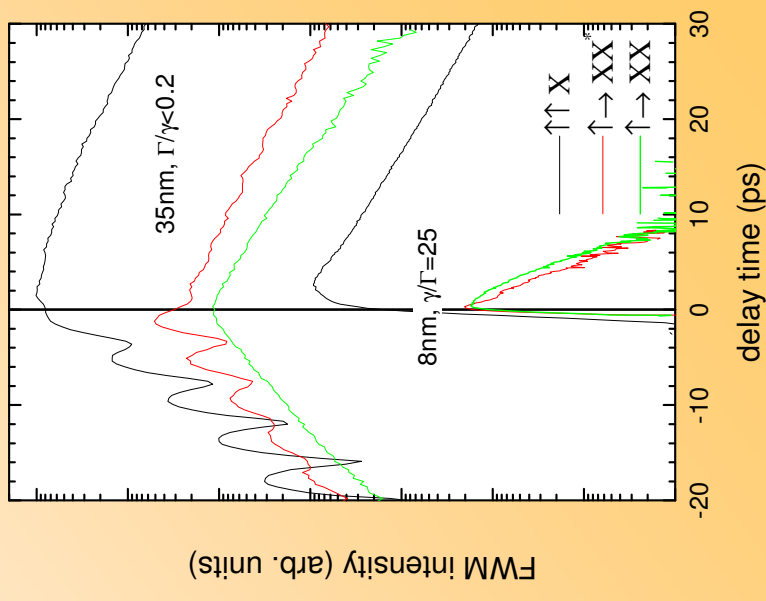
Testing the **biexciton binding energy distribution** within the exciton ensemble using a **spectrally selective excitation of the first-order polarization ( $k_1$ )**



Biexciton binding energy is

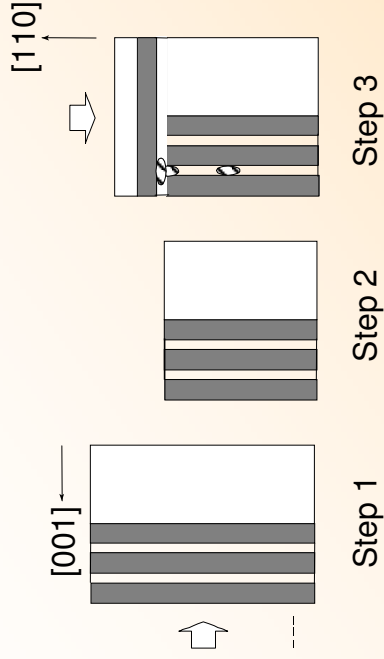
- ⇒ **inhomogeneously broadened** for each exciton energy
- ⇒ **increasing with decreasing exciton energy** (stronger localization)

The **inhomogeneous broadening** of the **biexcitonic binding energies** lead to a **fast decay** of the **X-XX** and **X-XX\*** **photon-echo** signal (rephasing is not complete)



# T-shaped quantum wires

## Growth Technique : Cleaved Edge Overgrowth

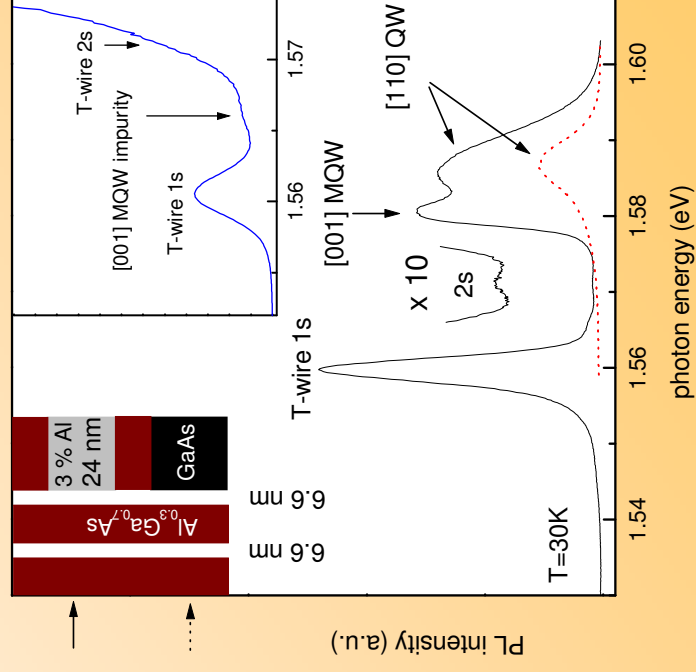


## Investigated 6.6 x 24 nm T-wire:

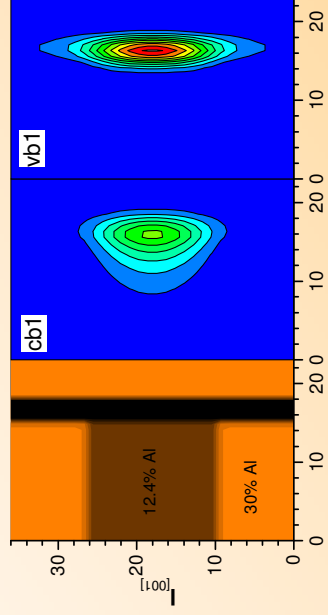
50 period [001] MQW, thickness 2.5mm  
**24nm  $\text{Al}_{0.03}\text{Ga}_{0.07}\text{As}$  well, 26nm  $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$  barrier**  
 Overgrown by **6.6nm GaAs well**

20 meV confinement

3.4 meV inhomogeneous broadening



## Confined states of an optimized structure with a 3nm overgrown GaAs QW



Langbein et al., Phys. Rev. B **60**, 16667 (1999)

## Exciton localization:

**Stokes-shift** between absorption  
and PL at low temperatures.

**Strongly varying homogeneous linewidth**  
within the inhomogeneous broadening:  
**phonon-assisted relaxation**

## Exciton binding energy:

Excitation of exciton series up to the continuum edge:

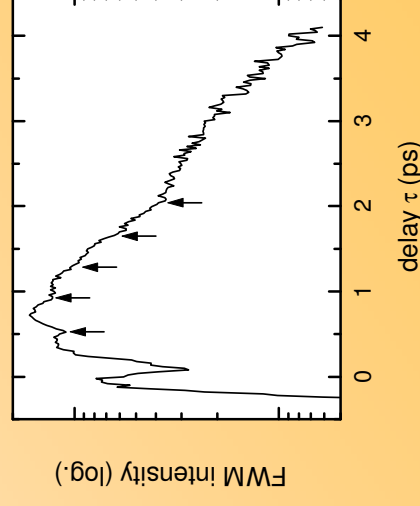
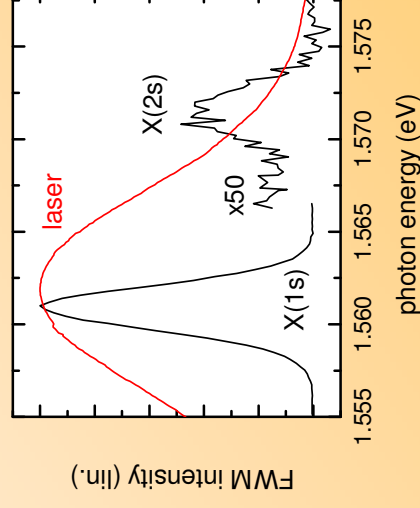
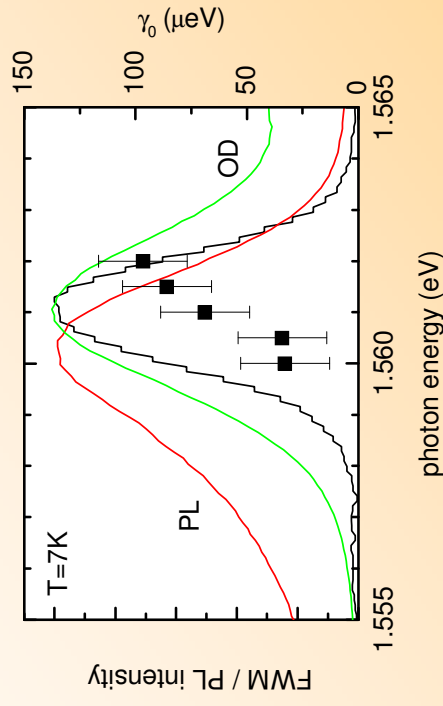
high-energy **side-line**, quantum beats

⇒ Identification of the 1s-2s beating with

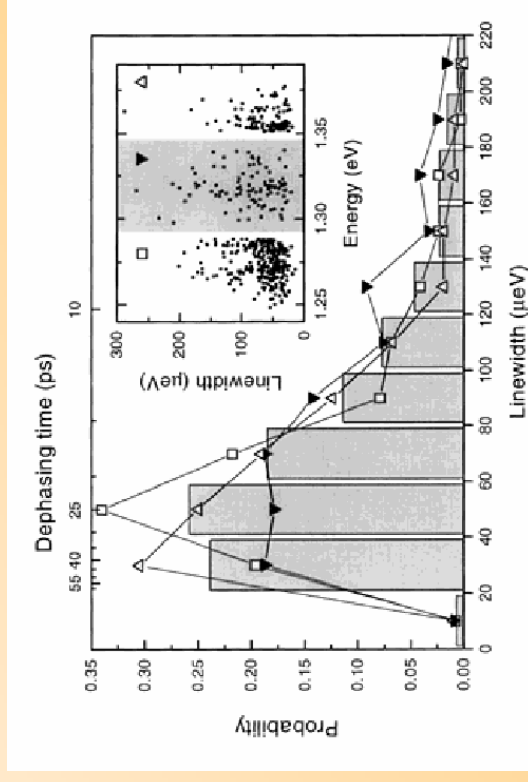
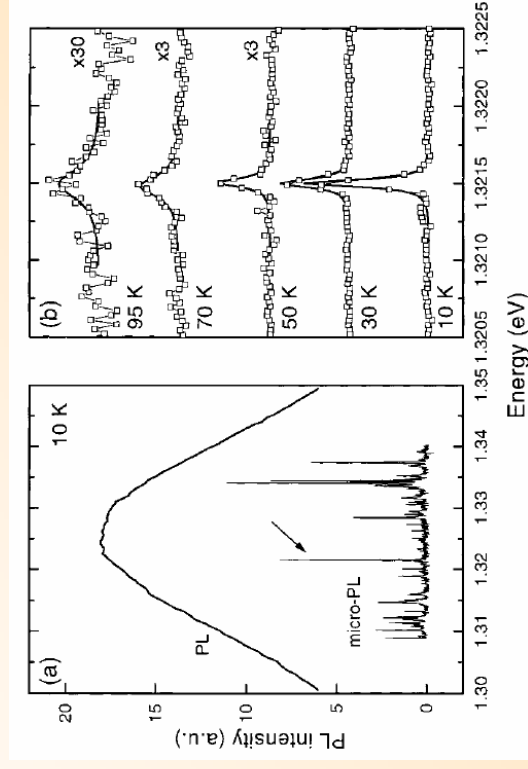
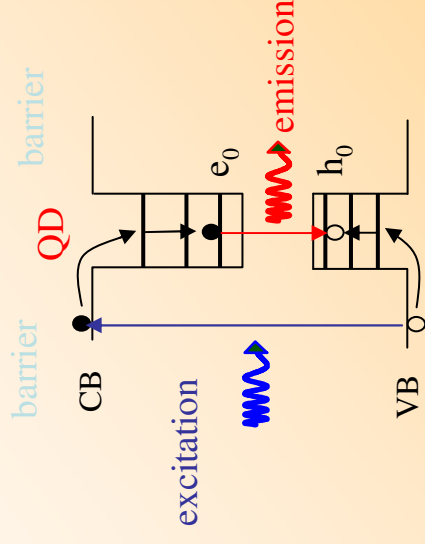
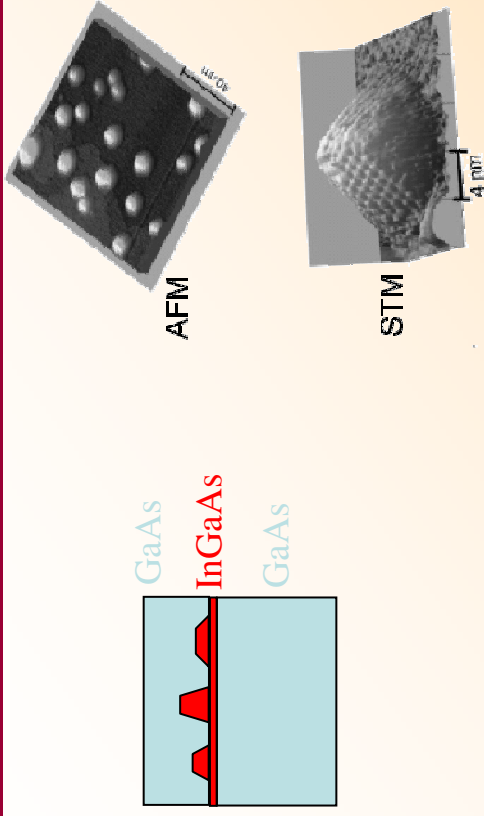
$$E_{1s-2s} = 10.5 \pm 0.4 \text{ meV}$$

From fractional-dimensional model:

$$E_X = 11.9 \pm 0.4 \text{ meV}$$



# In<sub>x</sub>Ga<sub>1-x</sub>As self-organized quantum dots



K. Leosson, J.R.Jensen, J.M.Hvam and W. Langbein, phys. stat. sol. **221**, 49 (2000)

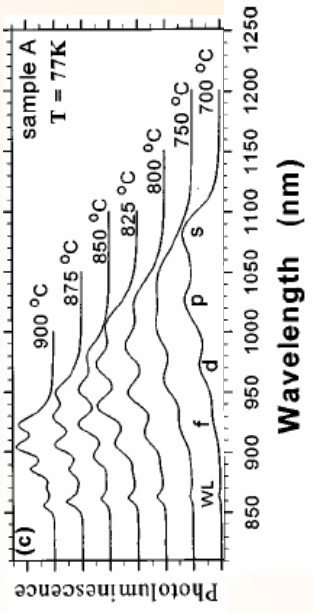
**Linewidth of  $\sim 50\mu\text{eV}$  (25ps dephasing time) but  $\gamma_{\text{rad}} \sim 0.66\mu\text{eV}$  (1ns lifetime)**



# Annealed InAs QDs

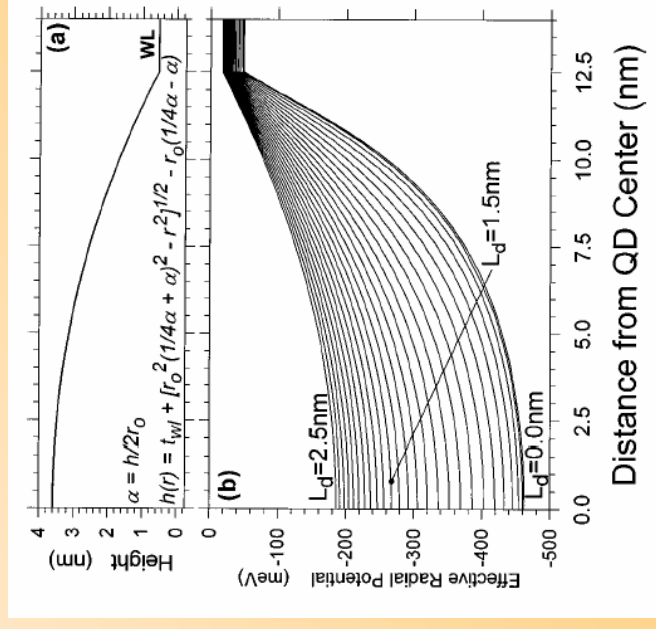
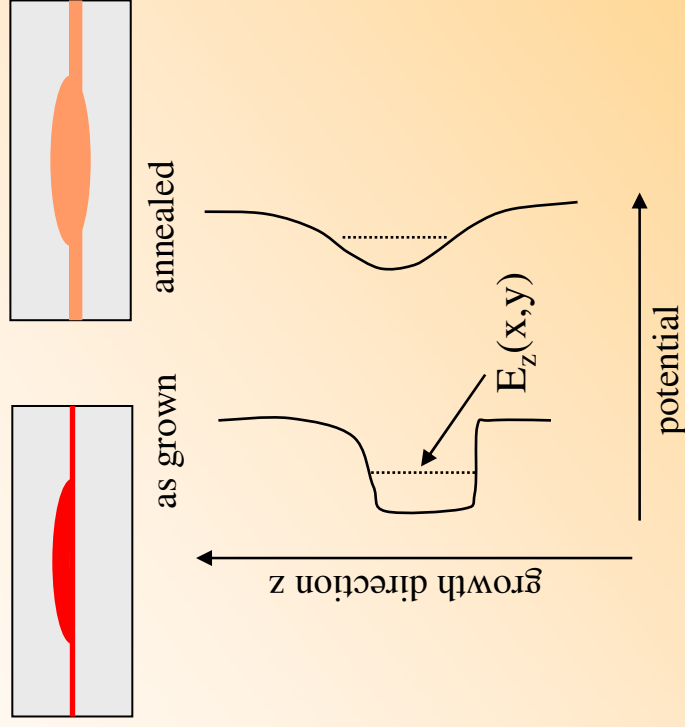
## Systematic study of confinement-dependent effects: Rapid thermal annealing

S. Fafard and C. Ni. Allen, Appl. Phys. Lett. **75**, 2374 (1999)



Thermally activated In-diffusion changes confinement potential

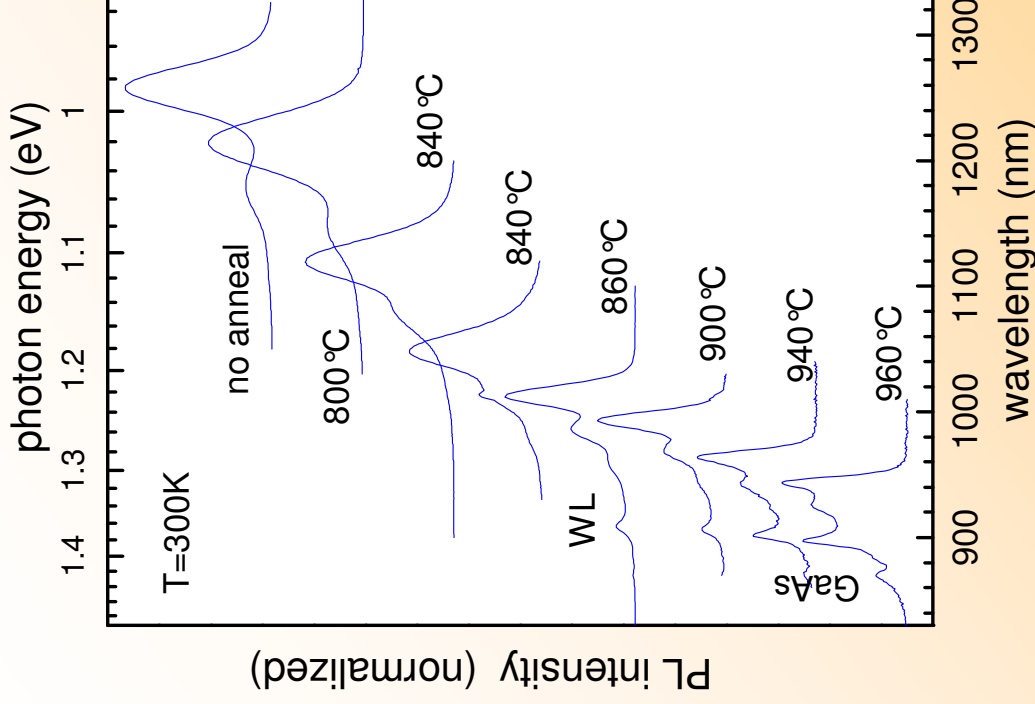
Vertical confinement larger than in-plane confinement  $\Rightarrow$   
In-plane confinement determined by the potential created by the confinement energy in growth direction





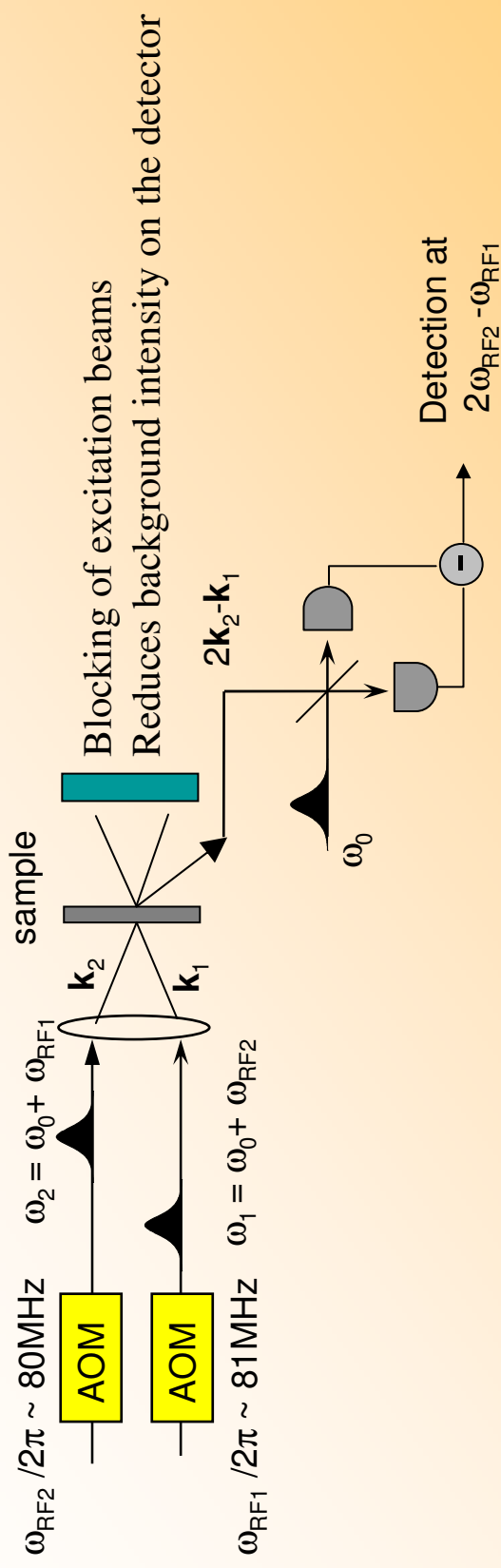
# Annealed InAs QDs

Sample growth & annealing: V. Stavarache, D. Reuter, A. D. Wieck, University of Bochum, Germany



- 10 layers of InAs QDs in GaAs matrix, 100nm spacing between layers (no coupling)
- **Rapid thermal annealing** for 30 seconds
- Tuning of **transition energy** from 0.985 to 1.315 eV
- Tuning of **confinement energy** (QD ground state to wetting layer) from **340meV to 65meV**
- Tuning of **intersublevel spacing** from 70meV to ~30 meV

# Combination of directional and frequency selection



Experiments on single layers of self-organized InGaAs QDs:

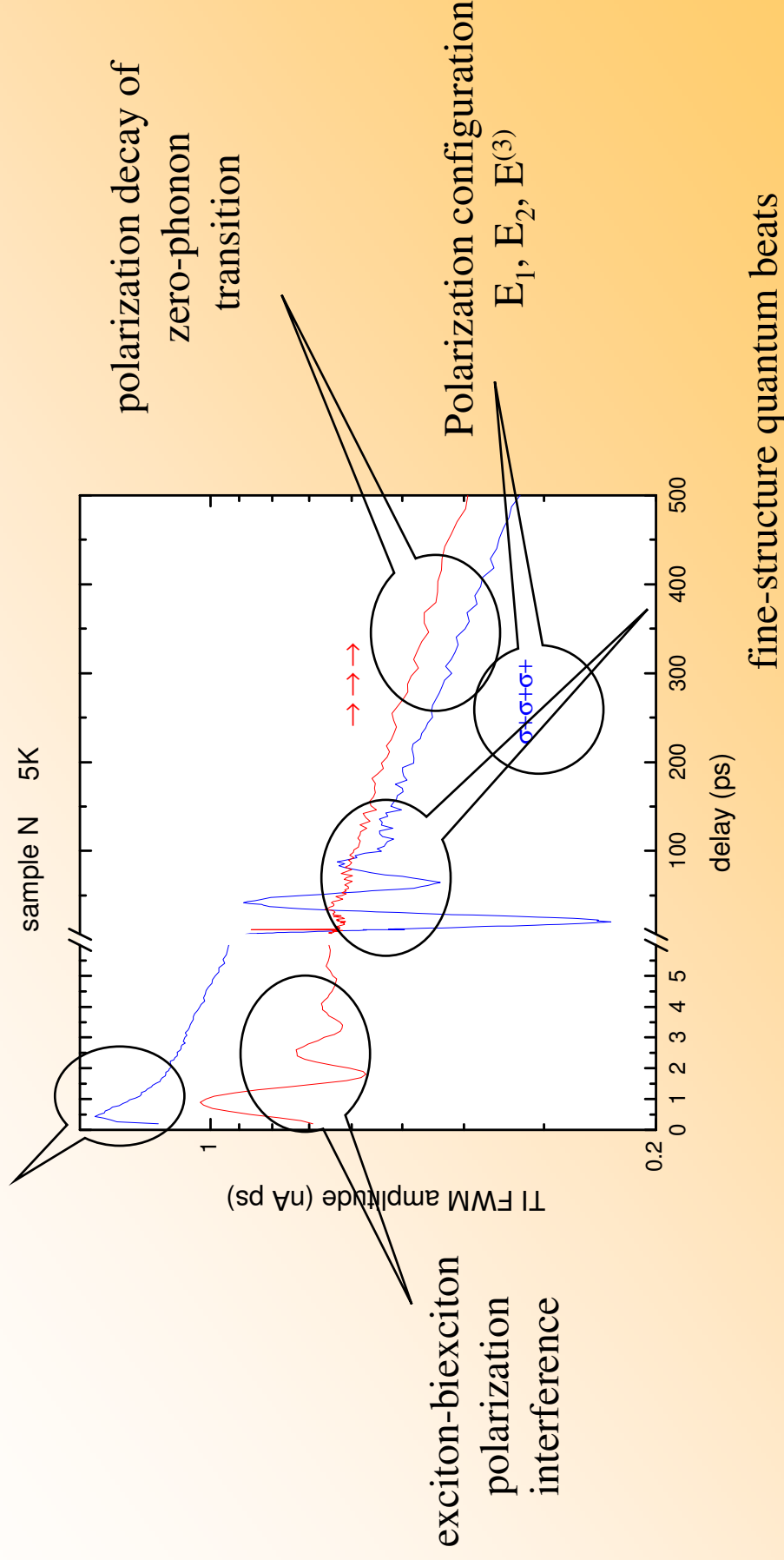
- Input power  $\sim 1\text{mW}$  (third-order regime)
- Excitation spot size  $\sim 30\mu\text{m}$
- Detection of coherent signal **shot-noise limited** ( $10^{-18} \text{ W/Hz}^{0.5}$ )
- typical FWM intensity less than  $10^{-10}$  of incident intensity
- excitation and detection polarizations freely adjustable

Langbein et al. Phys. Rev. B **69**, 161301R, and **70**, 033301 (2004); Borri et al. **71**, 115328 (2005)

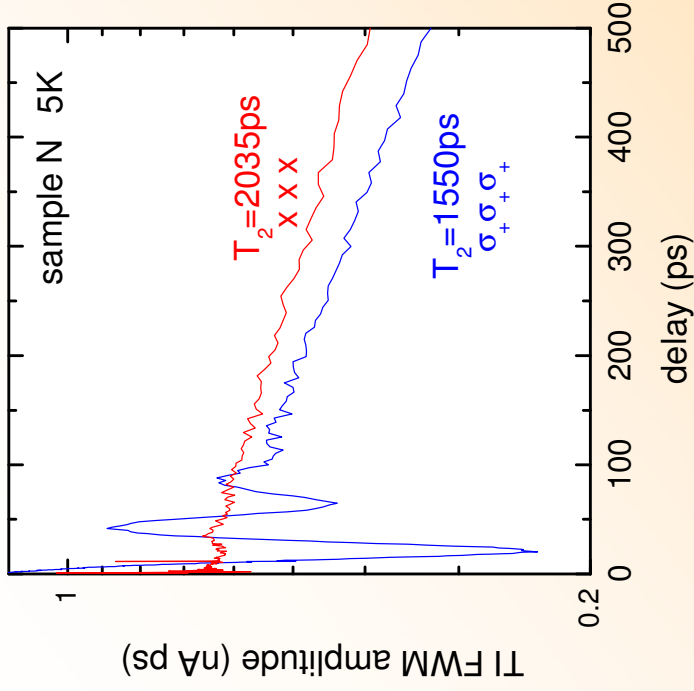
# Features visible in FWM dynamics

## Polarization-dependent FWM:

polarization decay of  
phonon-assisted transitions

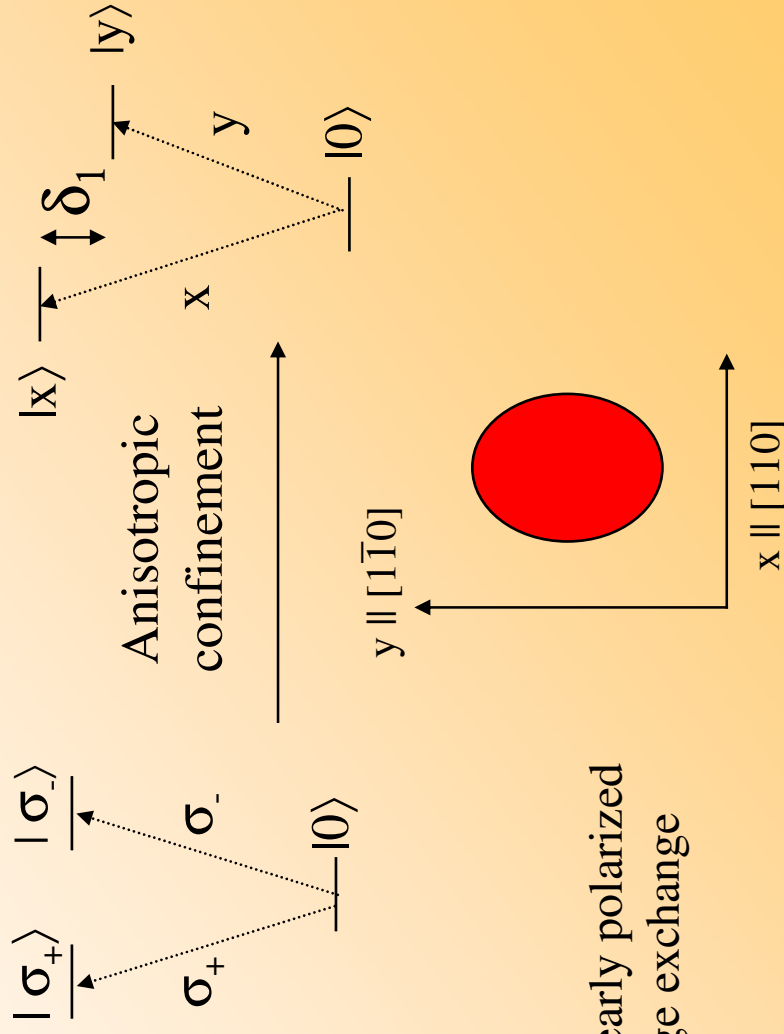


# Fine-structure splitting of excitons



Polarization-dependence reveals exciton fine-structure:

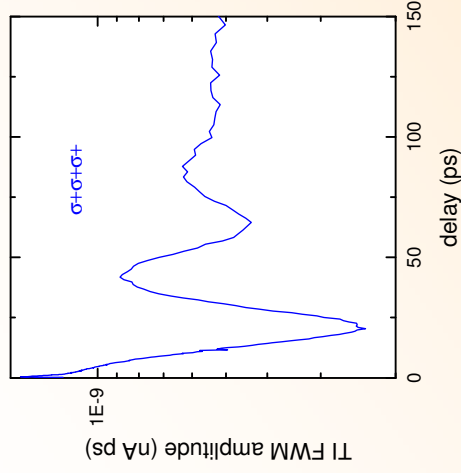
- oscillations for circular polarization ( $\sigma_+ \sigma_+ \sigma_+$ )
- no oscillations for linear polarization ( $x x x$ )



$\delta_1$ : Splitting between two linearly polarized exciton states by long-range exchange

# Fine-structure beat - circular excitation

Excitation:  $|\sigma_+\rangle = \frac{1}{\sqrt{2}}(|x\rangle + i|y\rangle)$  Dynamics:  $\frac{1}{\sqrt{2}}(e^{-i\omega t/2}|x\rangle + ie^{i\omega t/2}|y\rangle) = \cos(\omega t/2)|\sigma_+\rangle + i\sin(\omega t/2)|\sigma_-\rangle$   $\omega = \delta_1 / \hbar$



only  $|\sigma_+\rangle$  creates FWM polarization, since pulse 2 is  $\sigma_+$  polarized

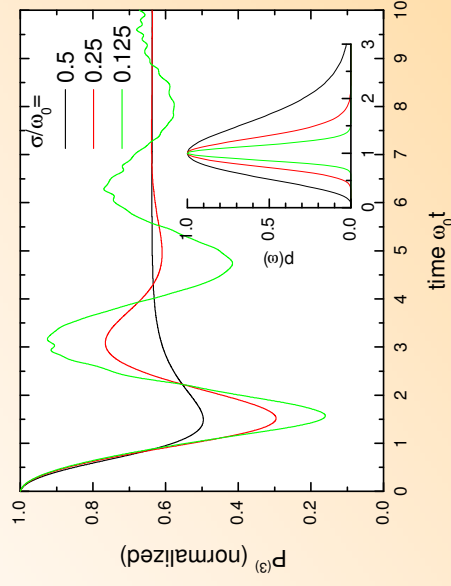
$$P_{echo}^{(3)} \propto |\cos(\omega t/2)|^2 \propto 1 + \cos(\omega t)$$

Damping of oscillations observed

$\Rightarrow$  QD ensemble exhibits a distribution  $p(\omega)$  of  $\omega$

$$P_{echo}^{(3)} \propto 1 + \int p(\omega) \cos(\omega t) d\omega$$

Consider  $p(\omega) \propto \omega^{n-1} \exp(-n\omega / \omega_0)$  of average  $\omega_0$  and variance  $\sigma = \omega_0 / \sqrt{n}$



Fit to measured data determines

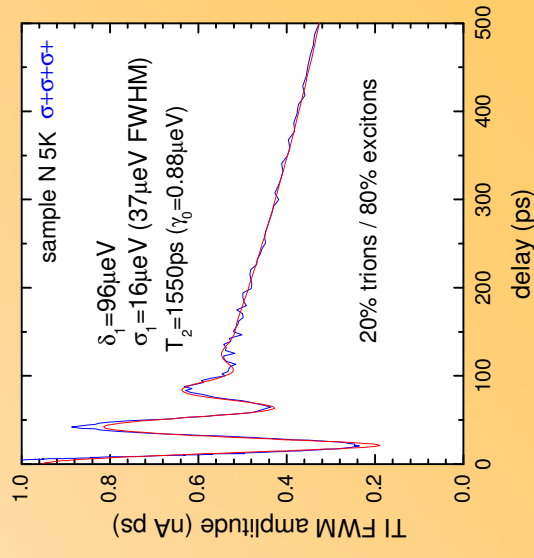
average fine-structure splitting  $\langle \delta_1 \rangle$

and its variance  $\sigma_1 = \sqrt{\langle \delta_1^2 \rangle - \langle \delta_1 \rangle^2}$

Observed oscillation amplitude

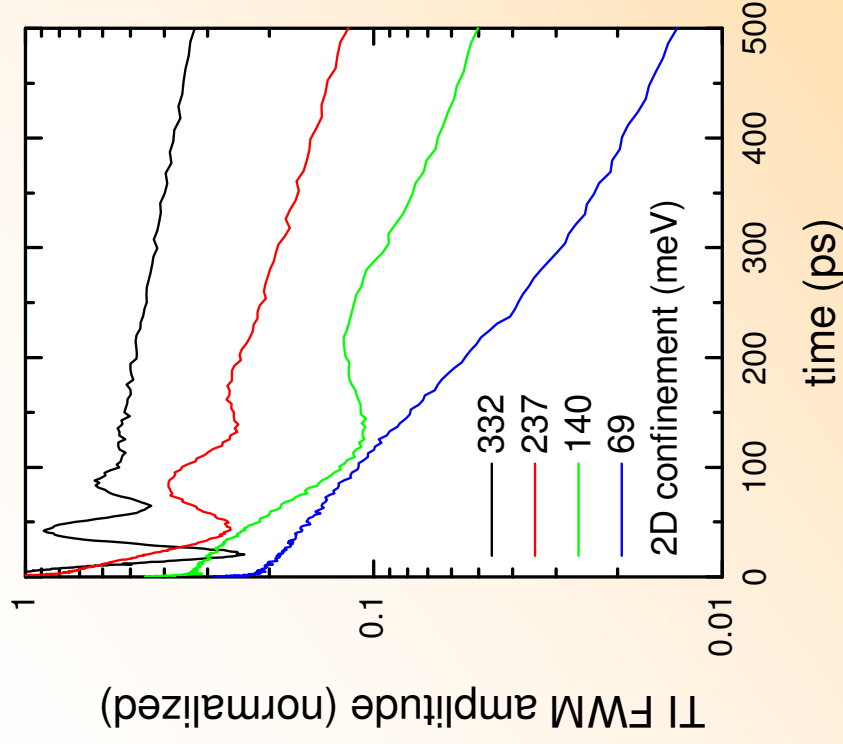
smaller than model:

Charged QDs are present

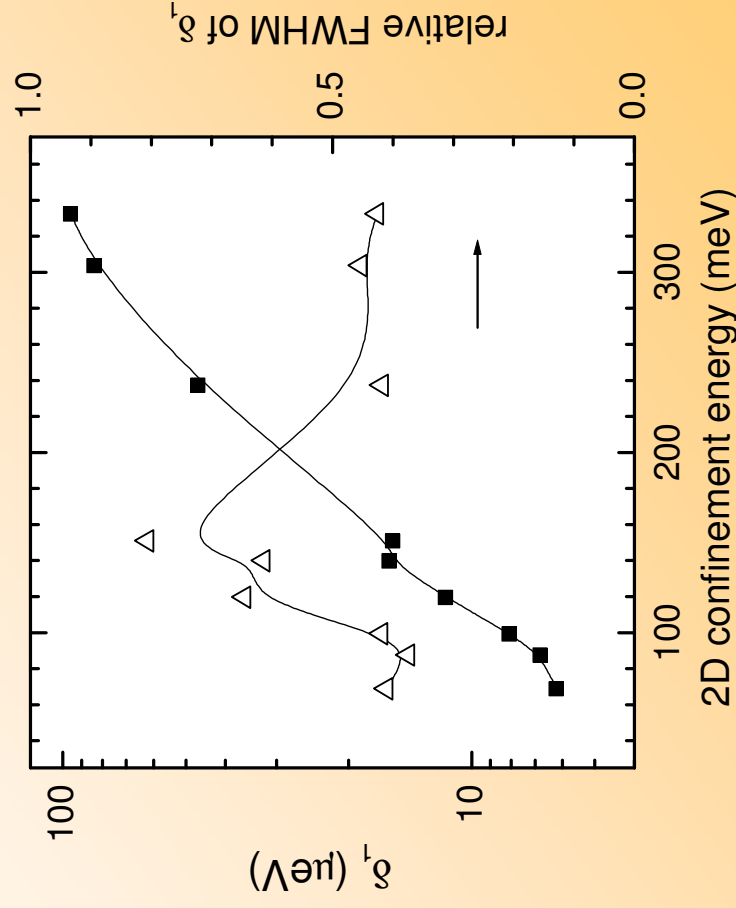


# Fine-structure splitting versus annealing

W.Langbein, P. Borri, U.Woggon, V. Stavarache, D. Reuter, A. D. Wieck, Phys. Rev. B 69, 161301R (2004)



Strong dependence decrease of  
fine-structure splitting  
with annealing

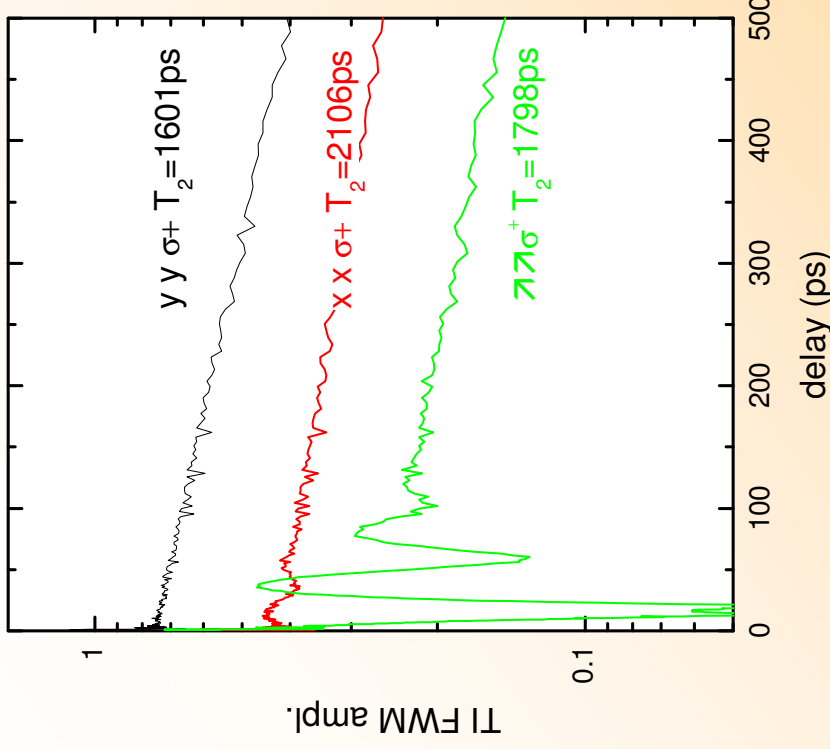


Relative width of inhomogeneous  
distribution rather constant

See also Tartakovskii et al. Phys. Rev. Lett. 93, 057401 (2004)

# Dephasing of fine-structure split exciton states

W. Langbein, P. Borri, U. Woggon, V. Stavarache, D. Reuter, A. D. Wieck, Phys. Rev. B, 70, 033301 (2004)



Fine-structure beat suppressed for [110] and [1-10] polarized excitation:

- ⇒ Dipole anisotropy of wavefunctions along (110)
- Different polarizations show different decay rates.
- Since  $kT \gg \delta_1$  this is not spin relaxation
- ⇒ radiative lifetimes are different
- ⇒ oscillator strengths are different  $\gamma_r \propto \mu^2$

$$\frac{\mu_y}{\mu_x} = \sqrt{\frac{\gamma_y}{\gamma_x}} = 1.147 \pm 0.005$$

This assumes radiatively limited dephasing  
We can cross-check by the FWM signal strength

$$E^{(3)} \propto \mu^4$$

and we find

$$\frac{\mu_y}{\mu_x} = \sqrt[4]{\frac{E_y^{(3)}}{E_x^{(3)}}} = 1.149 \pm 0.01$$

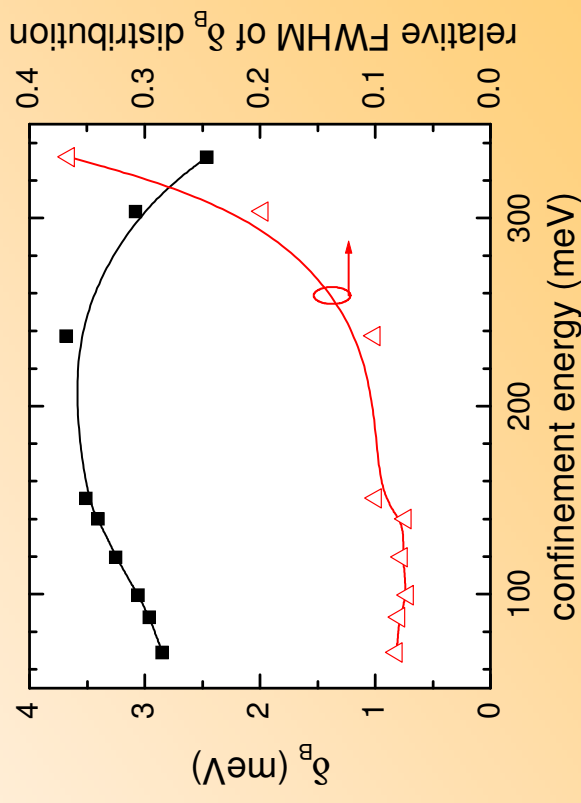
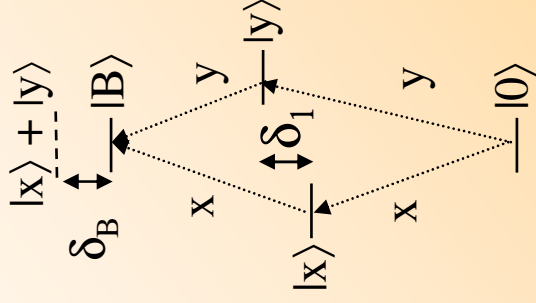
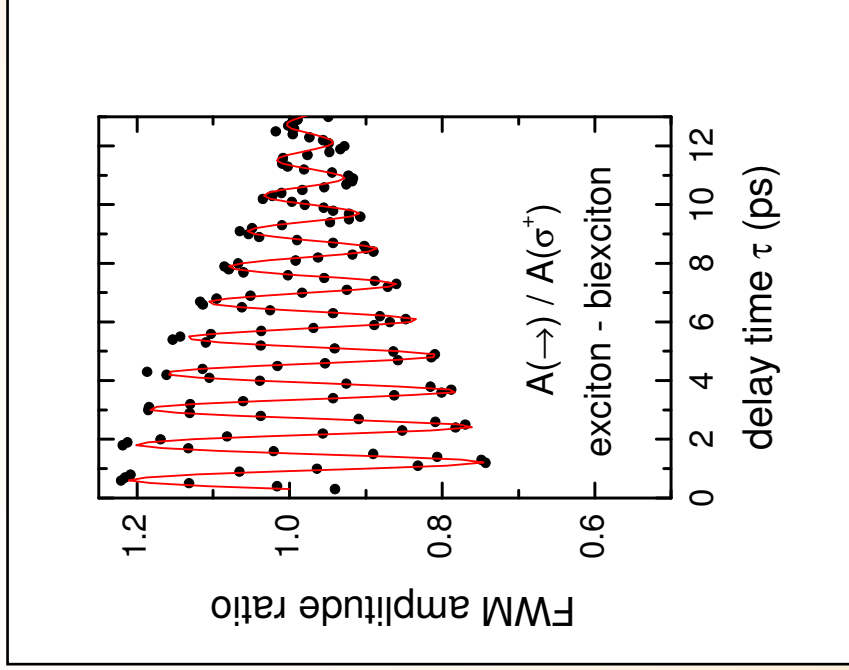
Radiative lifetime limited dephasing



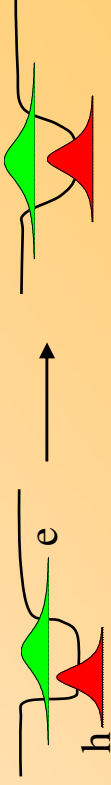
# Biexciton binding energy

W. Langbein, P. Borri, U. Woggon, V. Stavarache, D. Reuter, A. D. Wieck, Phys. Rev. B 69, 161301R (2004)

$\delta_B$ : Difference between the sum of the two finestructure-split exciton energies and the biexciton energy  $\delta_B = E_x + E_y - E_B$



binding energy rather insensitive to annealing  
 created and distribution decreased by initial annealing:  
 symmetrizing of the vertical confinement



# Acoustic-phonon assisted transitions in QDs

- Small spatial extension of the electron-hole pair
- ⇒ Single pair creates a large local density ( $10^{19}\text{cm}^{-3}$ )
  - ⇒ electronic binding of atoms disturbed
  - ⇒ **lattice equilibrium position is shifted**
  - ⇒ **phonon-assisted optical transitions** exist, following the **Franck-Condon principle** for absorption/emission of acoustic phonons.
  - ⇒ Each phonon mode creates a series of discrete lines on each side of the **zero-phonon line (ZPL)**.

Compared to bi-atomic molecules:

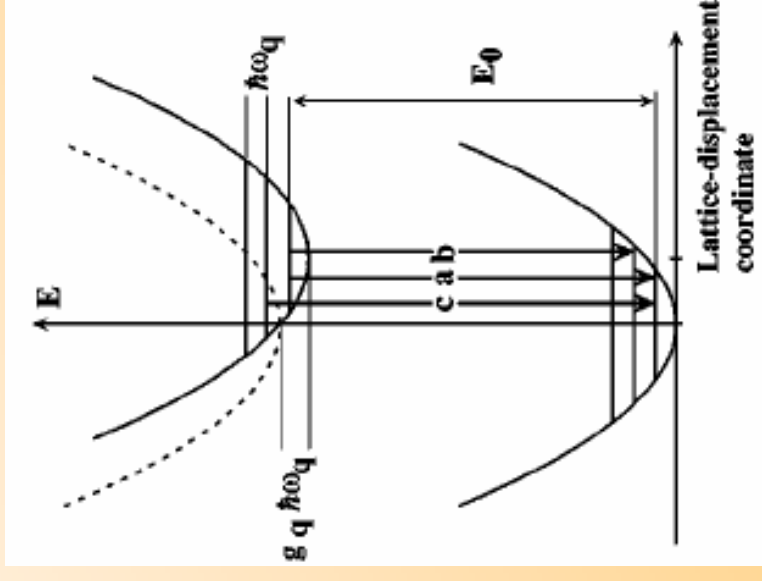
- **zero-phonon transition has relevant strength**
- **phonons are a continuum**

Theoretical description: independent boson model:

$$H = \mathcal{E}_0 B^\dagger B + \sum_{\vec{q}} \hbar \omega_{\vec{q}} a_{\vec{q}}^\dagger a_{\vec{q}} + \sum_{\vec{q}} M_{\vec{q}} (a_{\vec{q}}^\dagger + a_{-\vec{q}}) B^\dagger B$$

$H$  commutes with excitonic density  $B^\dagger B$  (pure dephasing)

B. Krummheuer *et al.* PRB **65** 195313 (2002), R. Zimmermann and E. Runge ICPS26 (2002)



# Exciton-acoustic phonon interaction in QDs

Deformation potential coupling with acoustic phonons of dispersion:  $\omega_{\vec{q}} = qs$

$$M_{\vec{q}} = \sqrt{\frac{\hbar\omega_{\vec{q}}}{2s^2\rho V}} \int d\vec{r}_e d\vec{r}_h |\Psi(\vec{r}_e, \vec{r}_h)|^2 (D_c e^{i\vec{q}\vec{r}_e} - D_v e^{i\vec{q}\vec{r}_h})$$

$D_c, D_v$ :  
deformation potential coupling constants

Non-exponential decay of the first-order polarization:

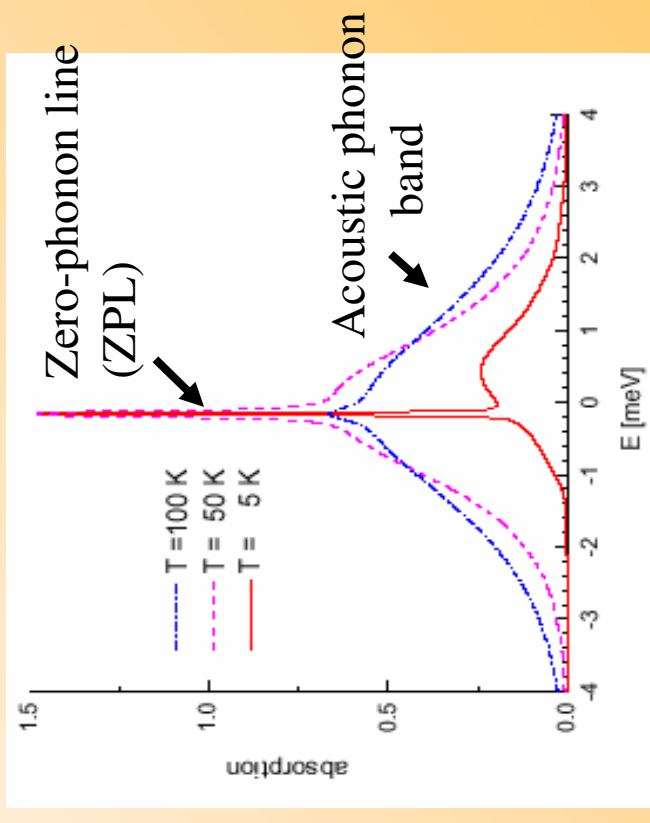
$$P^{(1)}(t) = i \exp(-i(\omega_0 - \Delta)t + R(t) - S)$$

$\Delta$ : polaron shift

$R(t=0) = S$ : Huang-Rhys factor

$$R(t) = \sum_{\vec{q}} \frac{M_{\vec{q}}^2}{\hbar\omega_{\vec{q}}^2} [(2N_{\vec{q}} + 1)\cos(\omega_{\vec{q}}t) - i\sin(\omega_{\vec{q}}t)]$$

$$N_{\vec{q}} = \frac{1}{e^{\hbar\omega_{\vec{q}}/k_B T} - 1}$$

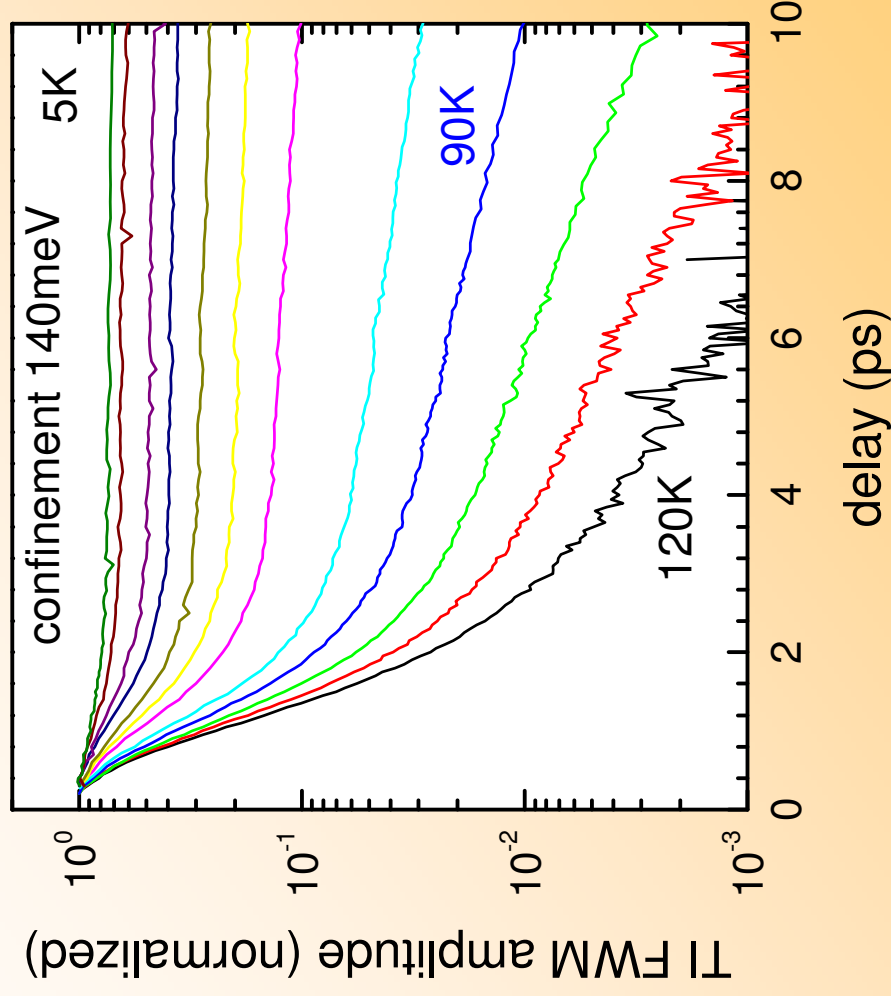


R. Zimmermann and E. Runge ICPS26 (2002)

$$\text{Absorption spectrum: } a(\omega) \propto \text{Im} \int_0^{\infty} dt P^{(1)}(t) e^{i\omega t}$$

# Phonon-assisted transitions: experiments

Fast initial decay of FWM due to dephasing of **phonon-assisted transitions**

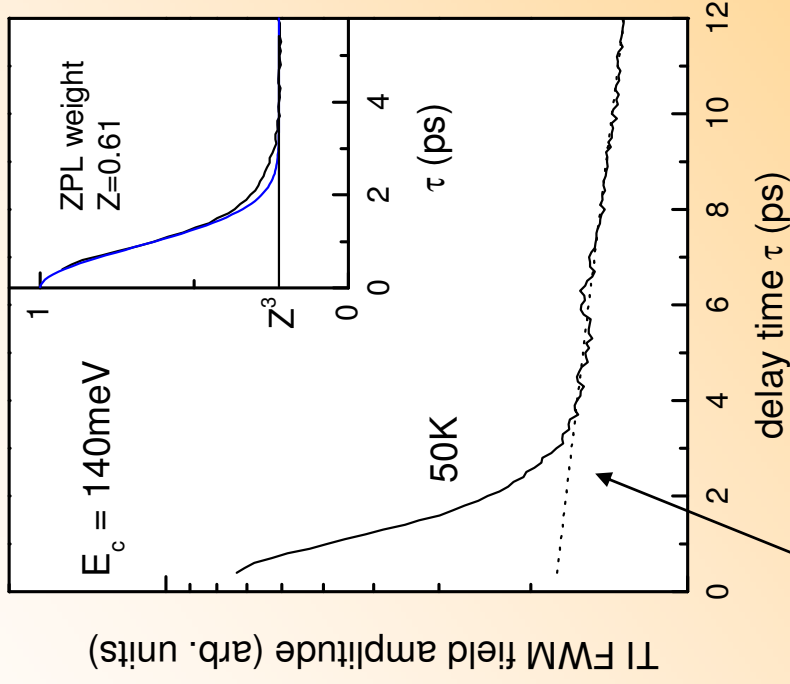


# Weight of the zero-phonon line

Zero-phonon line weight: Area fraction of the ZPL in the absorption spectrum  $Z = \exp(-S)$   
 (For negligible ZPL dephasing: equal to the normalized polarization amplitude for  $t \rightarrow \infty$ )

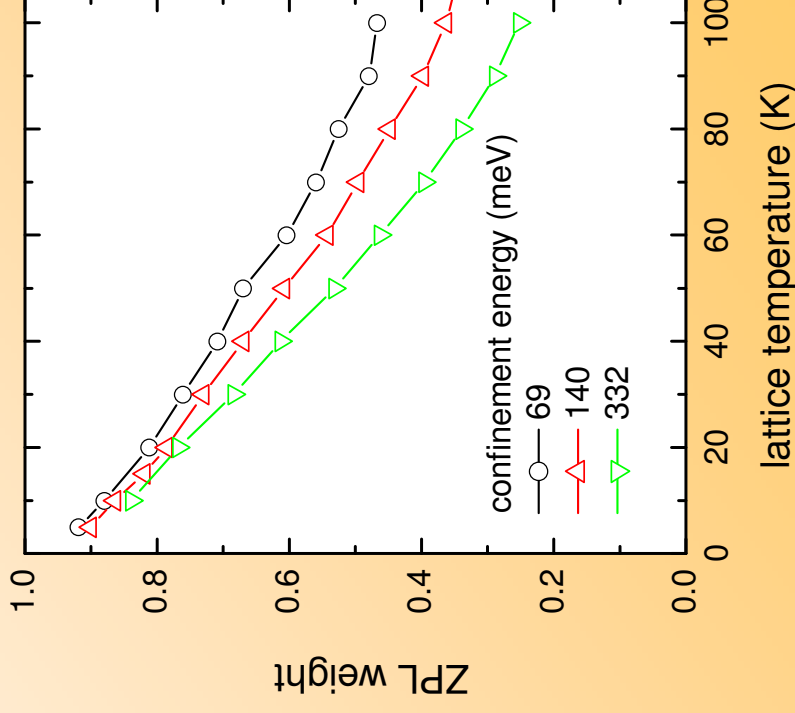
The asymptotic value of the TI FWM

photon echo is  $Z^3 = \exp(-3S)$   
 A. Vagov *et al.* PRB **66** 165312 (2002)



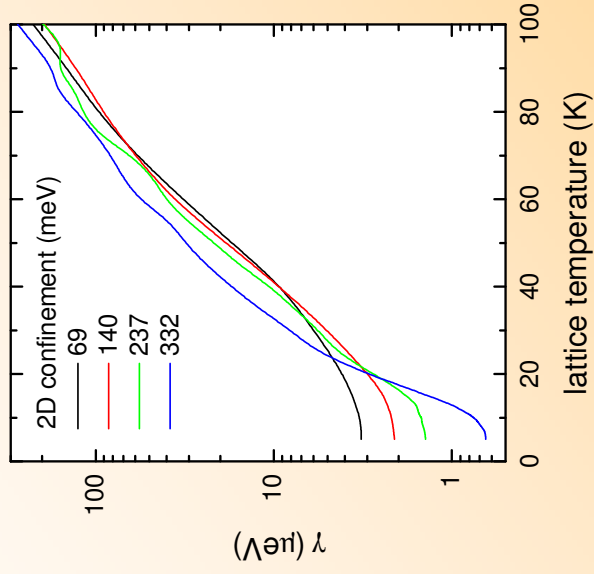
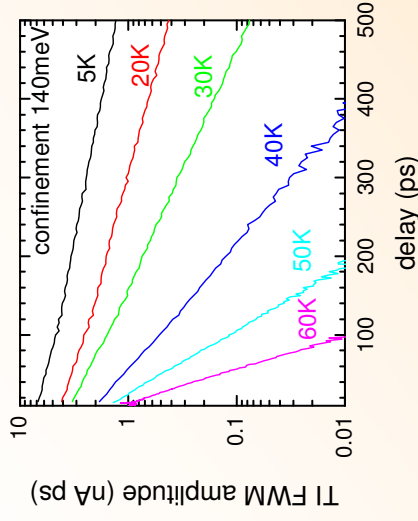
Correction for exponential ZPL decay

ZPL weight decreases with increasing temperature and decreasing extension of the exciton wavefunction, as expected by the model



# Dephasing of zero-phonon transition versus temperature

Zero-phonon line polarization:  
exponential decay rate  $\gamma$



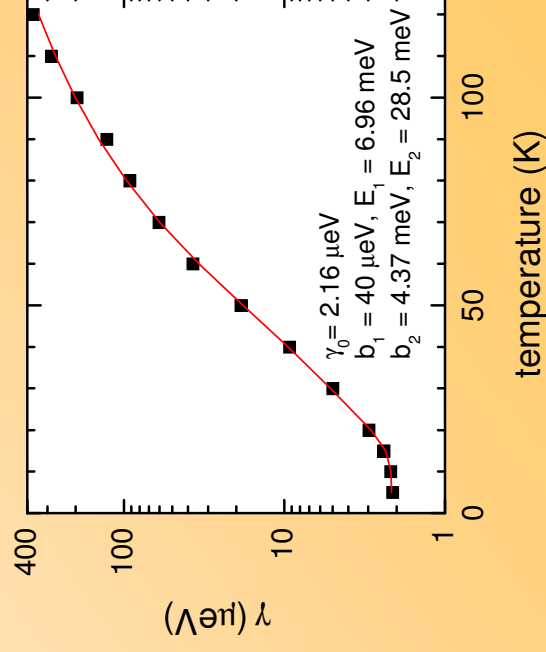
Temperature-dependence fitted by

$$\gamma(T) = \gamma_0 + \frac{b_1}{\exp\left(\frac{E_1}{kT}\right) - 1} + \frac{b_2}{\exp\left(\frac{E_2}{kT}\right) - 1}$$

natural (radiative) linewidth

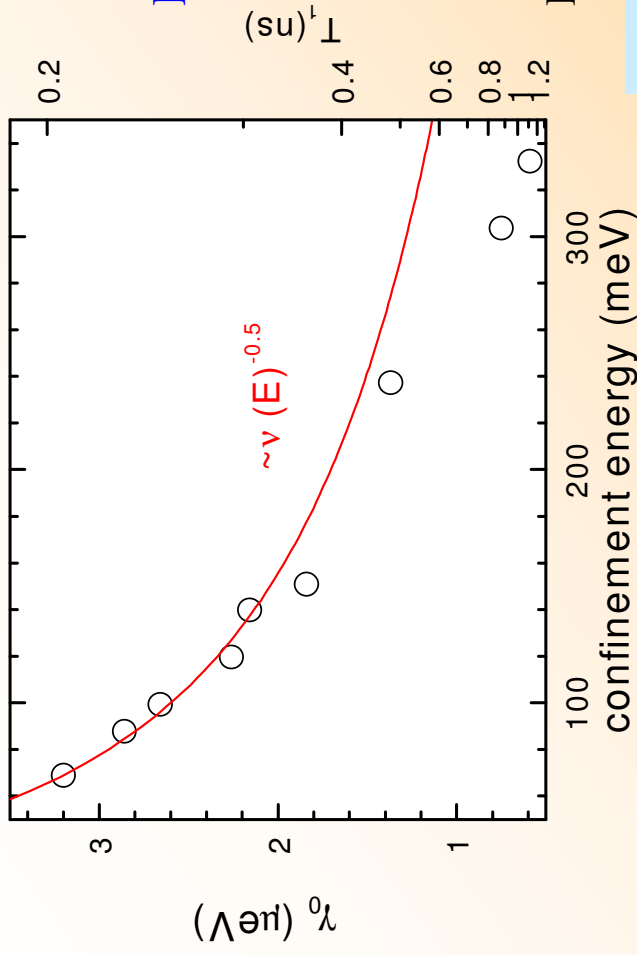
acoustic phonons ?

optical phonons ?



Systematic change  
with annealing

# Low-temperatures: radiatively limited dephasing times



Decrease of dephasing rate with increasing confinement:

Radiative lifetime changes from 200ps to 1ns

Radiative rate is proportional to the coherence area  $A$ :

Parabolic in-plane confinement of „rigid“ exciton:

$$V(r) \propto \omega^2 \Rightarrow \omega \propto \sqrt{E_{\text{conf}}}$$

$$A \propto l^2 \propto \omega^{-1} \propto 1/\sqrt{E_{\text{conf}}}$$

Deviation for weak annealing (i.e. large confinement):

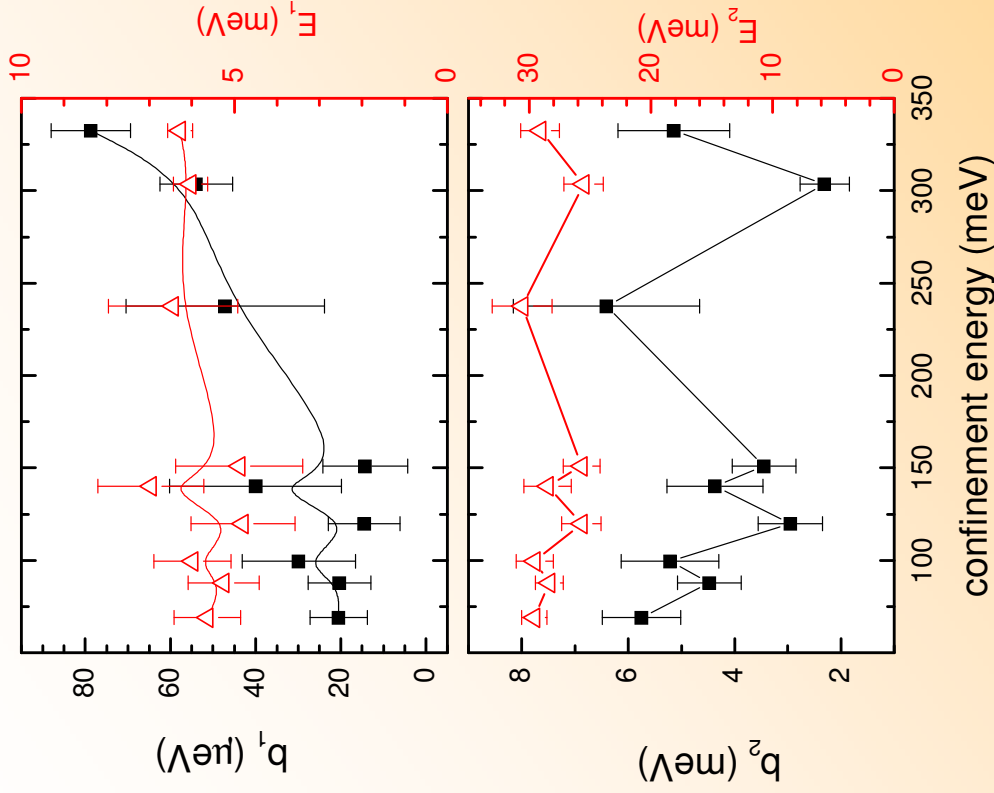
Reduced e-h overlap by asymmetric vertical confinement of electron and hole in the

exciton





# ZPL dephasing : phonon interaction parameters

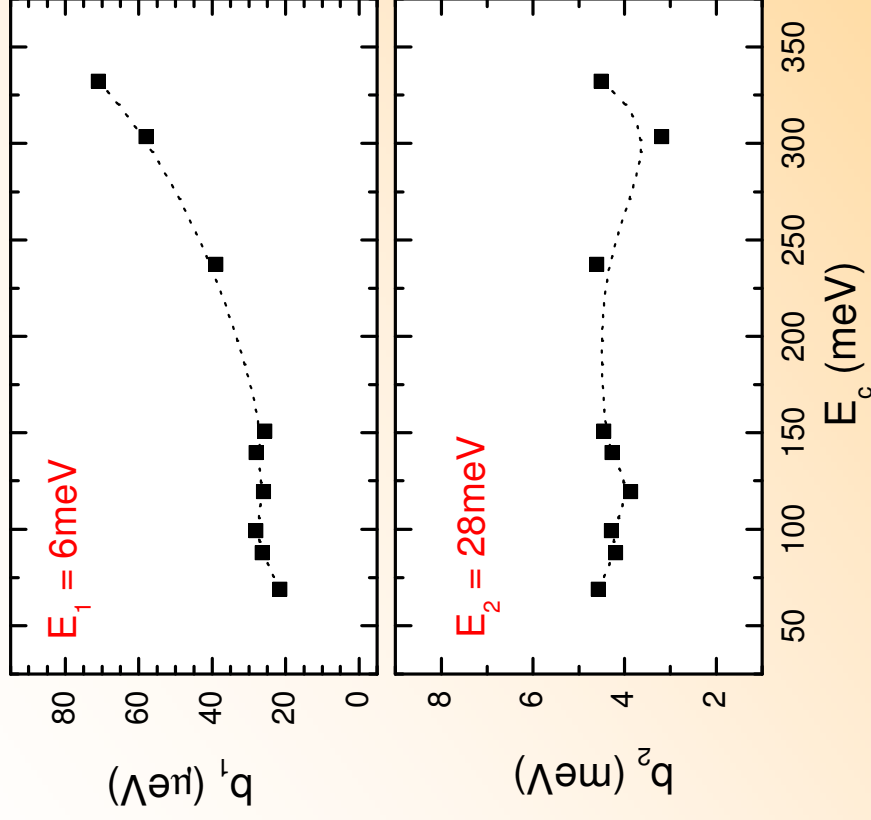


$$\gamma(T) = \gamma_0 + \frac{b_1}{\exp\left(\frac{E_1}{kT}\right) - 1} + \frac{b_2}{\exp\left(\frac{E_2}{kT}\right) - 1}$$

Both activated parts show an activation energy **nearly independent** of confinement energy !

Contradicts the importance of **single phonon absorption** to excited electronic levels for the dephasing !

# ZPL dephasing : phonon interaction parameters



- Activation energies independent of  $E_c$   
6meV: acoutical phonons  
28meV: optical phonons

No significant influence of electronic level spacing: phonon absorption is not the dominant process

- Systematic dependence of  $b_1$  on  $E_c$

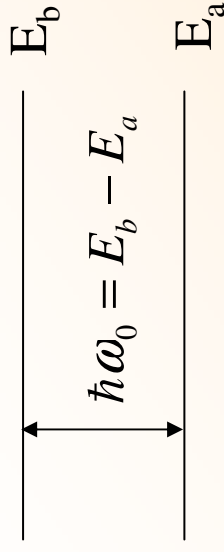
Stronger coupling to acoustic phonons with decreasing size, similar to the behaviour of the ZPL weight

- Also ZPL dephasing is due to elastic phonon interaction ?

P. Borri *et al.*, Phys. Rev. B submitted

# Optical Rabi oscillations

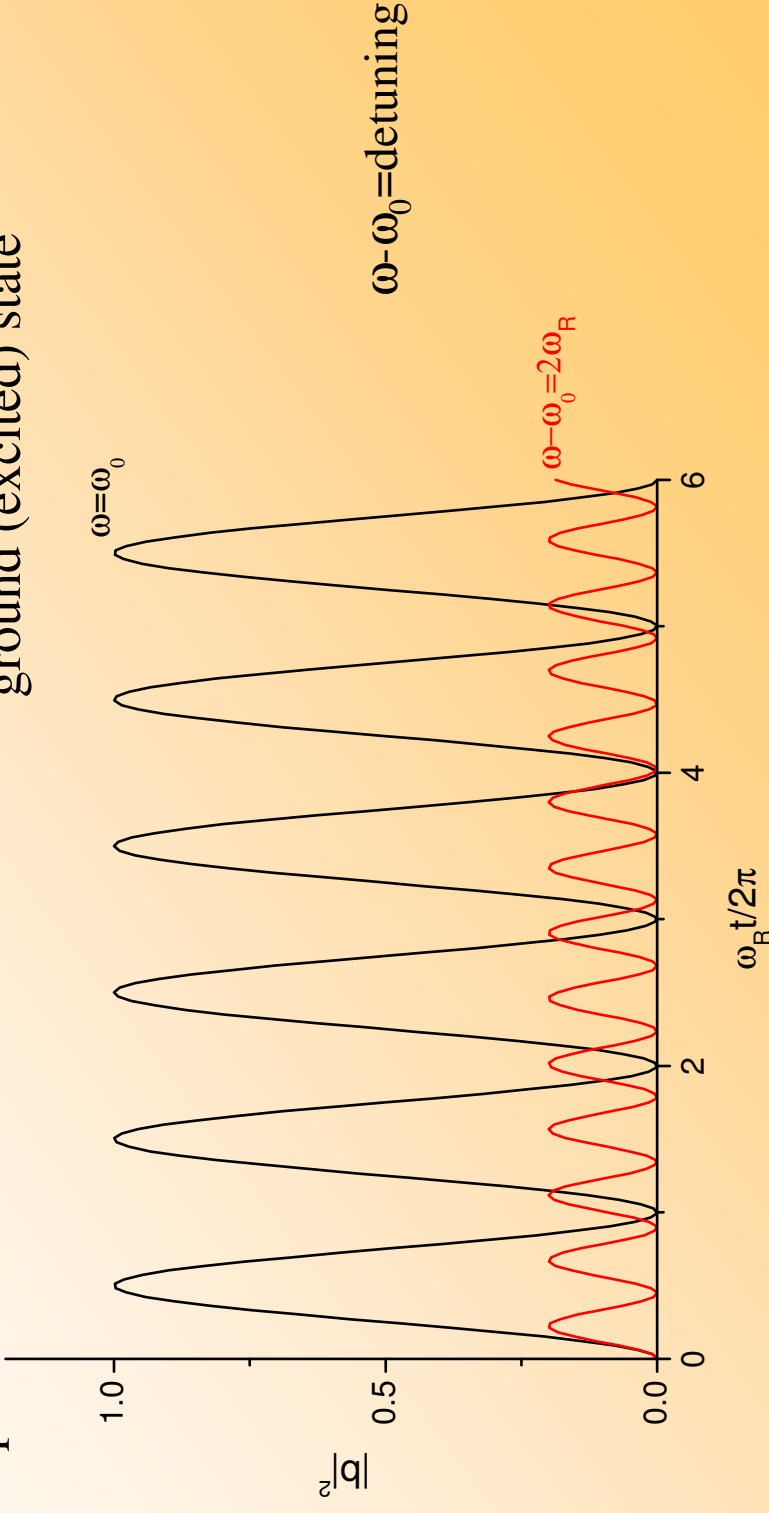
Two-level system in presence of a time-dependent electric field  $\vec{E} = \Theta(t)\vec{E}_0 \cos \omega t$



$$\omega_R = \frac{\vec{\mu} \cdot \vec{E}_0}{\hbar} \quad : \text{ Rabi frequency}$$

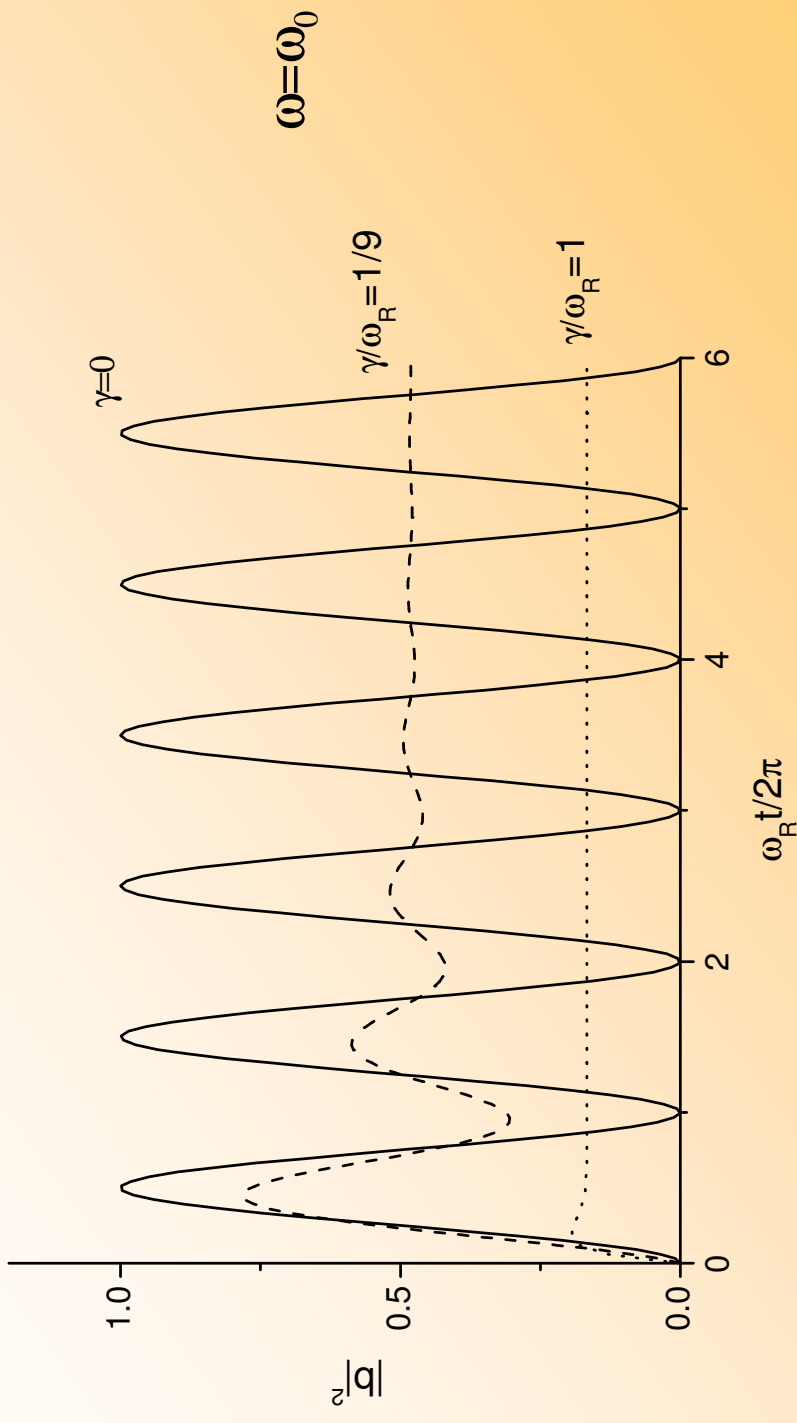
$|a|^2 (|b|^2)$  : occupation probability of the ground (excited) state

$\vec{\mu}$  : transition dipole moment



# Optical Rabi oscillations

The effect of a radiative damping  $\gamma=1/T_2$  of the polarization ( $T_2$  dephasing time):



Population flopping over many periods is possible in systems with long dephasing times and large transition dipole moments:  $\gamma/\omega_R \ll 1$ .

# Rabi oscillations versus pulse area

Pulsed excitation: e.g.  $E_0 = E_{00} \exp(-(t/t_0)^2)$

Pulse area: time-integrated Rabi frequency  $\theta = \int_{-\infty}^{+\infty} \frac{\vec{\mu} \cdot \vec{E}_0}{\hbar} dt$

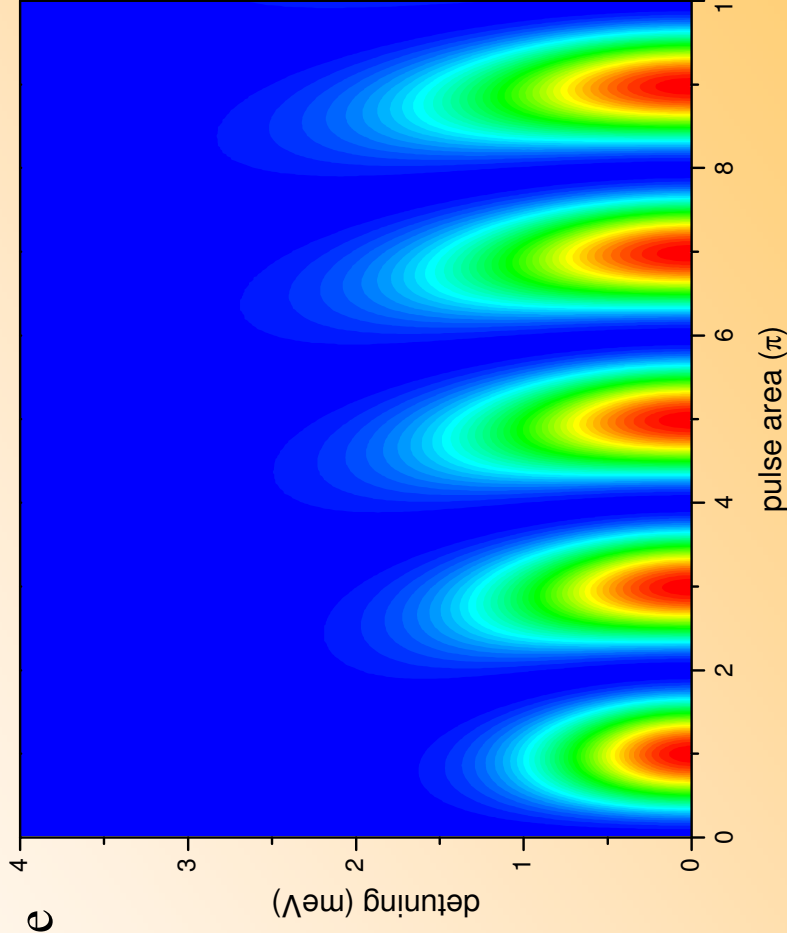
Population

difference after pulse

( $t \gg t_0$ )

$$\Delta N = |b|^2 - |a|^2$$

$t_0 = 1 \text{ ps}$  ( $\Delta E_{\text{HWHM}} = 0.75 \text{ meV}$ )



$\gamma = 0$

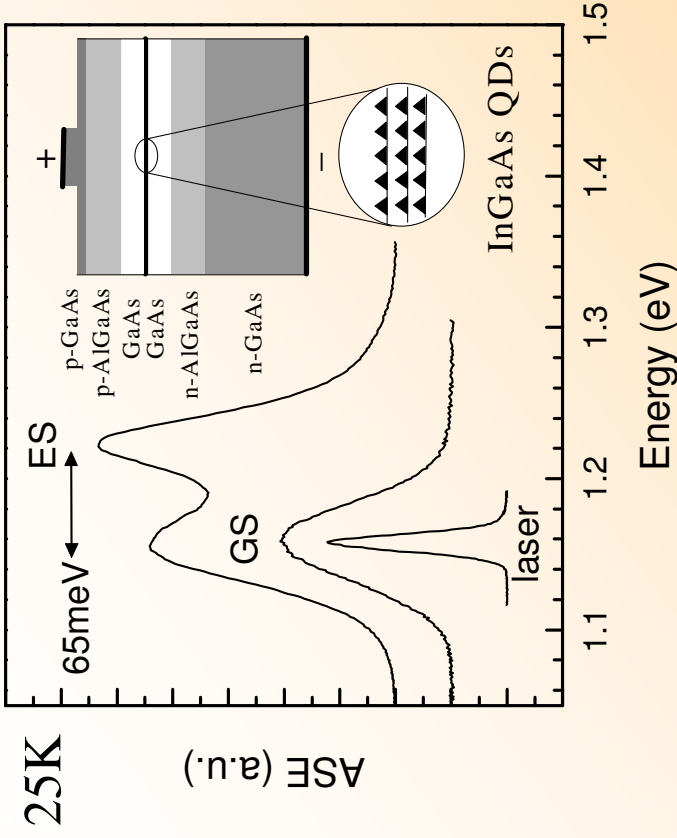
blue = -1

red = +1

Initial conditions:

$$\Delta N = -1 \quad (t \ll -t_0)$$

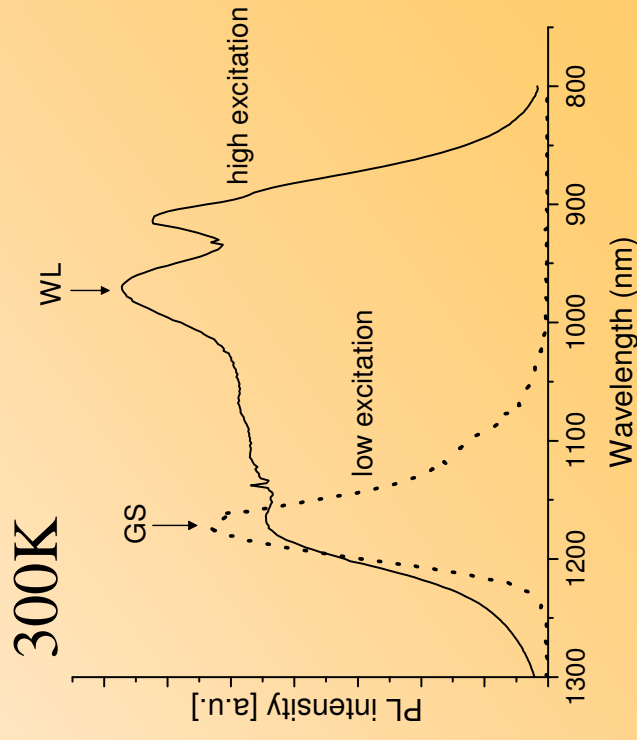
# InGaAs quantum dots: investigated sample



Inhomogeneous broadening: **60meV**

GS-ES separation: **65meV**

GS-WL separation **~220meV**



ridge waveguide  $5 \times 500 \mu\text{m}$   
 3 stacked QD layers  
 35nm GaAs spacers  
 areal dot density  $\sim 2 \times 10^{10} \text{cm}^{-2}$

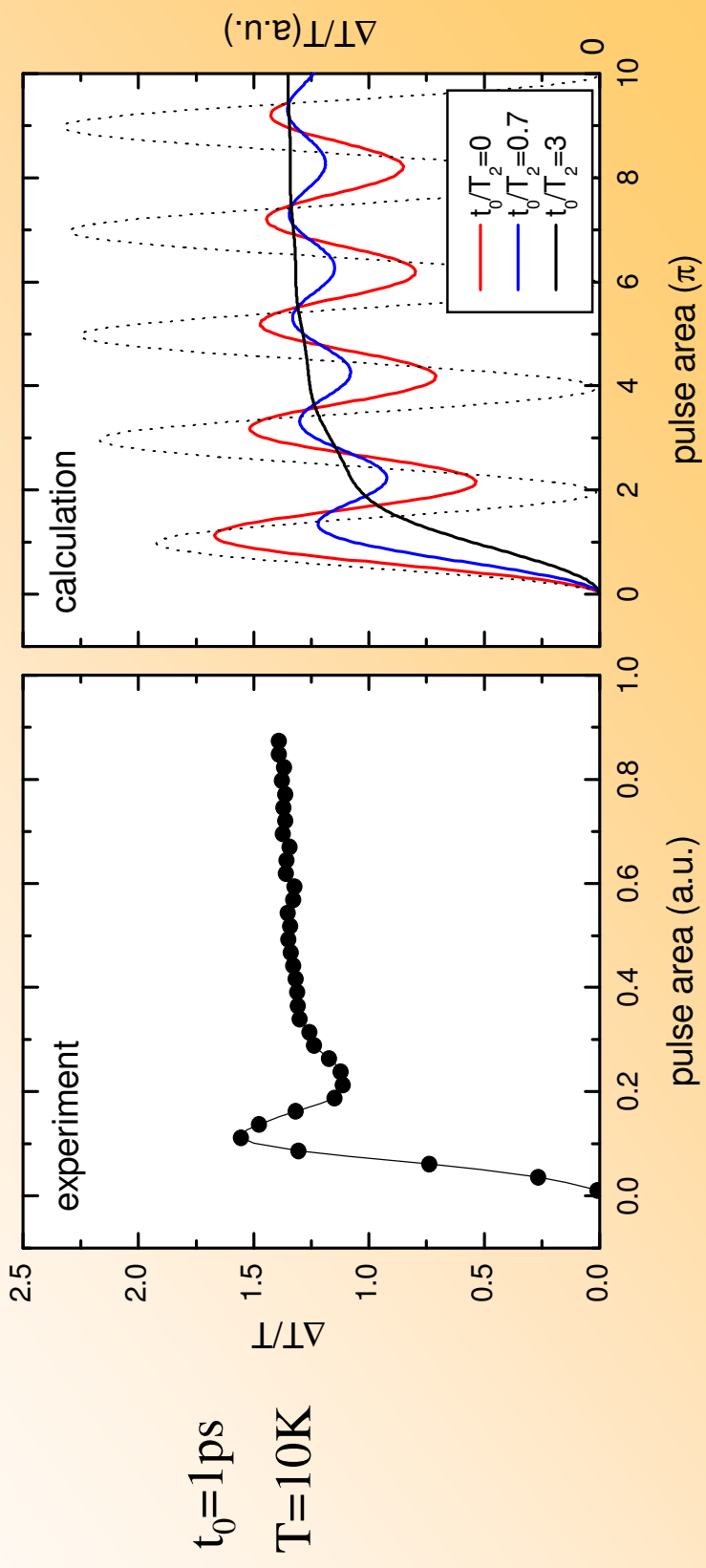
# Differential transmission

Absorption coefficient of an inhomogeneously broadened ensemble:

$$\alpha(\omega) = \int_{-\infty}^{+\infty} \sigma_0 \frac{1/T_2}{(\omega - \omega_\xi)^2 + (1/T_2)^2} \Delta N(\omega_c - \omega_\xi, \theta_{pump}) f(\omega_\xi) d\omega_\xi \xrightarrow{T_2 \rightarrow \infty} \sigma_0 \Delta N(\omega_c - \omega, \theta_{pump}) f(\omega)$$

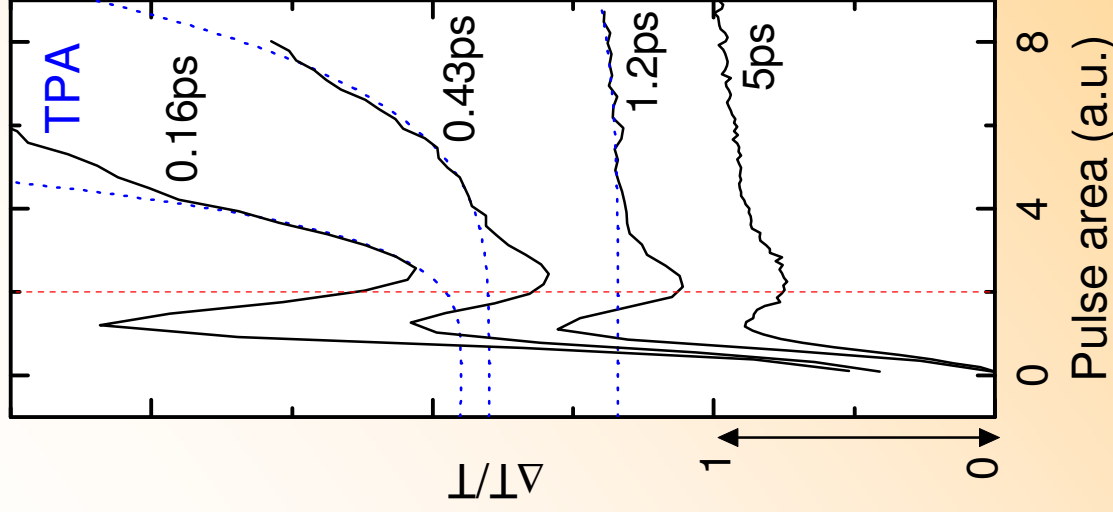
↑ Lorentz.    pop. diff.    ↑ inh. distrib.

$\alpha$  is probed by a weak probe pulse after the pump: differential transmission intensity of the probe





# Study of the damping versus pulse area: Biexciton and dephasing



Differential transmission intensity using **different pulse durations**

We measured  $E_{XX}=3\text{meV}$  biexciton binding energy. If the pulse spectral width  $\Delta E \ll E_{XX}$  the X-XX transition is out of resonance. However, for all pulses the damping of the oscillations remains. When  $\Delta E \sim E_{XX}$  the oscillation **changes period**.

If  $t_0 > T_2$  dephasing is important. We observe a **quenching of the amplitude** of the oscillations with increasing  $t_0$ . For short  $t_0$  two-photon absorption (TPA density  $\sim (\theta/t_0)^4 t_0$ ) in the waveguide covers the Rabi oscillations.

# Calculations: Optical Bloch Equations

**Basic equations:**

$$i\hbar \frac{\partial \Psi}{\partial t} = (H_0 + V)\Psi$$

$$i\hbar \frac{\partial \Psi_i}{\partial t} = H_0 \Psi_i = E_i \Psi_i$$

$$\Psi_i = \varphi_i e^{-\frac{iE_i t}{\hbar}}$$

$$\Psi = \sum_i a_i \Psi_i$$

$$\rho_{ij} = a_i^* a_j$$

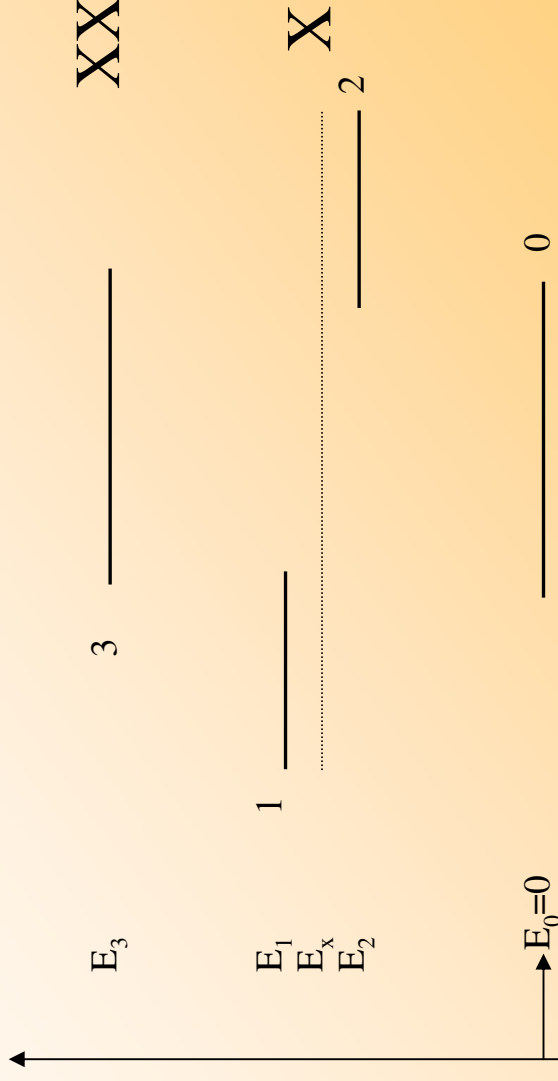
$$\frac{d\rho_{ij}}{dt} = -\frac{i}{\hbar} \sum_k (\rho_{ik} V_{jk} - \rho_{jk}^* V_{ik}^*)$$

$$V_{ij} = e^{\frac{i(E_i - E_j)t}{\hbar}} (-\vec{\mu}_{ij} \cdot \vec{E})$$

**Dipole-allowed transitions:**

Only  $V_{10}$ ,  $V_{20}$ ,  $V_{32}$ ,  $V_{31}$  (and c.c.) are different from zero.

**Considered 4-level system:**



$$\Delta = E_1 - E_2 \text{ (exciton fine-structure splitting)}$$

$$-E_{XX} = E_3 - 2E_x \text{ (biexciton binding energy)}$$

## Damping rates:

populations

$$\frac{d\rho_{00}}{dt} = \dots + (\rho_{11} + \rho_{22})g_{1X}$$

$$\frac{d\rho_{11}}{dt} = \dots + \rho_{33} \frac{g_{1XX}}{2} - \rho_{11}g_{1X}$$

$$\frac{d\rho_{22}}{dt} = \dots + \rho_{33} \frac{g_{1XX}}{2} - \rho_{22}g_{1X}$$

$$\frac{d\rho_{33}}{dt} = \dots - \rho_{33}g_{1XX}$$

coherences

$$\frac{d\rho_{10}}{dt} = \dots - \rho_{10}g_{2X}$$

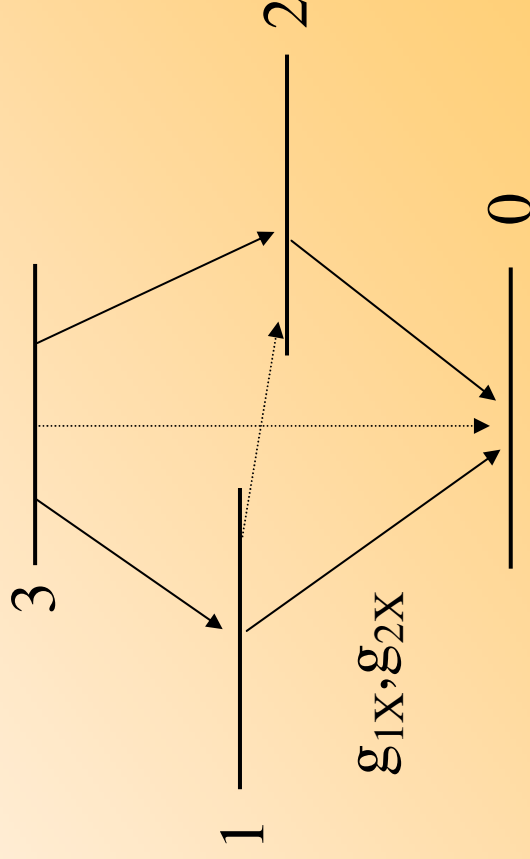
$$\frac{d\rho_{20}}{dt} = \dots - \rho_{20}g_{2X}$$

$$\frac{d\rho_{31}}{dt} = \dots - \rho_{31}g_{2XTXX}$$

$$\frac{d\rho_{32}}{dt} = \dots - \rho_{32}g_{2XTXX}$$

$$\frac{d\rho_{30}}{dt} = \dots - \rho_{30}g_{2XX}$$

$$\frac{d\rho_{21}}{dt} = \dots - \rho_{21}g_{2XTX}$$



# Chosen parameters for calculations

Fine structure splitting  $\sim 100\mu\text{eV}$  (important only for  $t_0 \geq 5\text{ps}$ )

Simplified calculation with  $\Delta=0$ :

$$\vec{\mu}_{01}\vec{E} = \vec{\mu}_{13}\vec{E}$$

Linearly polarized field

$$\vec{\mu}_{02}\vec{E} = 0$$

along 0-1-3 transitions

$$\vec{\mu}_{23}\vec{E} = 0$$

and same 0-X and X-XX

dipole

$$E_{XX}=3\text{meV}$$

## Dephasing:

At 10K we measure  $1/g_{2X}=500\text{ps} \gg t_0$ .

However, the polarization decay is *non-exponential* with an initial fast dephasing time of  $\sim 1.5\text{ps}$  (non-Lorentzian lineshape).

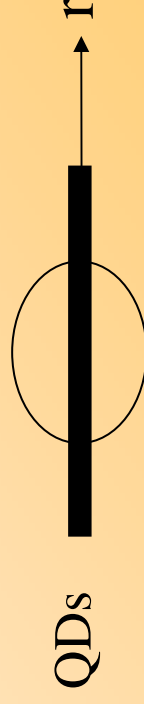
## Population lifetimes

$$(g_{1X})^{-1} = 1\text{ns} \gg t_0$$

are not important

**Pulse field with  $TE_0$  spatial profile:**

$$E_0(t, r) = E_{00}e^{-(t/t_0)^2} \Psi_{TE_0}(r) = E_{00}e^{-(t/t_0)^2} \text{sech}(r/r_0)$$

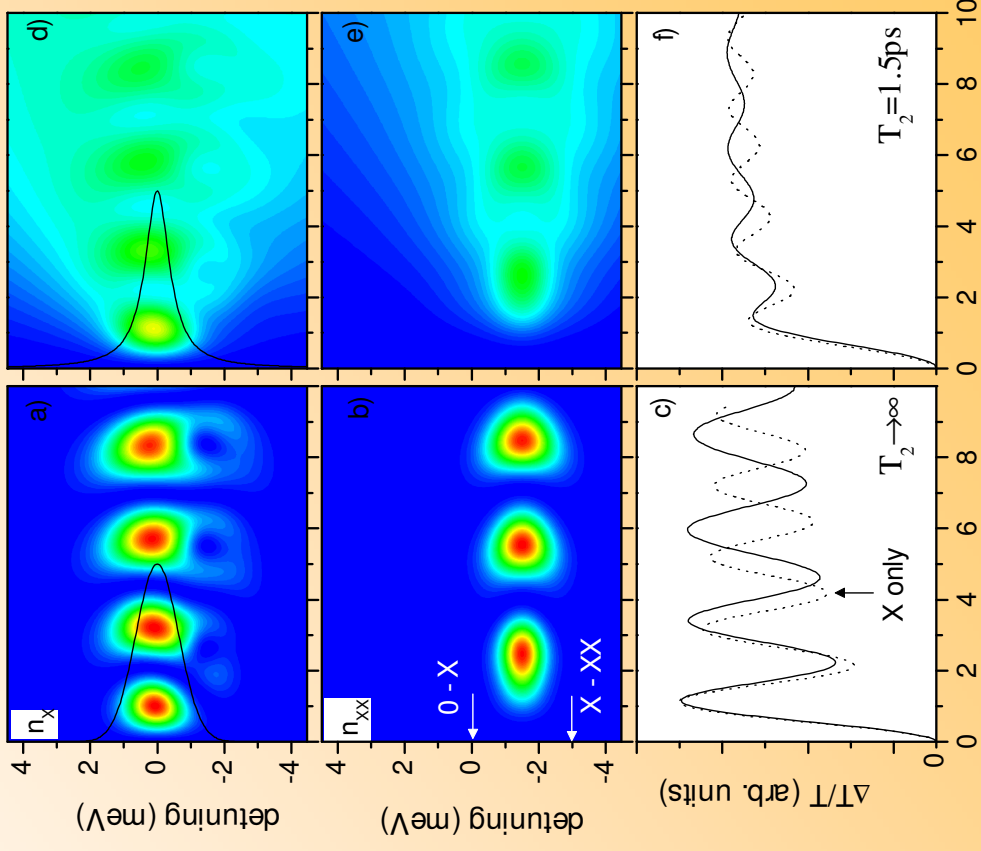


# Calculations

$$\frac{\Delta T}{T} \propto \int_{-\infty}^{+\infty} d\omega |E_{probe}(\omega)|^2 \int_0^{+\infty} \Psi_{TE_0}^2(r) dr \int_{-\infty}^{+\infty} [L(\omega - \omega_\xi)(n_X - n_0) + L(\omega - (\omega_\xi - \omega_{XX}) - \omega_{XX})(n_{XX} - n_X)] f(\omega_\xi) d\omega_\xi$$

Example:  $t_0 = 1\text{ps}$

Solutions of OBE at  $t \gg t_0$ : functions of  $\omega_c - \omega_\xi, \theta_{pump}(r)$



The biexciton population oscillates with a **different period** compared to the exciton population.

The dephasing reduces the **amplitude** of the oscillations.

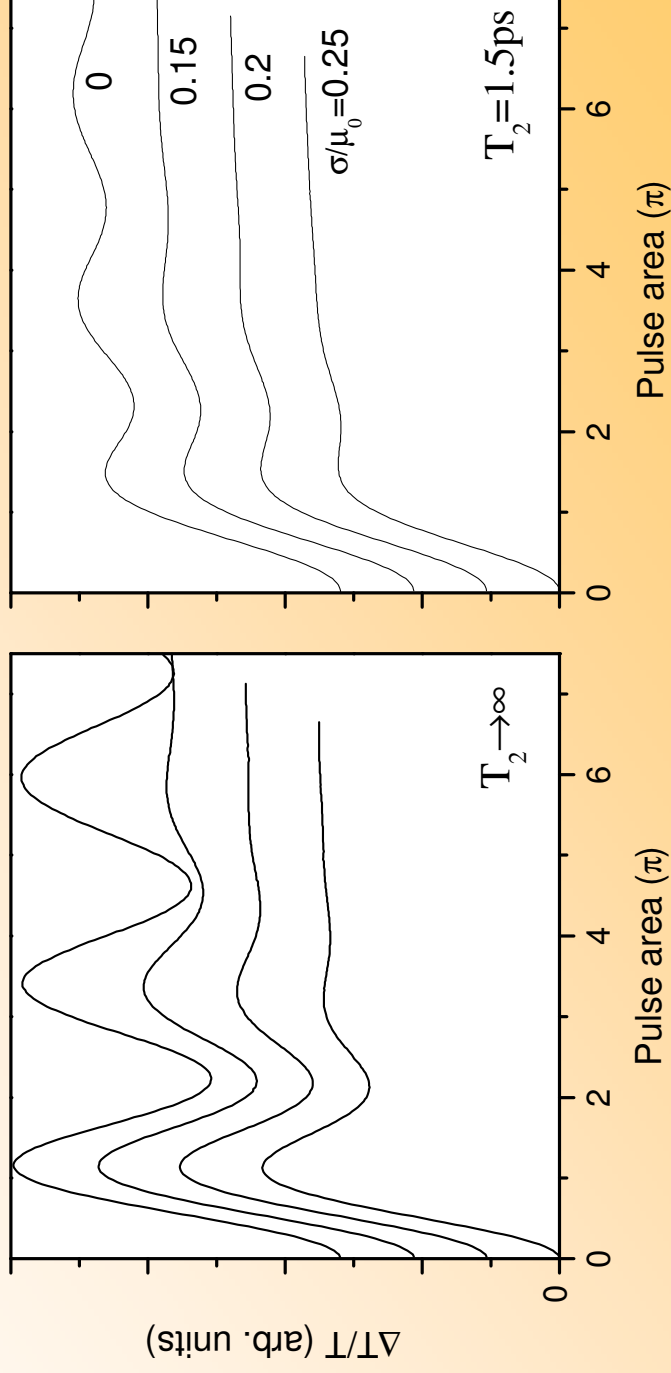
**Many oscillation periods are present**, even when the averaging over the inhomogeneous broadening and the spatial mode profile are included.

# Distribution of transition dipole moments

$$\frac{\Delta T}{T} \propto \int_{-\infty}^{+\infty} d\omega |E_{probe}(\omega)|^2 \int_0^{+\infty} \Psi_{TE_0}^2(r) dr \int_{-\infty}^{+\infty} P(\mu) d\mu \int_{-\infty}^{+\infty} \alpha(\omega, \omega_\xi, \mu, \theta(E_{pump}, \mu)) f(\omega_\xi) d\omega_\xi$$

$$P(\mu)\mu^2 = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\mu-\mu_0)^2}{2\sigma^2}}$$

$\alpha \propto \sigma_0 \propto \mu^2$

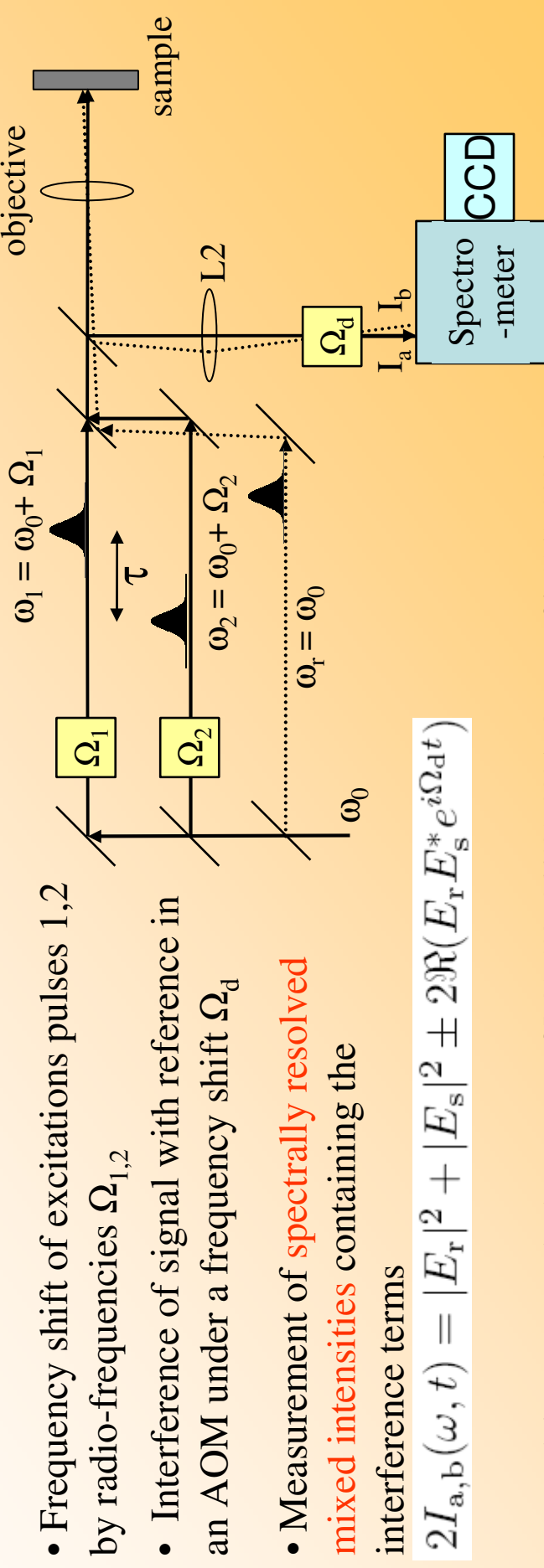


# How to measure FWM from individual localized states ?

States are typically much smaller than light wavelength : **k is undefined**  
 FWM consist of several sharp resonances: Single channel detection is inefficient  
 Solution:

- Use time-invariance for FWM selection
- Upgrade the balanced detection to multichannel

## Heterodyne spectral interferometry (HSI)



W.Langbein, B.Patton et al., Phys. Rev. Lett. **95**, 017403 ; 266401 (2005), Opt. Lett. **31**, 1151 (2006)



# How to measure FWM from individual localized states ?

- Balanced measurement of interference

$$I_d(\omega) = I_a - I_b = 2 \int_0^T \Re(E_r E_s^* e^{i\Omega_d t}) dt$$

filters the **signal of frequency-shift  $\Omega_d$**  with a **bandwidth  $4/T$**

Choice of  $\Omega_d$  determines measured field: for  $\Omega_d = \Omega_{1,2}$  Pulse 1,2, for  $\Omega_d = 2\Omega_2 - \Omega_1$  FWM

- Retrieval of signal field in **phase and amplitude** using **spectral interferometry**

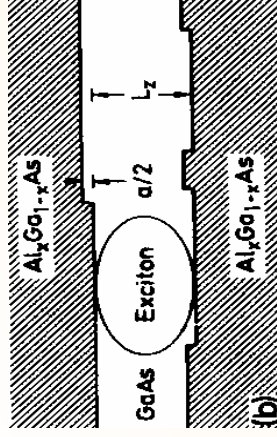
$$F(\Theta(t) F^{-1}(I_d(\omega))) = E_r^*(\omega) E_s(\omega) e^{i\Omega_d t}$$

- Balanced detection eliminates classical reference noise
- Multichannel detection enables measurement of all signal components at once

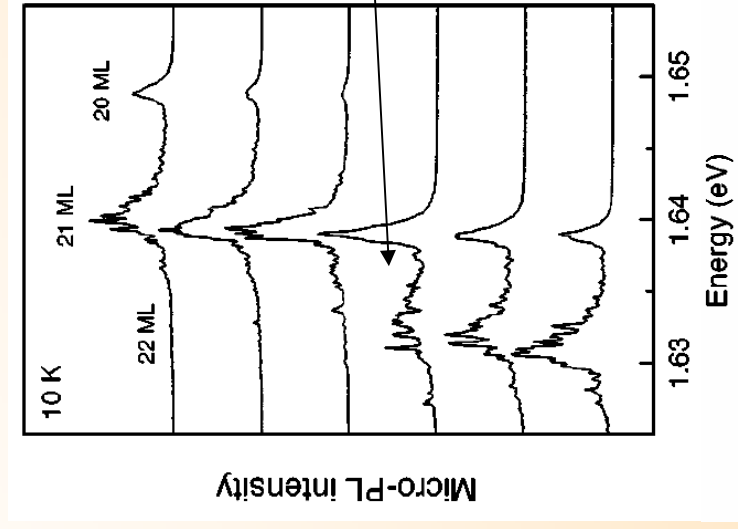
**This setup offers the best possible detection sensitivity and background suppression**

W.Langbein, B.Patton et al., Phys. Rev. Lett. **95**, 017403 ; 266401 (2005), Opt. Lett. **31**, 1151 (2006)

# Localized Excitons in GaAs quantum wells



Large islands form on the ALAs and GaAs surfaces when a **growth interruption** is applied. Excitons can be localized in these islands

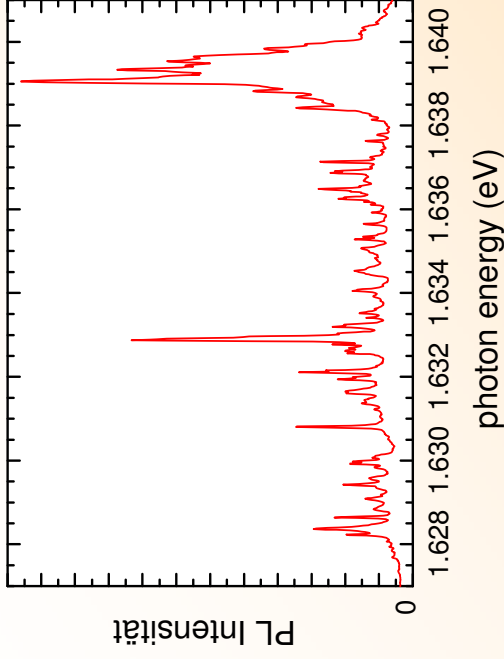


Thickness of the QW determines the spatial distribution of the localization potential due to the monolayer islands

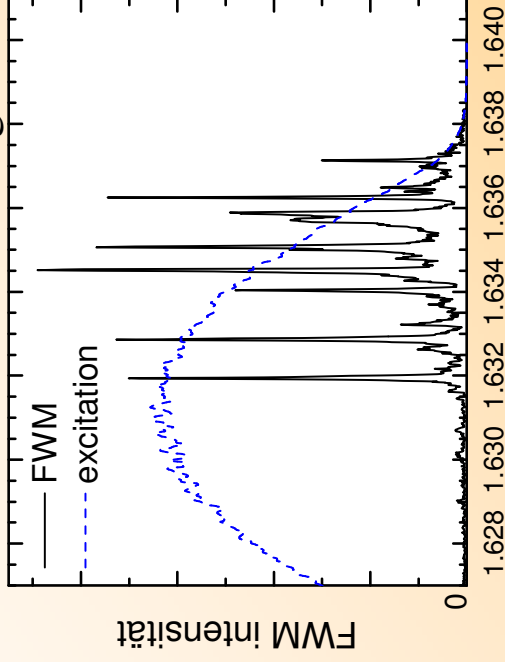
At the onset of a new monolayer, **few and small islands** are present, creating a small density of localized exciton states

# Spectrally resolved FWM from a few-state ensemble

PL of a  $(0.5\mu\text{m})^2$  region



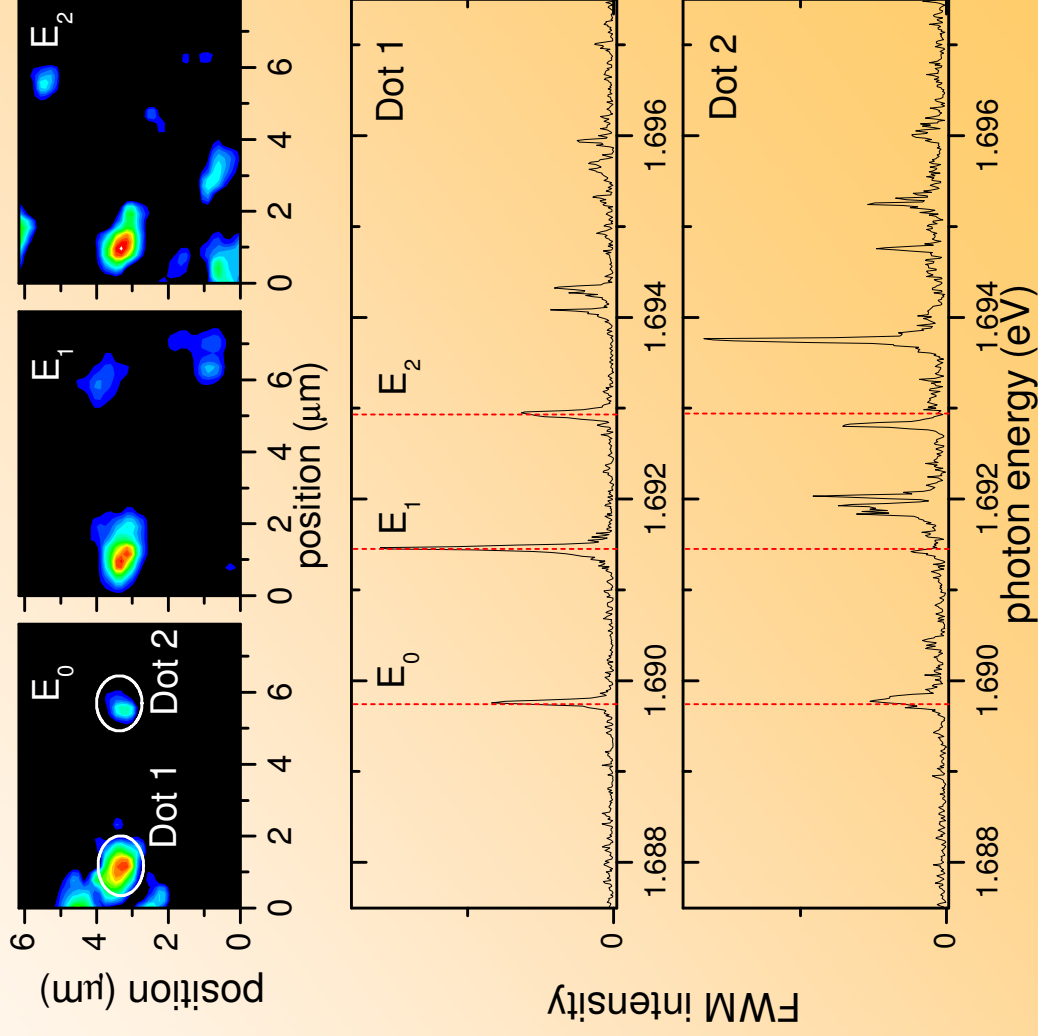
FWM of the same region



- Selection of the investigated region using a **confocal** excitation/detection (500nm resolution)
- Few localized exciton states emit in the range of the larger monolayer thickness
- Exciting laser selects only strongly localized states
- FWM intensities of states are **not related** to the PL intensities: **Phonon-assisted relaxation efficiency** determines the PL while the **oscillator strength** determines the FWM intensities & higher spatial resolution in FWM ( $\sim I^4$ )
- Narrow linewidth of single states (20-50 $\mu\text{eV}$  FWHM)

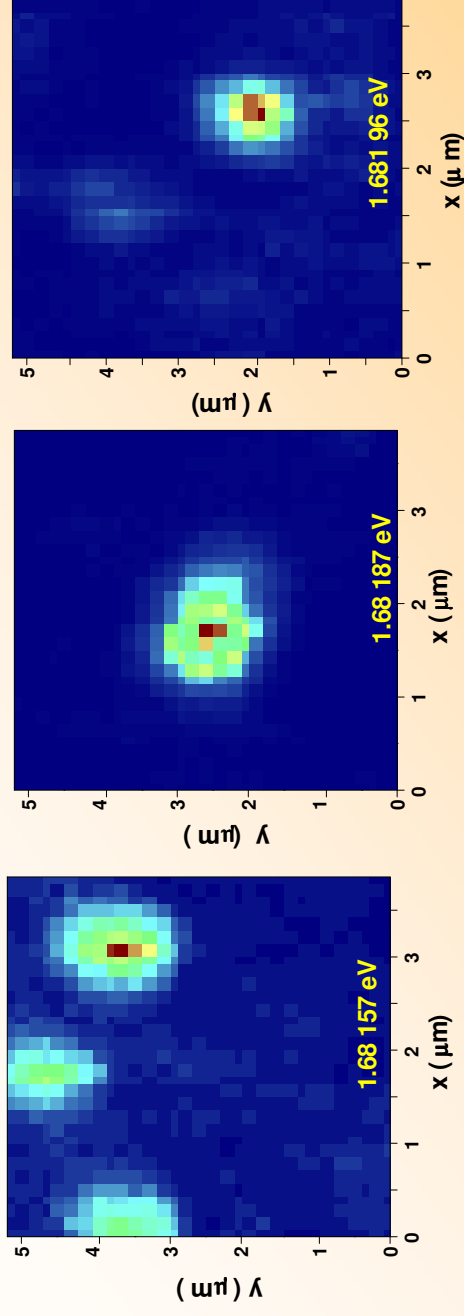
Imaging of non-linear response  
by scanning the optical focus  
over the sample surface:

Identification of spatial  
positions and distances of the  
excitonic states with 100nm  
accuracy

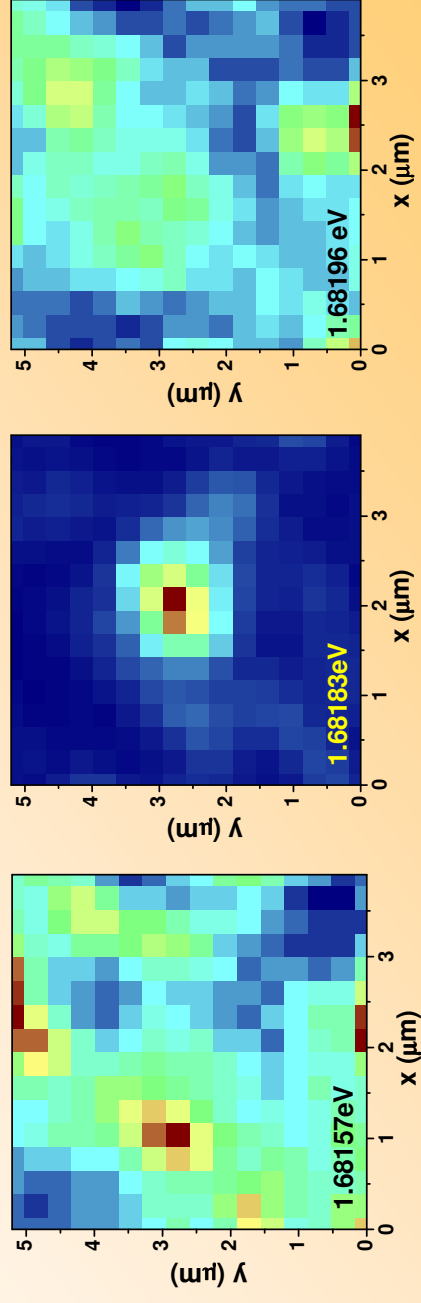


# XY mapping: FWM vs. PL

FWM



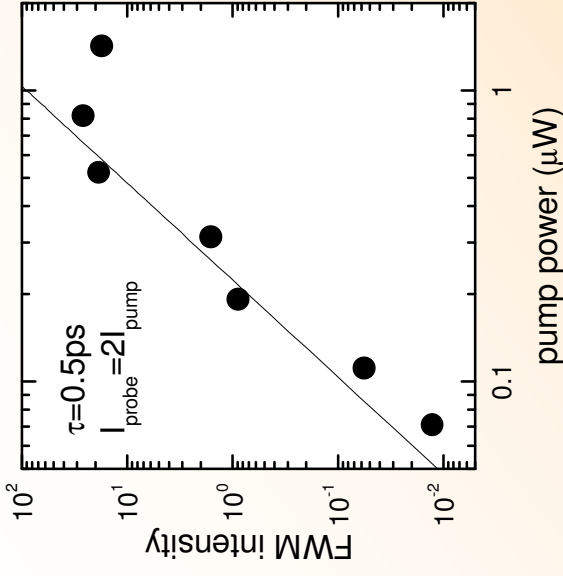
PL



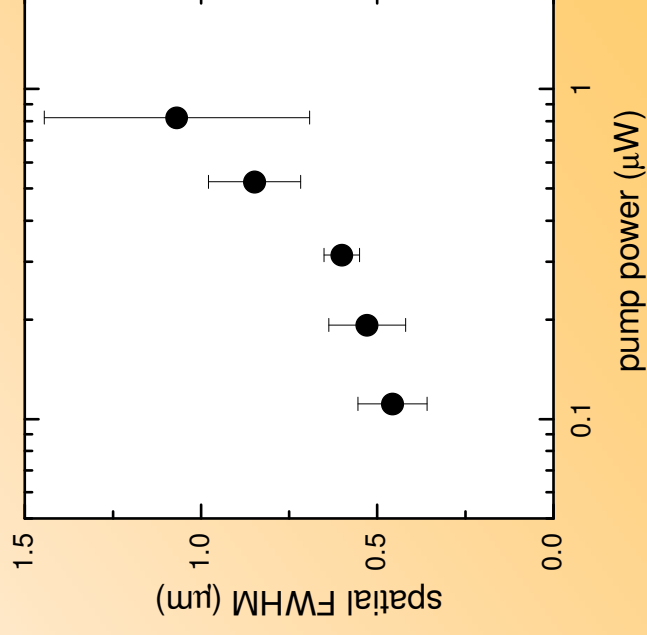
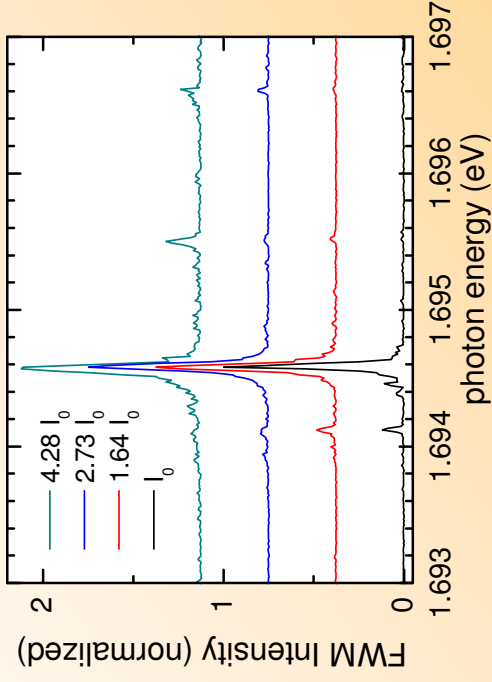
PL and FWM show mostly different states – as in spectral comparison

QDs in 5 nm QWs,  $T=7\text{K}$

# Intensity-scaling: third-order regime and above



- Detected FWM intensity scales with the third power of excitation intensity up to saturation
- FWM spectra do not change significantly
- Spatial resolution is improved by the nonlinearity, but deteriorates once saturation is reached



# Simulation of photon echo in small ensembles

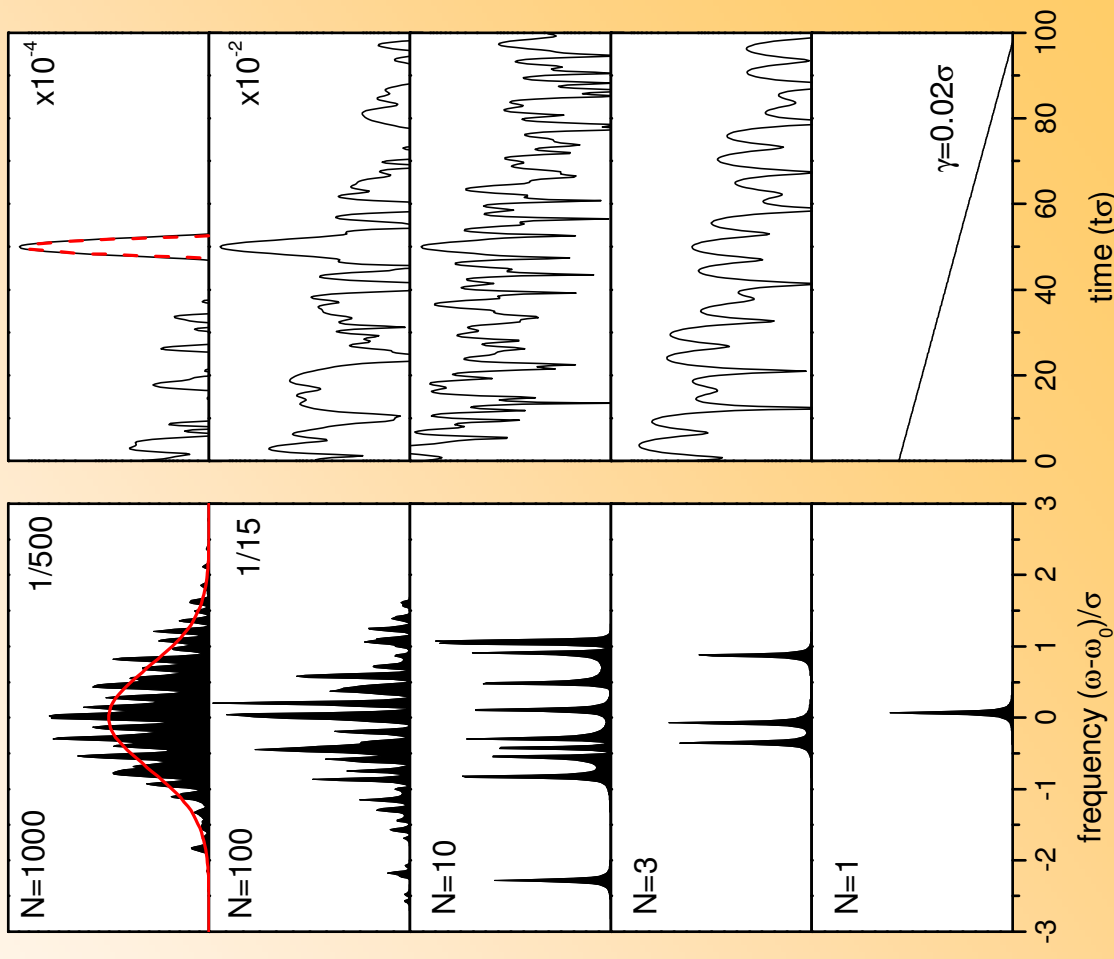
Photon echo formation of finite ensembles (size  $N$ ) of frequency variance  $\sigma$  for  $\tau\sigma=50$

$$E^{(3)}(t, \tau) = \sum_{k=1}^N \mu_k^4 E_1(\omega_k) E_2^2(\omega_k) e^{i\omega_k(t-\tau)} - \gamma_k(t+\tau)$$

In phase (e.g. at  $t=\tau$ ):  $E_{sum} = NE_0$

random phases:  $E_{sum} \approx \sqrt{N}E_0$

FWM intensity (arb. units)

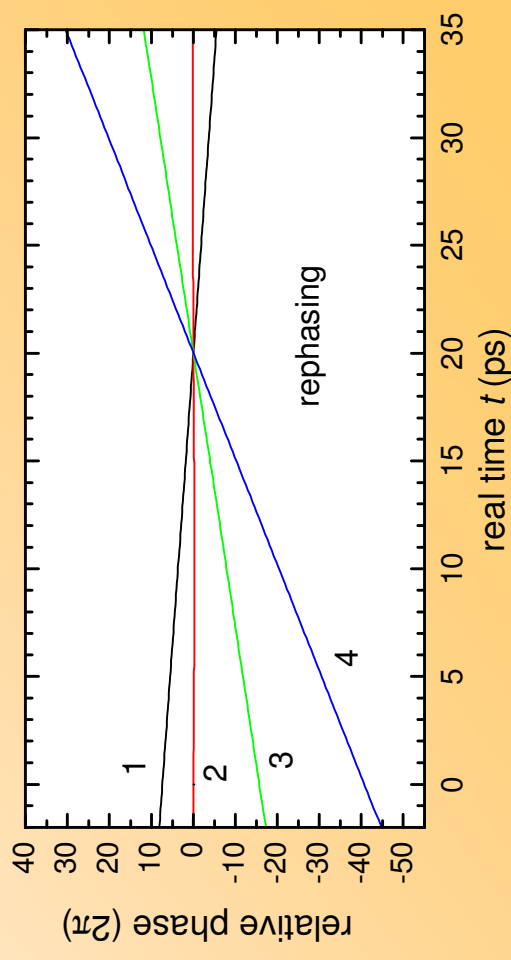
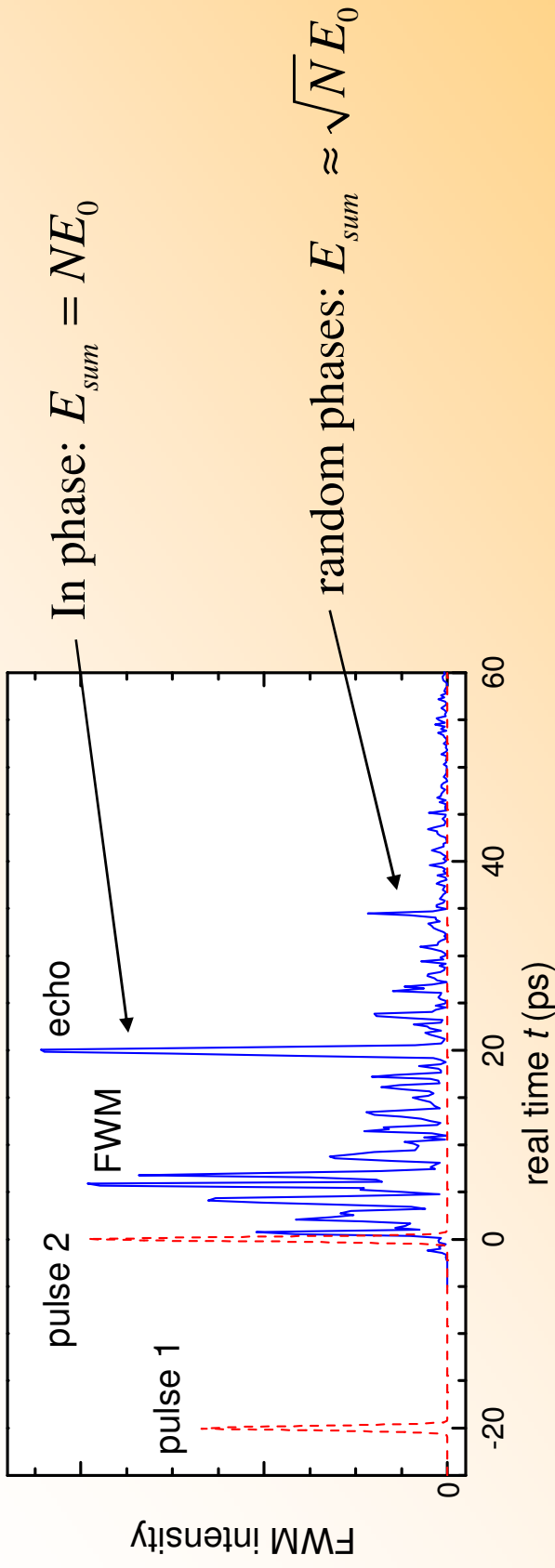


Intensity enhancement by a factor of  
 $N$  by superradiance



# Time-resolved FWM from a few-state ensemble

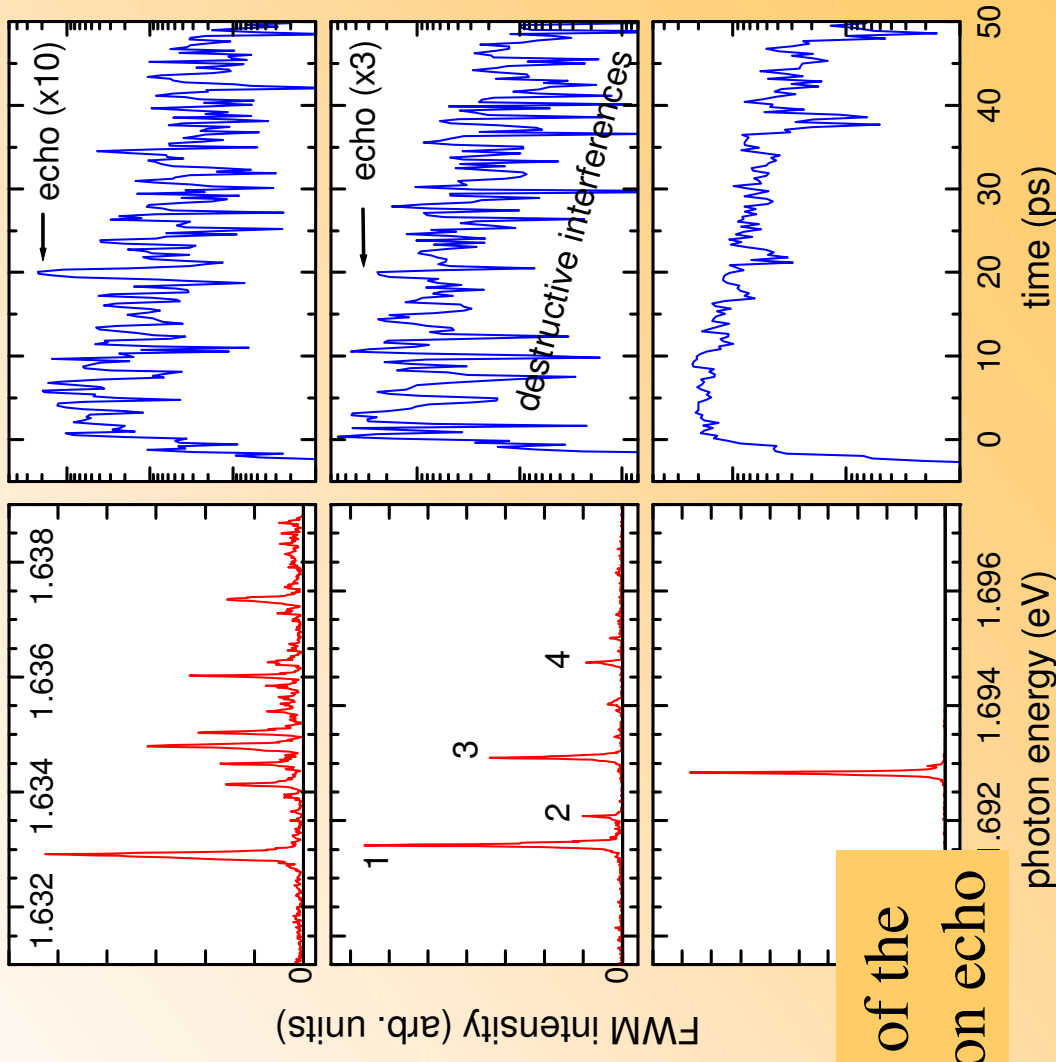
Photon echo formation of a finite ensemble at  $\tau=20$ ps



Measured phase evolution of  
the 4 contributing states:  
Direct observation of rephasing

# Measured echo formation versus N

- Select **ensemble size** using different sample positions
- Count **number of contributing states** in spectrally resolved FWM
- Observe **photon echo formation** in time-resolved FWM

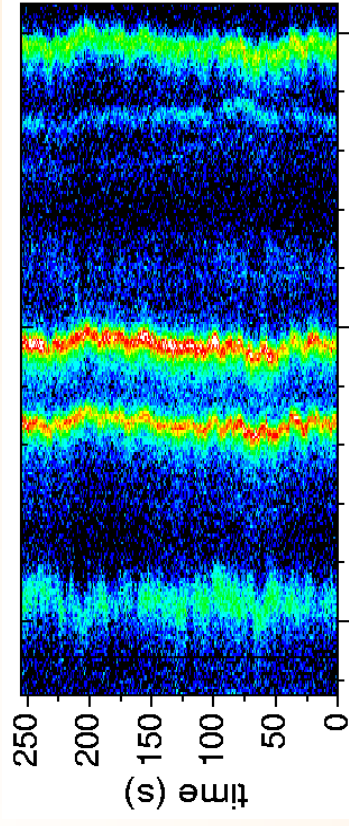


Experimental demonstration of the microscopic origin of a photon echo

W.Langbein and B.Patton, Phys. Rev. Lett. **95**, 017403 (2005)

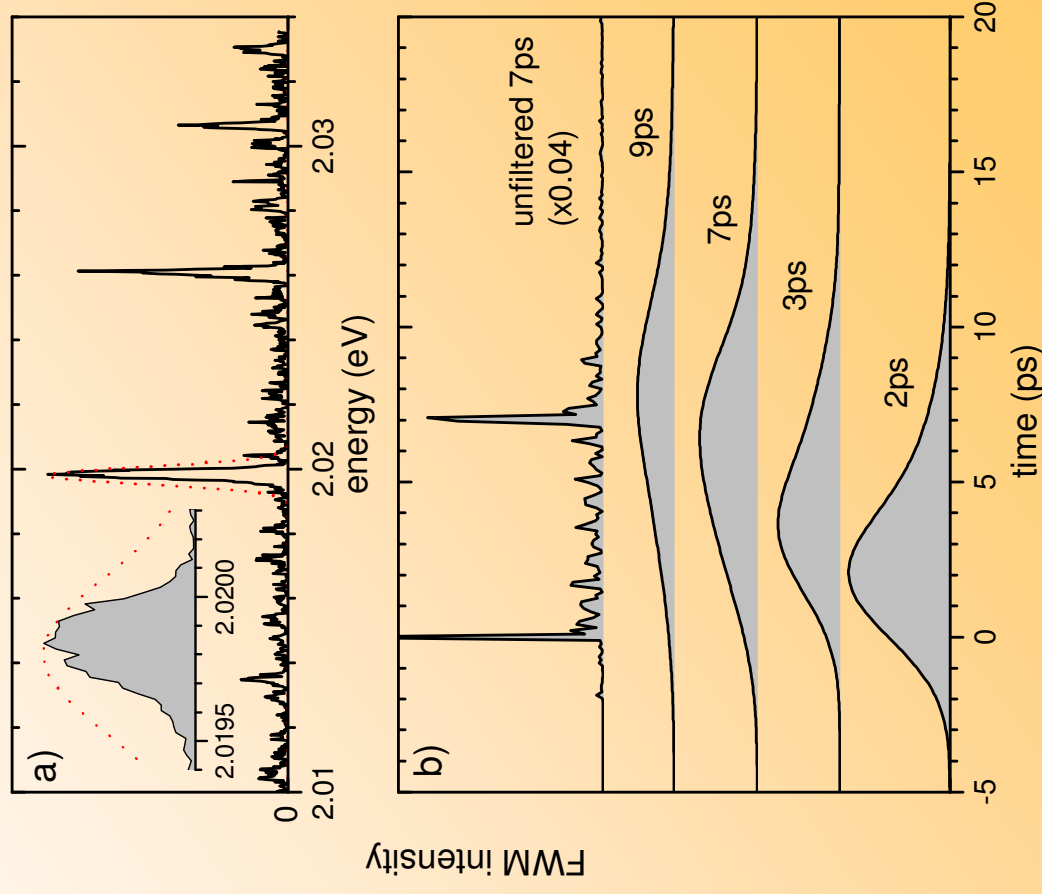
# Photon-echo from a single transition

Also a single exciton resonance can exhibit inhomogeneous broadening by slow spectral diffusion:



Time-ensemble of eigenenergies

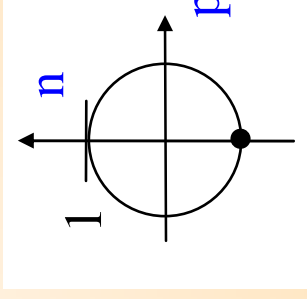
Measurements on epitaxial CdTe/ZnTe quantum dots (grown in the group of H. Mariette, Grenoble)



B.Patton et al, submitted to Phys. Rev. B

# Rabi Oscillations

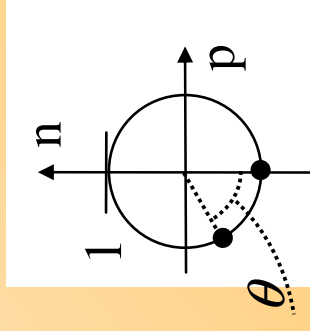
Represent the state-vector of a **two level** system as a position on a unitary sphere:  
**Bloch Sphere**



**n**: Population Inversion  
**p**: Polarisation

For resonant excitation, the rotation angle  $\theta$  (also called pulse area) is proportional to electric field  $\epsilon$  and transition dipole moment  $\mu$

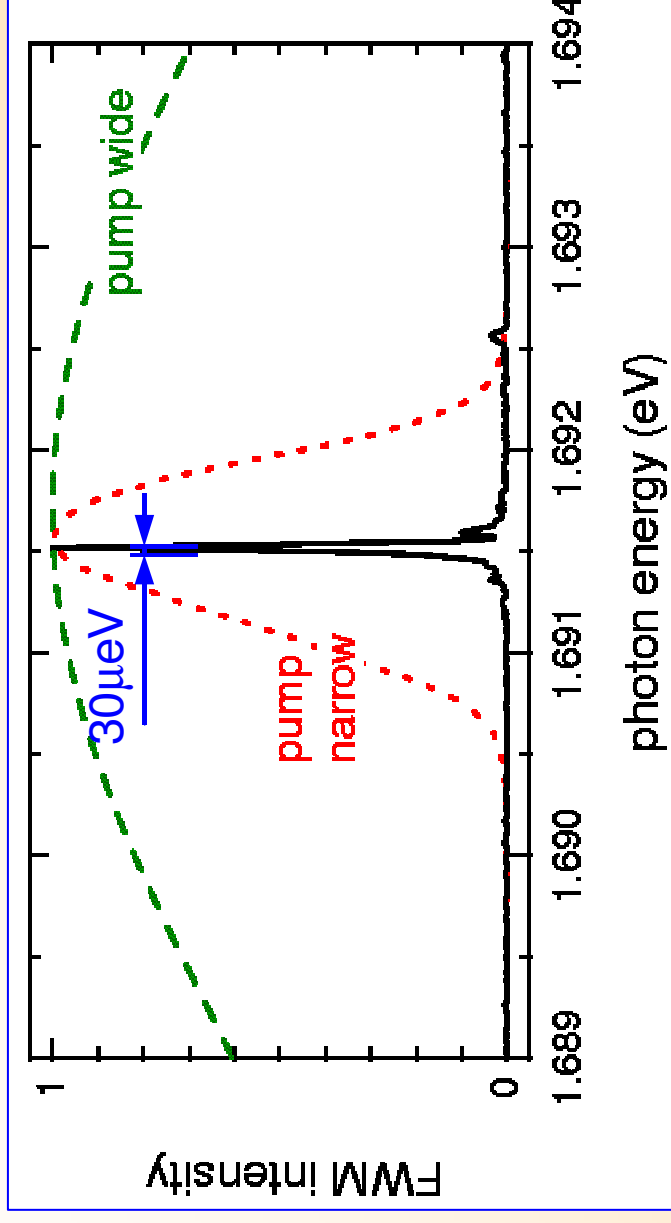
$$\theta = \frac{1}{\hbar} \int_{-\infty}^{\infty} \mu \epsilon(t) dt$$



FWM measures the projection of the state vector onto the **polarisation-plane**:  
Rabi oscillations have period of  $\pi$

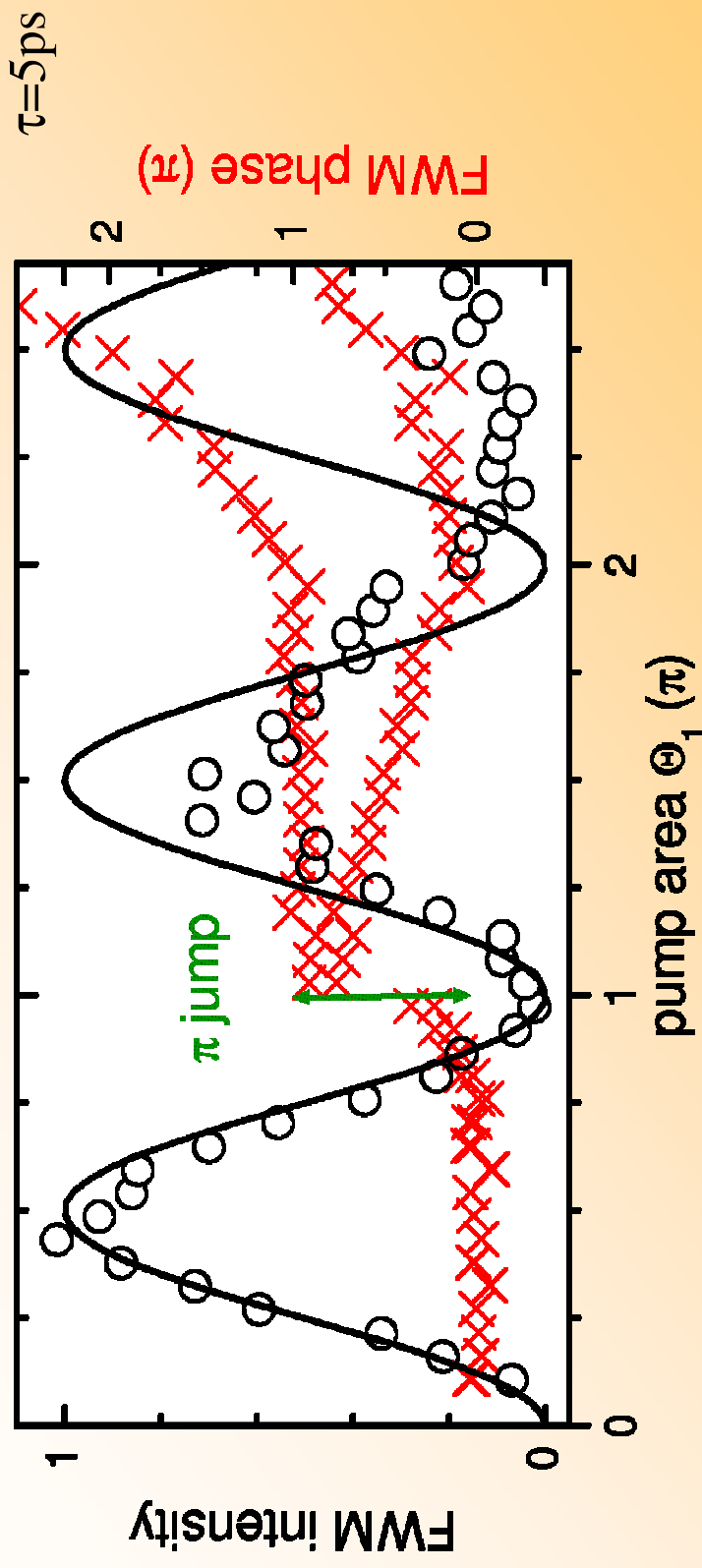
# Selection of an individual transition

$\tau=30\text{ps}$



- Identify region with isolated exciton transition using a spectrally wide pump
- Narrow spectrum down centered to transition.

# Rabi oscillations of a single transition



- Observed Rabi oscillations in the FWM intensity versus pump area
- **Observed phase** shows  $\pi$ -jump at top of Bloch sphere
- Deviations from ideal two-level system at higher pulse areas

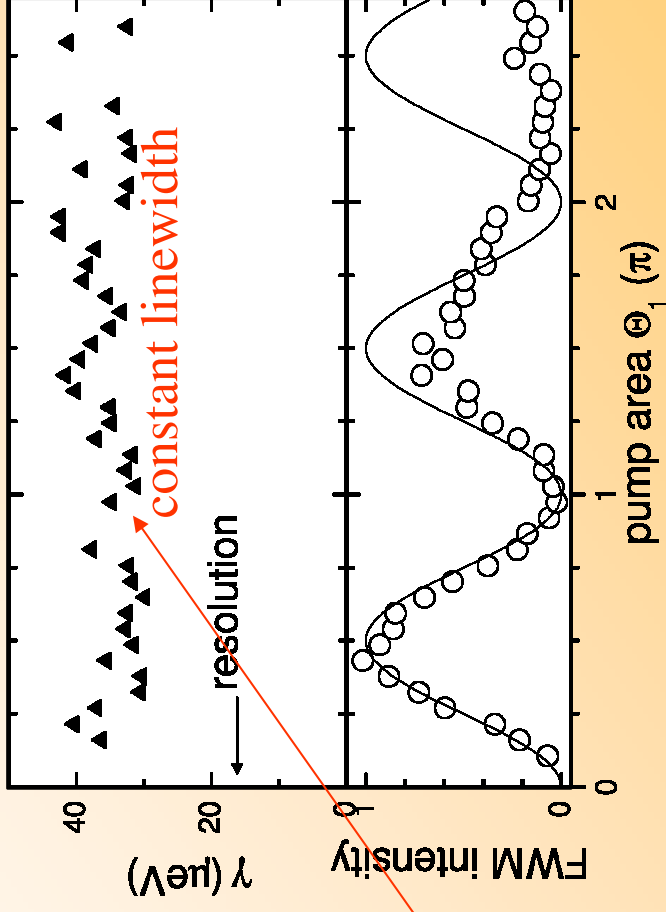
# Damping by Excitation Induced Dephasing ?

Deviations from the expected two-level behaviour are observed at large pulse areas.

**Excitation-Induced Dephasing** has been suggested as a source for such effects

- linewidth of third-order polarisation quantifies EID:

**Negligible EID observed**



**Interpretation: Presence of energetically close multiexcitonic transitions, to which the polarisation is transferred**



# Polarisation Transfer

Interpretation of observed polarization dynamics:

**Nearly resonant multiexcitonic resonances**

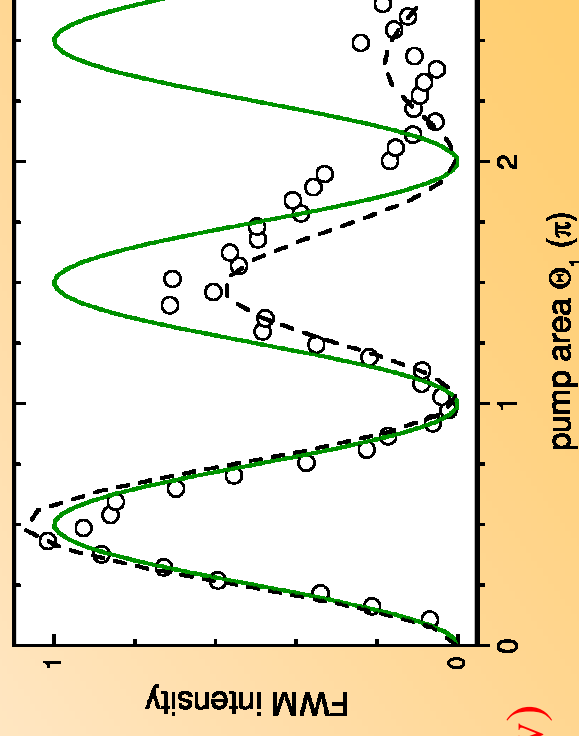
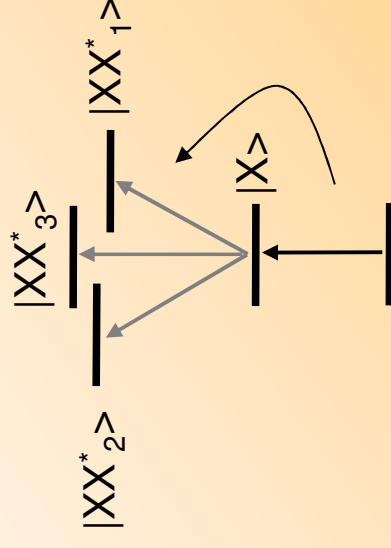
⇒ transferral of polarisation to coherently coupled multiexcitonic states

**Model calculation of FWM response:**

- No dephasing
- 3 nearly resonant biexciton states with -0.5, -0.6, -0.8meV binding energy and
- 30% of the exciton transition dipole moment (i.e. 9% oscillator strength)

**shows qualitative agreement (dashed line)**

**Biexcitonic FWM to weak to be detected (for now)**

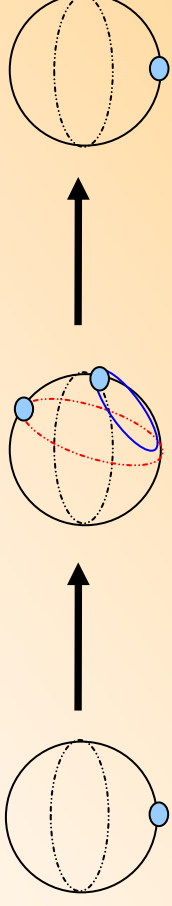


**Instead of EID, discrete non-resonant multiexcitonic states explain the observed polarization decay without line broadening**

# Off-resonant Rabi Oscillations

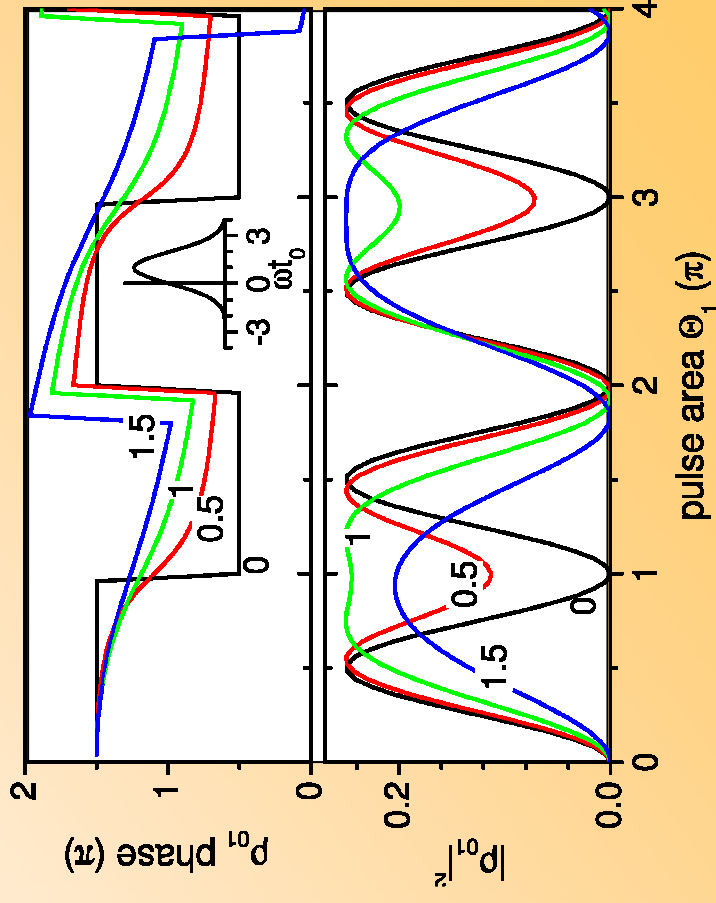
## Spectral detuning between excitation pulses and transition

Precession no longer purely about p-axis



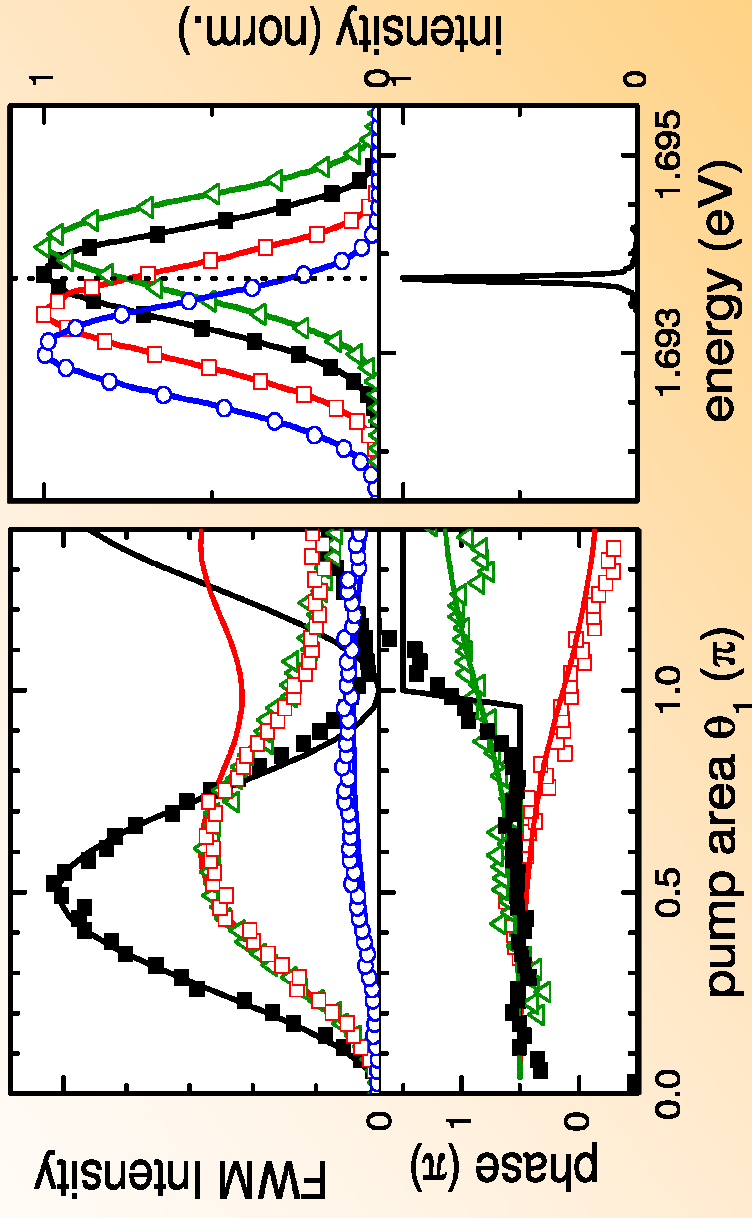
Bloch-sphere picture:

Numerically calculated polarization for a two-level system (no dephasing)



# Off-resonant Rabi oscillations: Experiment

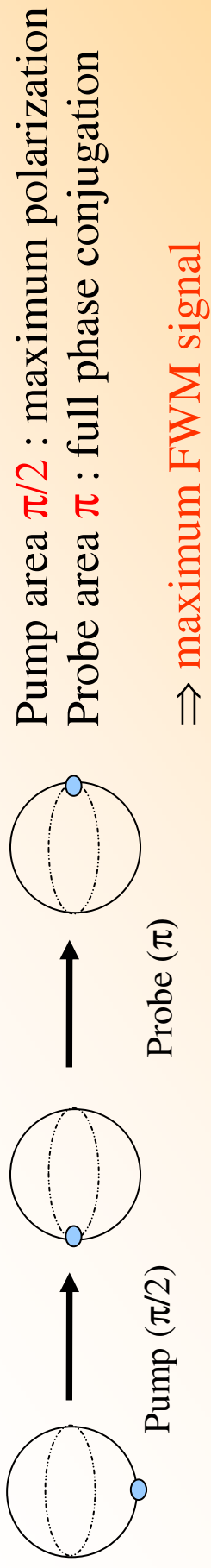
Measured FWM intensity and phase for different excitation detunings



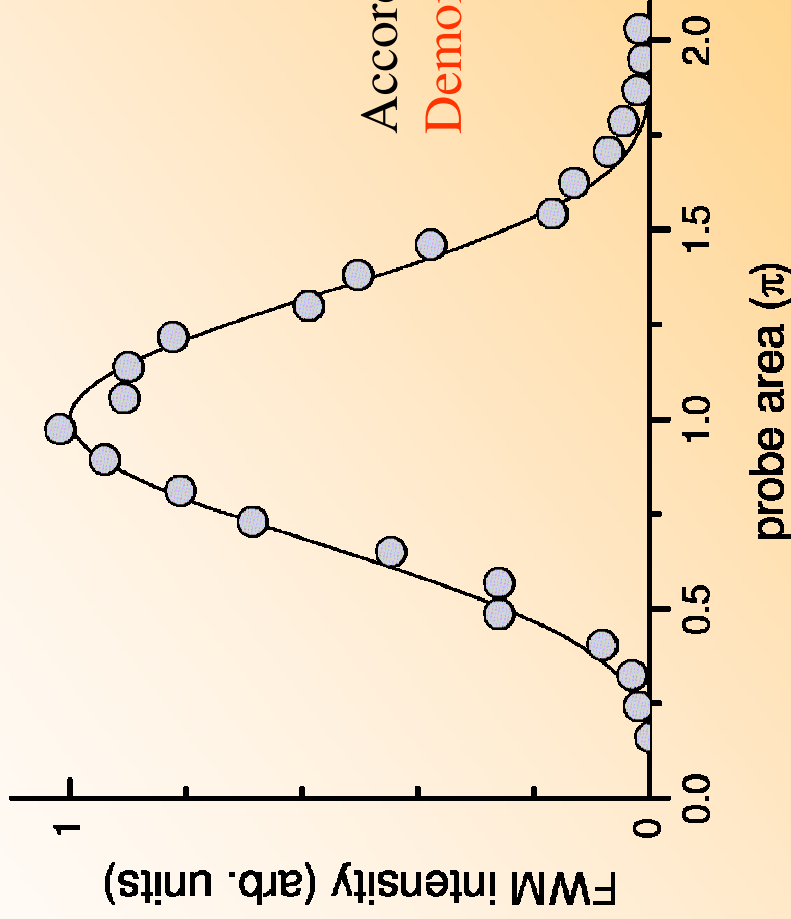
- FWM Intensity & Phase evolution in agreement with two-level calculation (lines)
- Deviations for large pulse areas, as discussed previously

Phase control via detuning demonstrated

# Coherent manipulation in FWM



$$\vec{P}^{(3)} \propto \chi^{(3)} \vec{E}_2 \vec{E}_2 \vec{E}_1^*$$

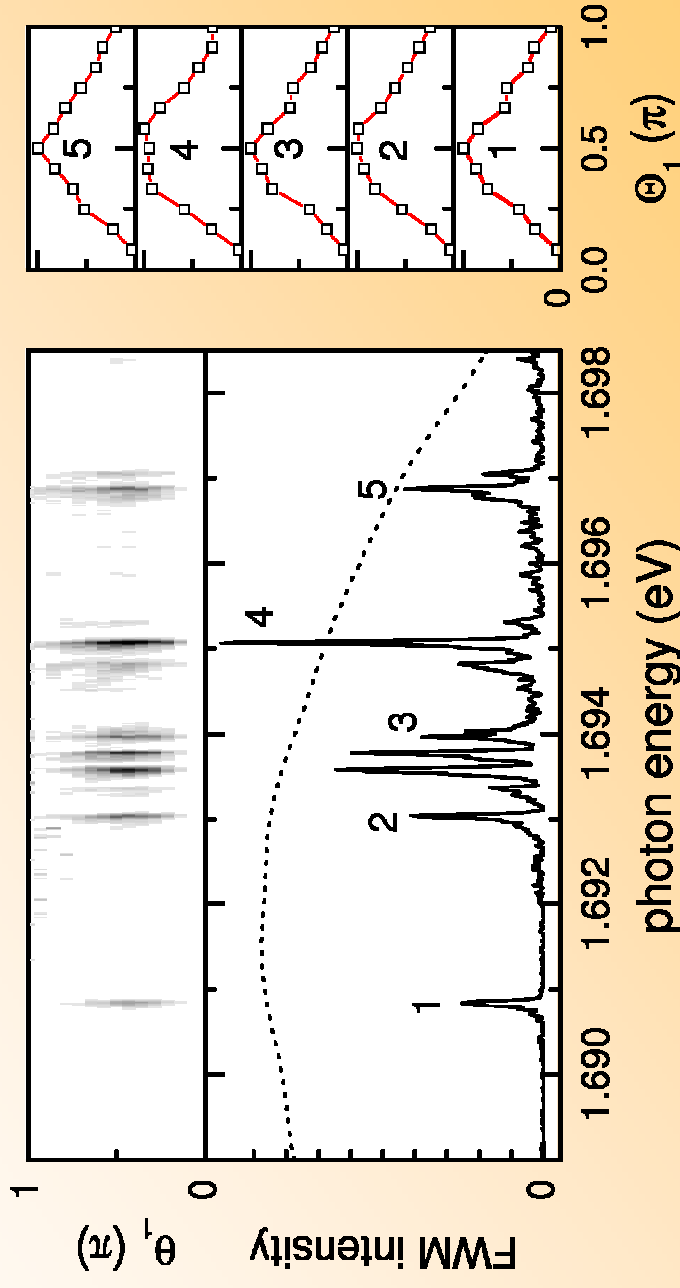


Analogous to  $\pi/2$  -  $\pi$  spin flip in NMR

According response observed in experiment:  
**Demonstration of two pulse coherent control**

# Multi-state Rabi oscillations

- Excitation of a **small exciton ensemble** by spectrally wide pulses:
- Observation of Rabi-Oscillations for all resonances
  - Coherent coupling between resonances is weak (inferred from 2D FWM)
  - Since the FWM intensity  $\sim \mu^8$ , different FWM intensities result from only small differences in the dipole moment  $\mu$  and thus in the pulse area



This observation shows the possibility of a simultaneous application of a gate operation on a set of two-level systems

# Two-dimensional spectroscopy

$\vec{P}^{(3)}(t, \tau)$  Measured by HSI

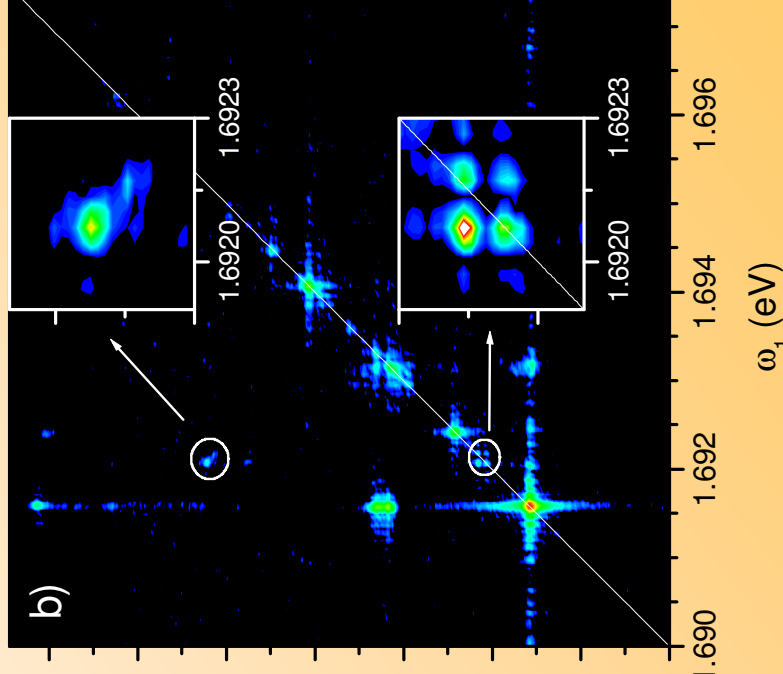
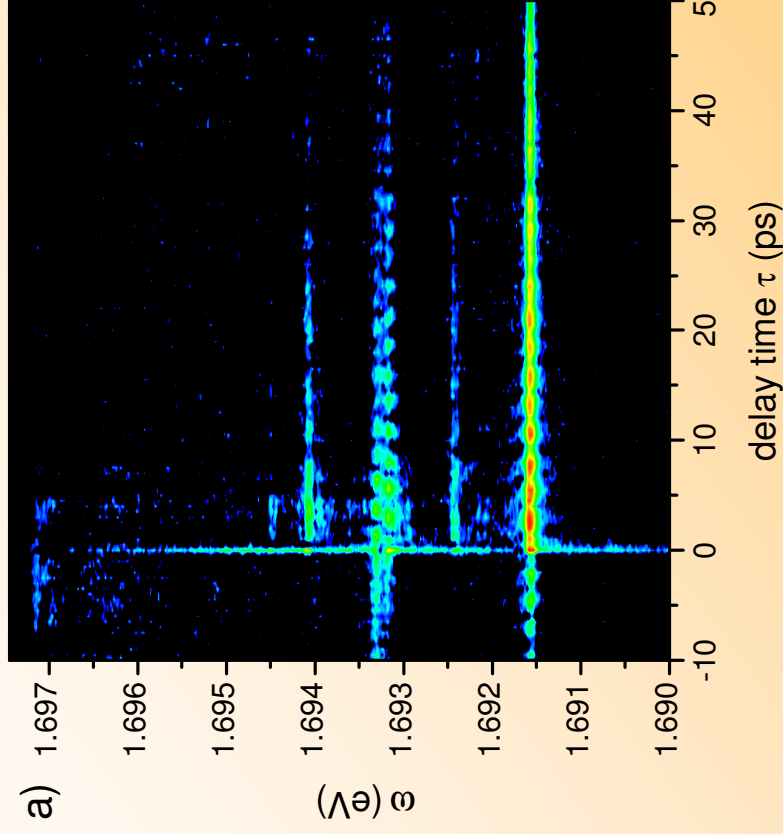
$$\vec{P}^{(3)}(\omega, \tau) = \int e^{i\omega t} \vec{P}^{(3)}(t, \tau) dt$$

$$I(\omega, \tau) = |\vec{P}^{(3)}(\omega, \tau)|^2$$

Transform delay time to  
first-order frequency  $\omega_1$

$$\vec{P}^{(3)}(\omega, \omega_1) = \int e^{i\omega_1 \tau} \vec{P}^{(3)}(\omega, \tau) d\tau$$

$$I(\omega, \omega_1) = |\vec{P}^{(3)}(\omega, \omega_1)|^2$$



# Conceptually similar to 2-dimensional nuclear magnetic resonance (NMR)

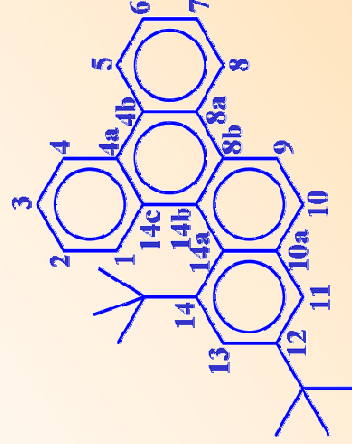
“The invention of multidimensional spectra was the major leap in NMR spectroscopy apart from the introduction of FT-NMR. Both techniques were acknowledged by a nobel prize”

- Aromatic part of the 2D COSY spectrum:
- protons at different positions in the molecule have different chemical shift
  - coherent coupling between protons  $\sim$  distance<sup>-6</sup> (magnetic dipole-dipole coupling)

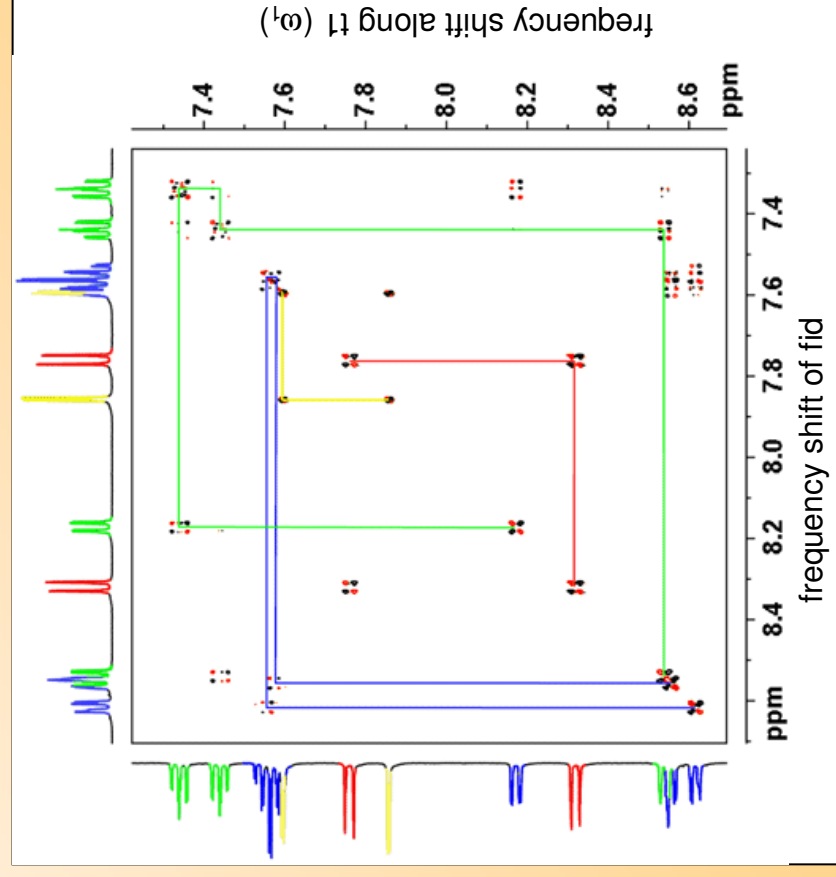
## Pulse sequence for gradient DQF-COSY



Structure of [12,14-dibutylbenzo\[g\]chrysene](http://dx.doi.org/10.1002/anie.201101010)



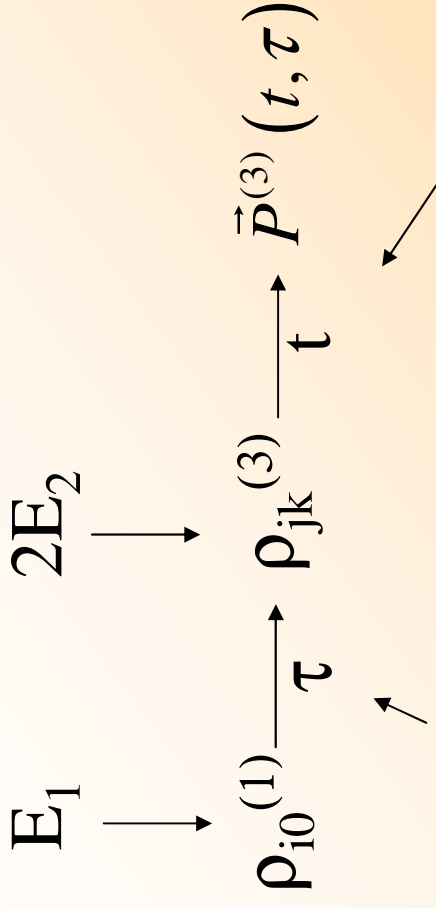
**blue** group of four protons is connected in the order 8.62 ppm to 7.55 to 7.59 to 8.56  
**green** group of four protons in the order 8.54 to 7.34 to 7.44 to 8.17  
**red** group or two protons, that correspond to H9 and 10 because they are the only group of two protons expected to have a three-bond coupling constant (8.9 Hz), are at 7.76 and 8.32 ppm.  
**yellow** group of two protons correspond to H11 and 13 because the coupling constant is small (1.9 Hz) and consistent with a four bond correlation.



<http://dx.ch.huji.ac.il/nmr/techniques/2d/cosy/cosy.html>



# Two-dimensional spectroscopy



Propagation of first order polarization

Propagation of third order polarization

$$\rho_{i0}^{(1)} \propto \exp((i\tilde{\omega}_{i0} - \gamma_i)\tau) \quad \rho_{jk}^{(3)} \propto \exp((i\tilde{\omega}_{jk} - \gamma_{jk})t)$$

neglect damping  $\gamma$  for the moment

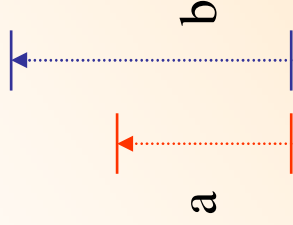
$$\vec{P}^{(3)}(\omega, \omega_1) = \sum_{i>0} \mu_{i0} \delta(\omega_1 + \omega_{i0}) \sum_{j>k} \mu_{jk} A_{i0,jk} \delta(\omega - \omega_{jk})$$

$A_{i0,jk}$  : Polarization coupling coefficient for the FWM process

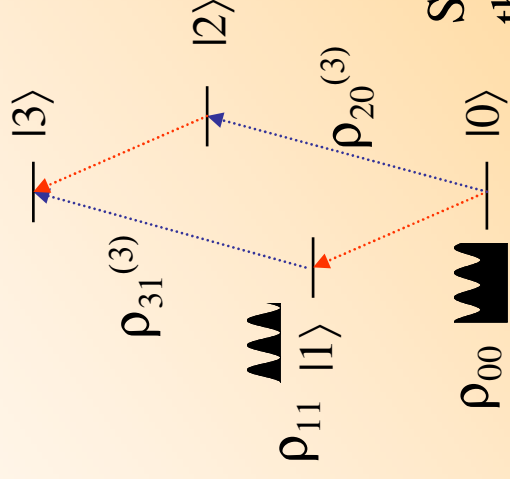
Uncoupled two-level systems:  $A_{i0,jk} = \mu_{i0}\mu_{jk}\delta_{i0,jk}$  is **diagonal**

# Uncoupled two level systems

Two uncoupled two-level systems

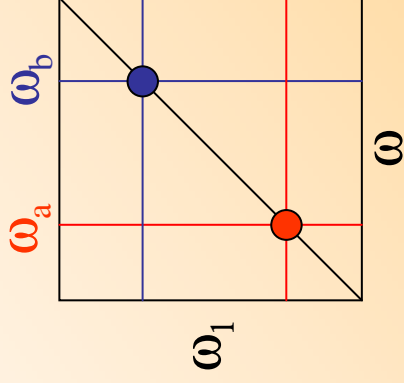


Four-level picture

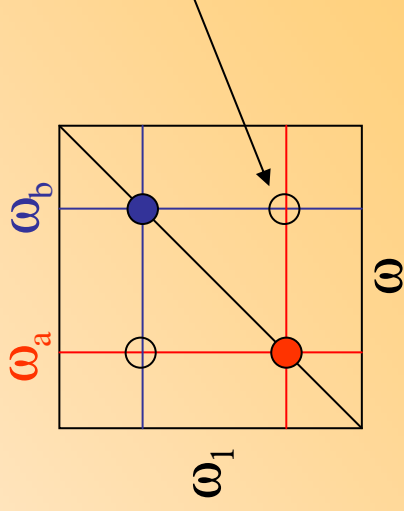


System initially in the ground state

2D FWM signature



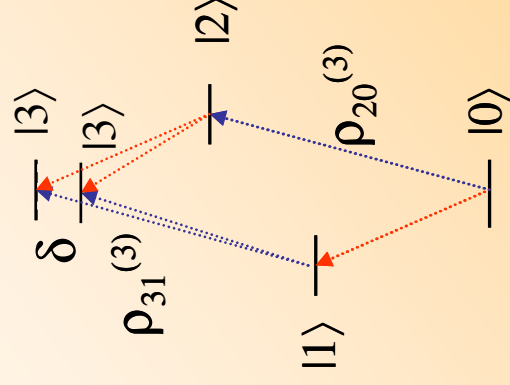
Cancellation of  $\rho_{20}^{(3)}$  and  $\rho_{20}^{(3)}$  (out of phase)



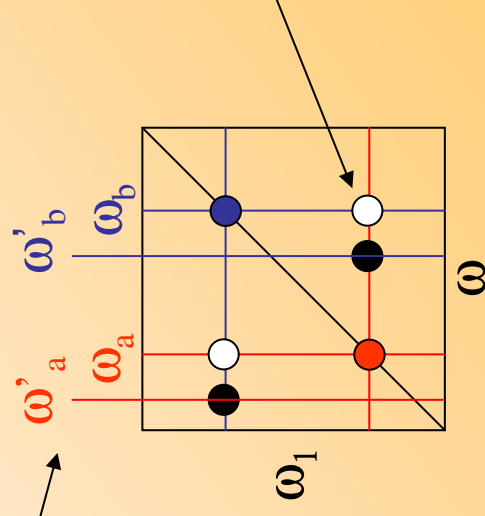
# Coupled two level systems

Renormalization  $\delta$  of two-exciton state  
(Static dipole-dipole, exchange)

$$\omega'_{a,b} = \omega_{a,b} - \delta$$



two-exciton states visible  
only in 3rd order frequencies

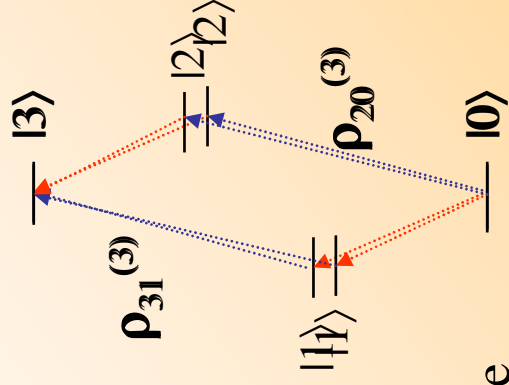


2D FWM signature

# Coupled two level systems

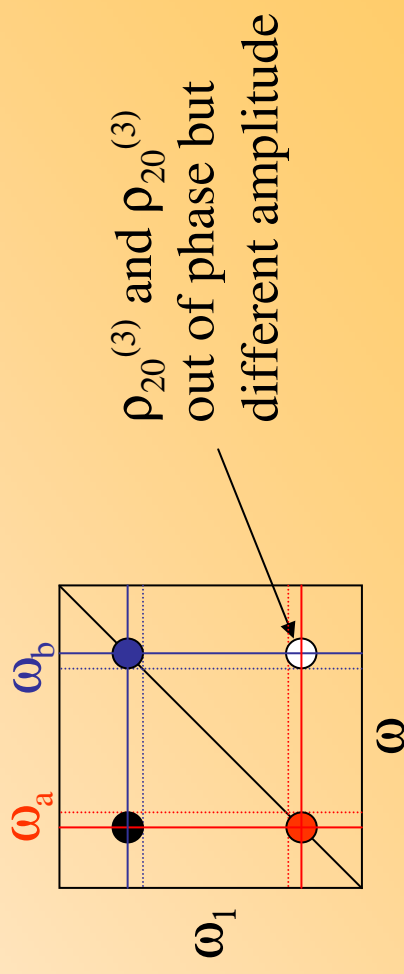
Coupling of exciton states by  $\delta_2$   
(transition dipole, Förster coupling)

$$\omega_{1,2} = \frac{\omega_a + \omega_b}{2} \pm \sqrt{(\omega_a - \omega_b)^2 + \delta_2^2}$$

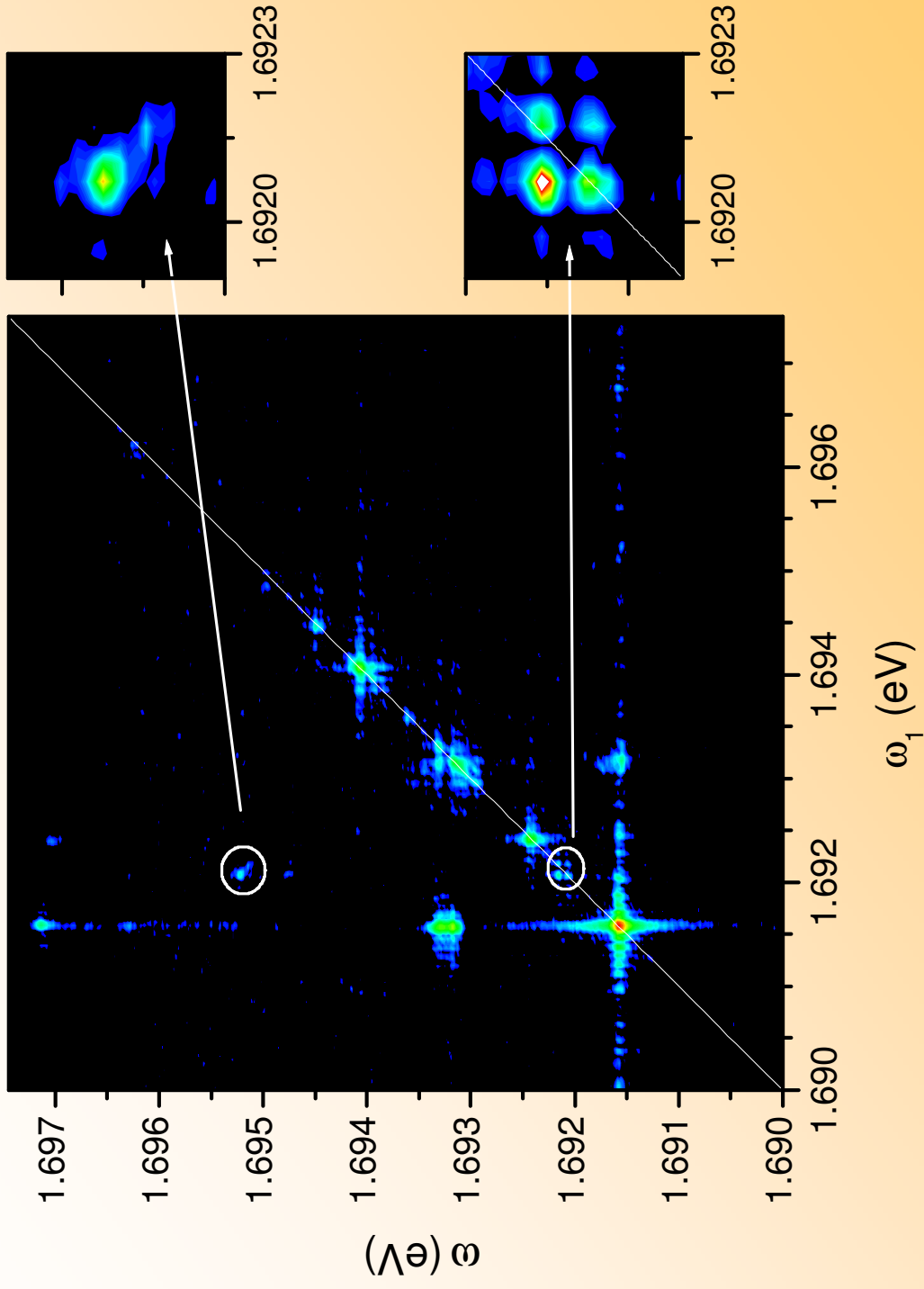


After coupling: different transition dipole moments of the states (super/subradiant superpositions)

2D FWM signature

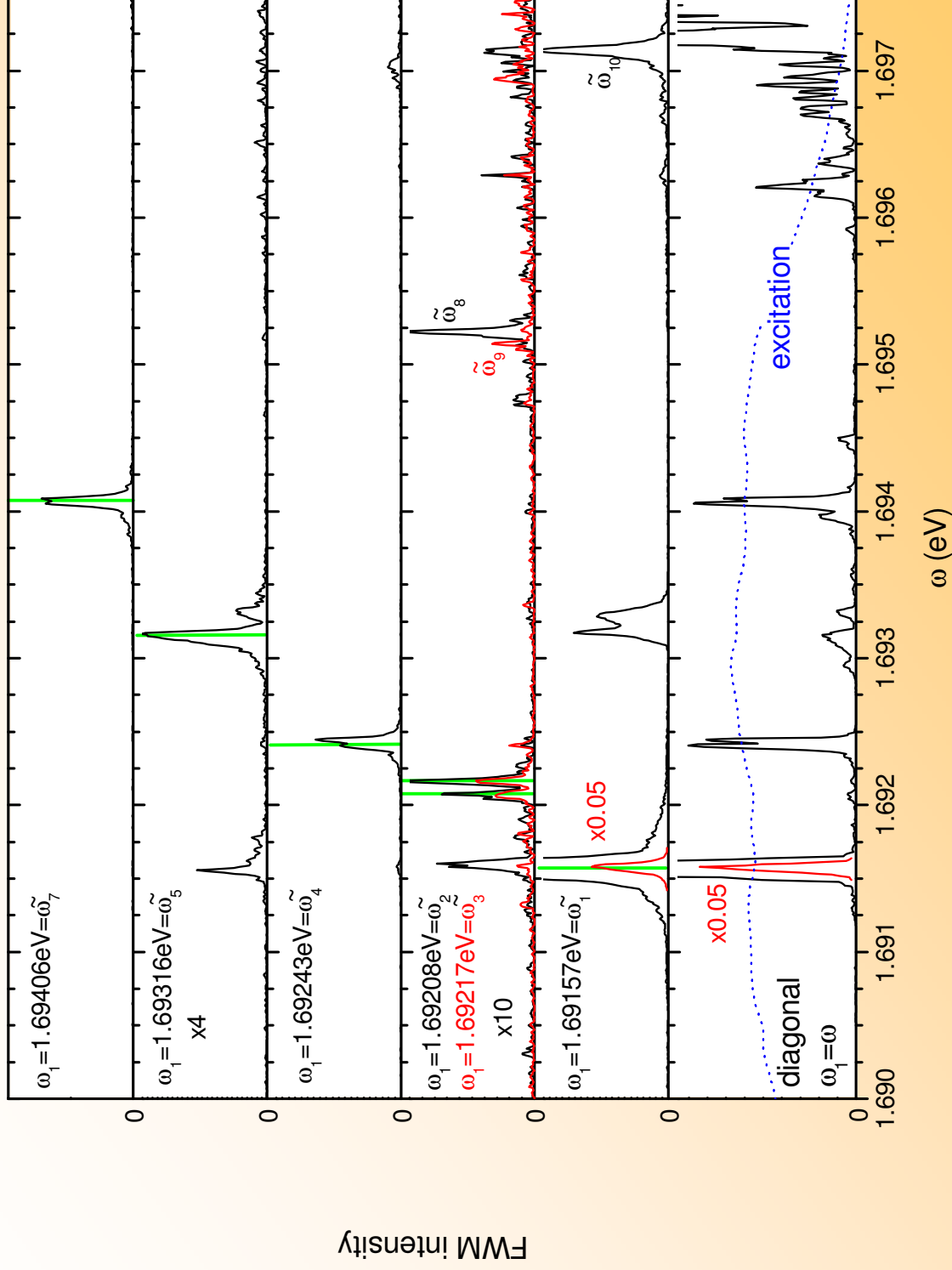


# Experimental data

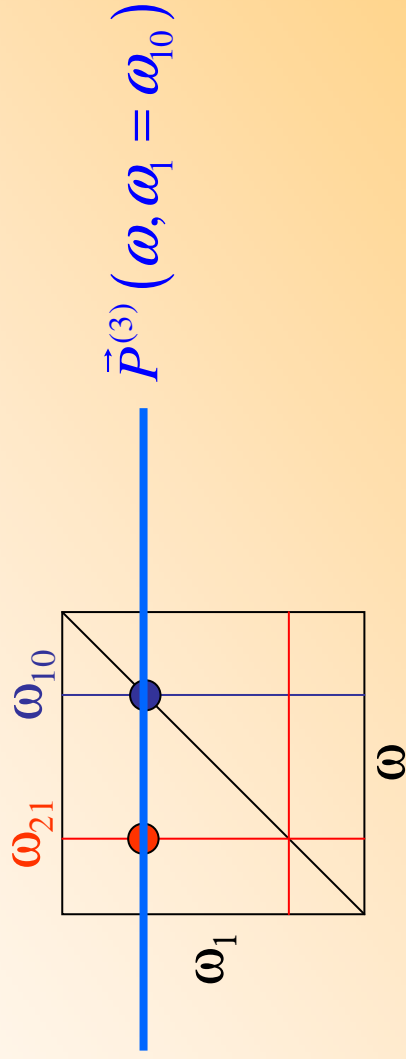
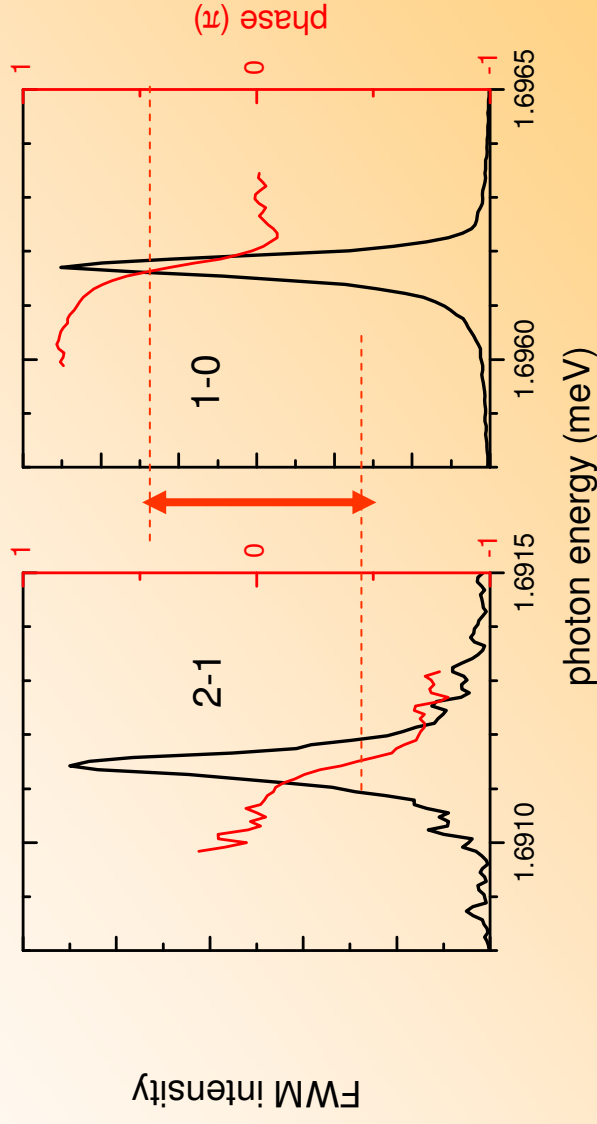
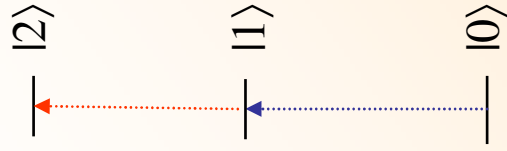


# Analysis of coherent coupling

green lines indicate diagonal position in each plot



# Exciton-Biexciton system



$\pi$  phase shift observed



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