Physics of Condensed Matter I

1100-4INZ`PC

"What do you expect, since 90% of all the scientists who ever lived are alive today?"

Faculty of Physics UW

Jacek.Szczytko@fuw.edu.pl

Dictionary

$$\vec{D} = \varepsilon \vec{E}$$

ε_0 **vacuum permittivity, permittivity of free space** (przenikalność elektryczna próżni)
ε_r relative permittivity (względna przenikalność elektryczna)
$\varepsilon = \varepsilon_0 \varepsilon_r$ permittivity (przenikalność elektryczna)

$$\vec{B} = \mu \vec{H}$$

μ_0 **vacuum permeability, permeability of free space** (przenikalność magnetyczna)

$$\mu_0 = 4\pi \cdot 10^{-7} \text{H/m}$$

μ_r relative permeability (względna przenikalność magnetyczna)
$\mu = \mu_0 \mu_r$ permeability (przenikalność magnetyczna)

magnetic susceptibility $\chi_m = \mu_r - 1$

electric field $\vec{E}$ and the magnetic field $\vec{B}$
displacement field $\vec{D}$ and the magnetizing field $\vec{H}$

Summary –Fermi golden rule

The probability of transition per unit time:

$$W(t) = W \qquad 0 \le t \le \tau$$

$$\boxed{P_{mn} = \frac{w_{mn}}{\tau} = \frac{2\pi}{\hbar}|\langle m|W|n\rangle|^2\delta(E_m - E_n)}$$

Transitions are possible only for states, for which $E_m = E_n$

$$W(t) = w^{\pm}e^{\pm i\omega t} \qquad 0 \le t \le \tau$$

$$\boxed{P_{nm} = \frac{w_{nm}}{\tau} = \frac{2\pi}{\hbar}\left|\langle n|w^{\pm}|m\rangle\right|^2 \delta(E_n - E_m \pm \hbar\omega)}$$

Transitions are possible only for states, for which $E_m = E_n \pm \hbar\omega$

The perturbation in a form of an electromagnetic wave:

$$\boxed{A_{nm} = \frac{{\omega_{nm}}^3 e^2}{3\pi\varepsilon_0\hbar c^3}|\langle m|\vec{r}|n\rangle|^2 = \frac{4\alpha}{3}\frac{{\omega_{nm}}^3}{c^2}|\langle m|\vec{r}|n\rangle|^2}$$

$$\boxed{P_{nm} = A_{nm}\delta(E_n - E_m \pm \hbar\omega)}$$

Summary –Fermi golden rule

The transition rate – the probability of transition per unit time – from the initial state $|i\rangle$ to final $|f\rangle$ is given by:

Szybkość zmian – czyli prawdopodobieństwo przejścia na jednostkę czasu – ze stanu początkowego $|i\rangle$ do końcowego $|f\rangle$ dane jest wzorem:

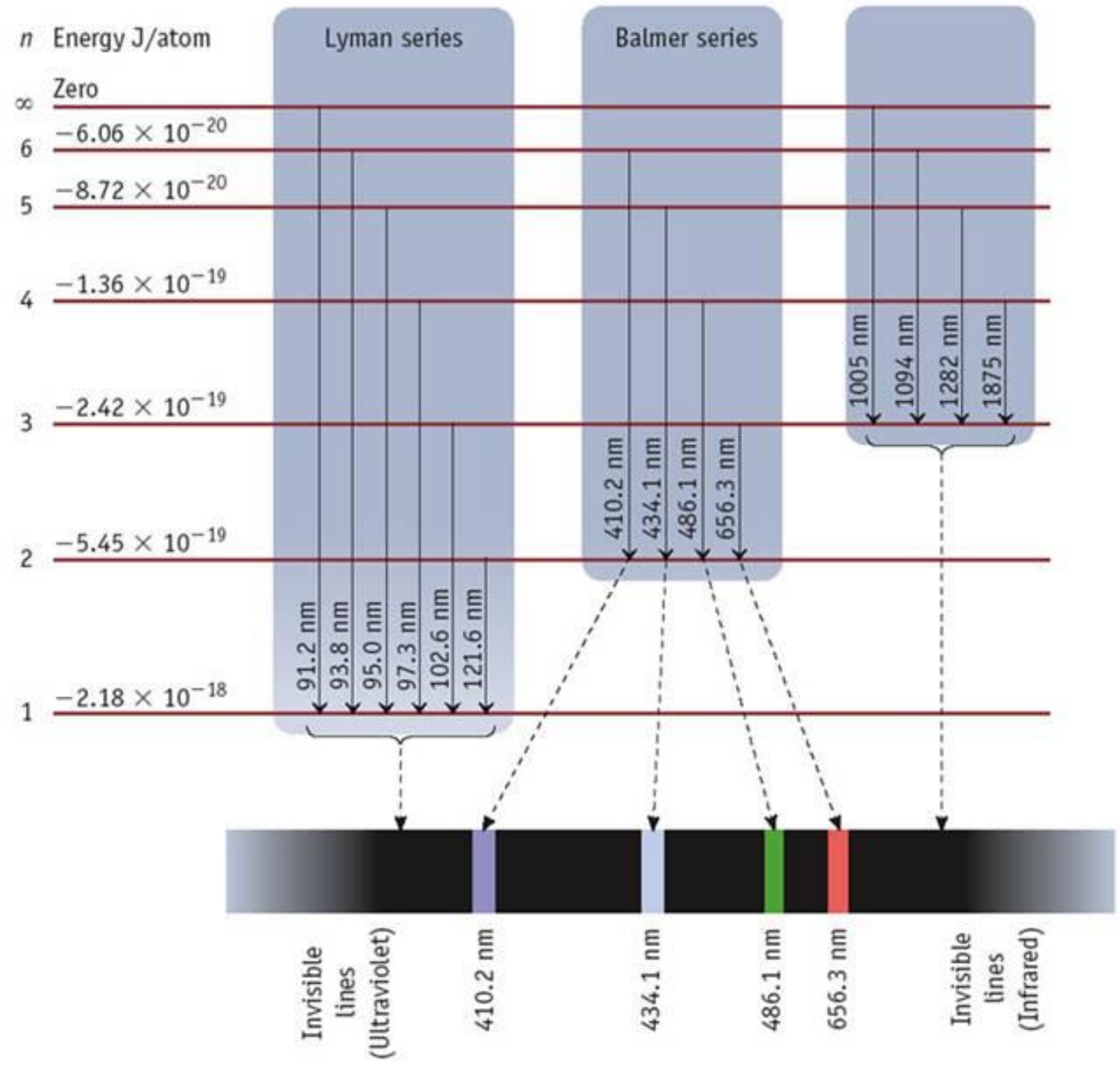
$$\boxed{P_{mn} = \frac{2\pi}{\hbar} |\langle f|W|i\rangle|^2 \rho\left(E_f\right)}$$

W - interaction with the field

$\rho\left(E_f\right)$ - the density of final states

Perturbation W does not have to be in the form of an electromagnetic wave.

Summary –Fermi golden rule



Hydrogen

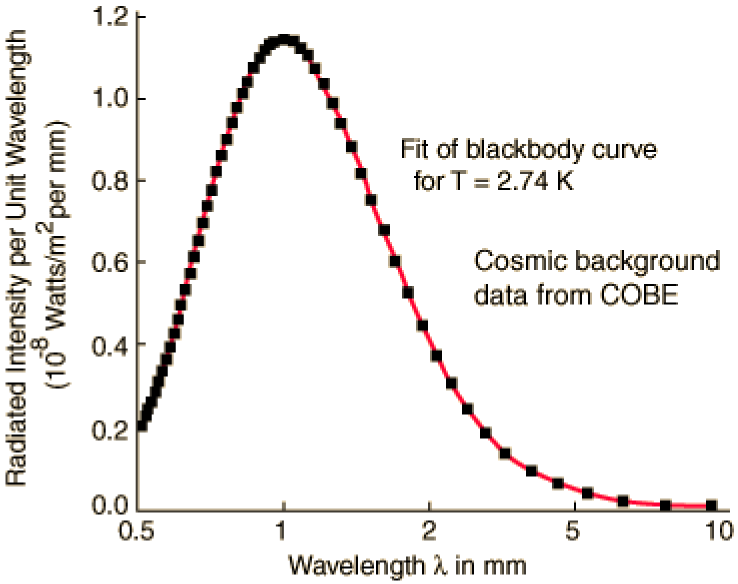
$$A_{nm} = \frac{4\alpha}{3}\frac{{\omega_{nm}}^3}{c^2}|\langle m|\vec{r}|n\rangle|^2$$

$$P_{nm} = A_{nm}\delta(E_n - E_m \pm \hbar\omega)$$

The history

- In the XIX century: the matter is granular, the energy (mostly e-m) is a wave

- Problems NOT solved
  - Black body radiation
  - Photoelectric effect
  - Origin of spectral lines of atoms



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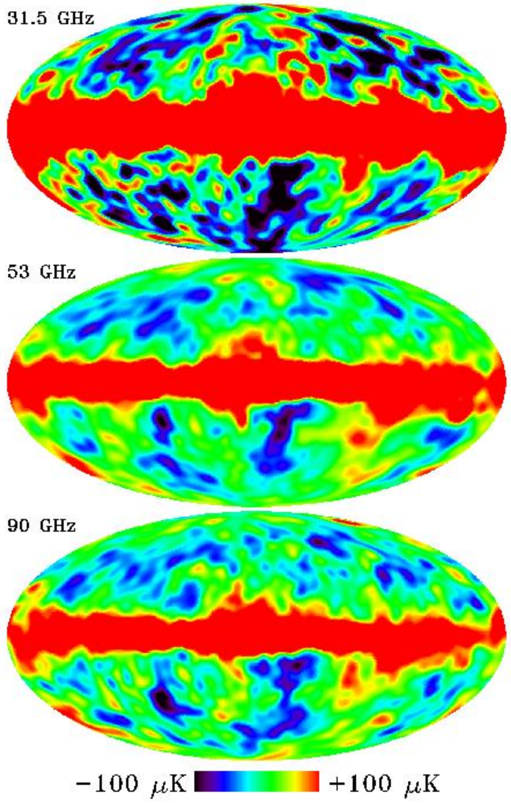
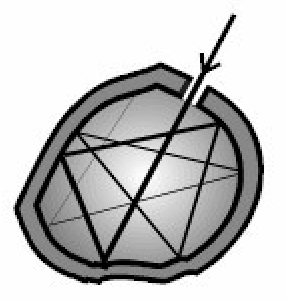


Ultraviolet catastrophe

Rayleigh–Jeans law



The spectral distribution of blackbody radiation:

Classically – The theorem of equipartition of energy: average energy of the standing wave is independent of frequency $\langle E \rangle = kT$

Radiation energy density (ρ) is the number of waves of a particular frequency range ($\nu\, d\nu$) times the average energy $\langle E \rangle$, divided by the volume of the cavity:

$$\rho(\nu, T)d\nu = \frac{8\pi\nu^2}{c^3} kTd\nu$$

The total radiation energy density at a given temperature is given by the sum of all frequencies:

$$\rho(T) = \int_0^\infty \rho(\nu, T)d\nu = \frac{8\pi}{c^3} kT \int_0^\infty \nu^2\, d\nu = \infty$$

Ultraviolet catastrophe

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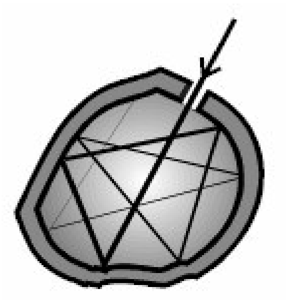

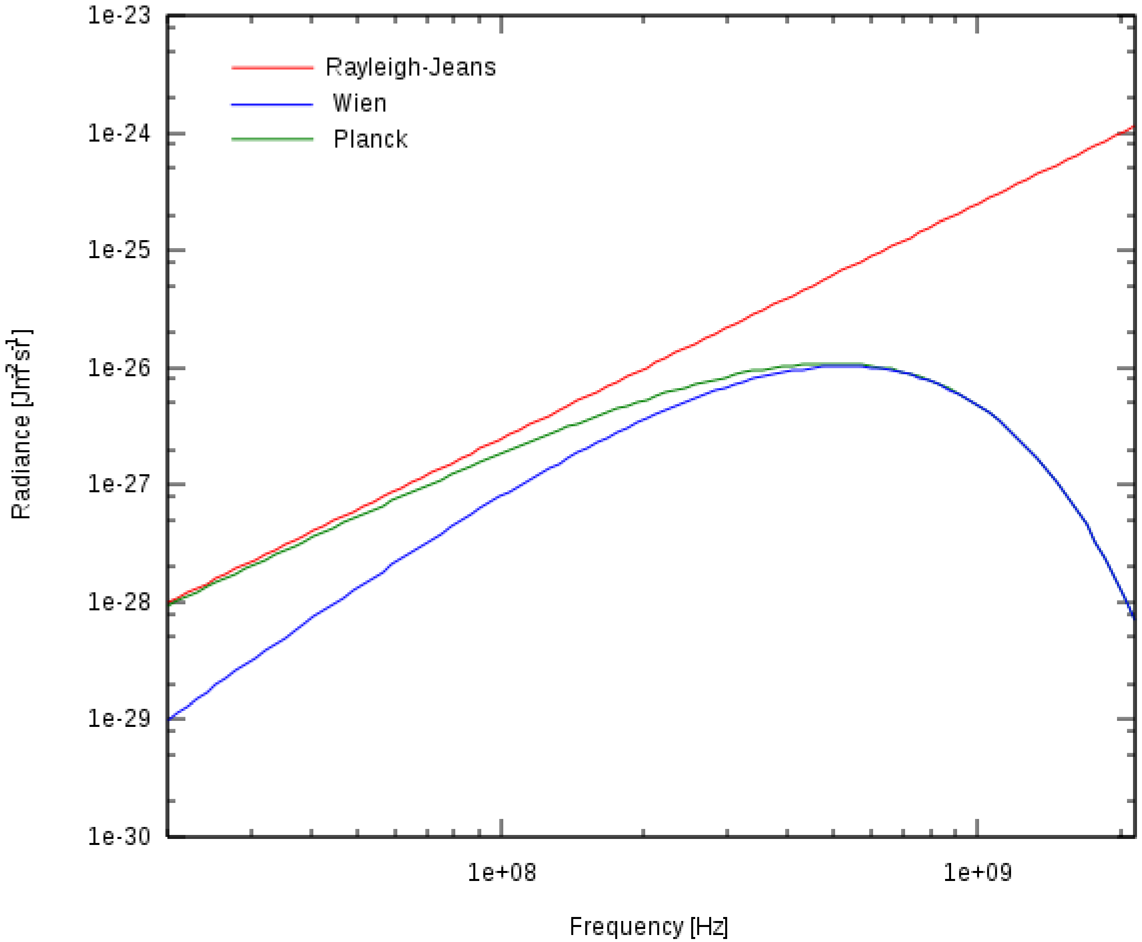

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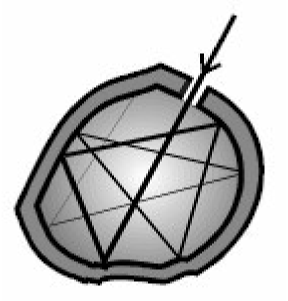
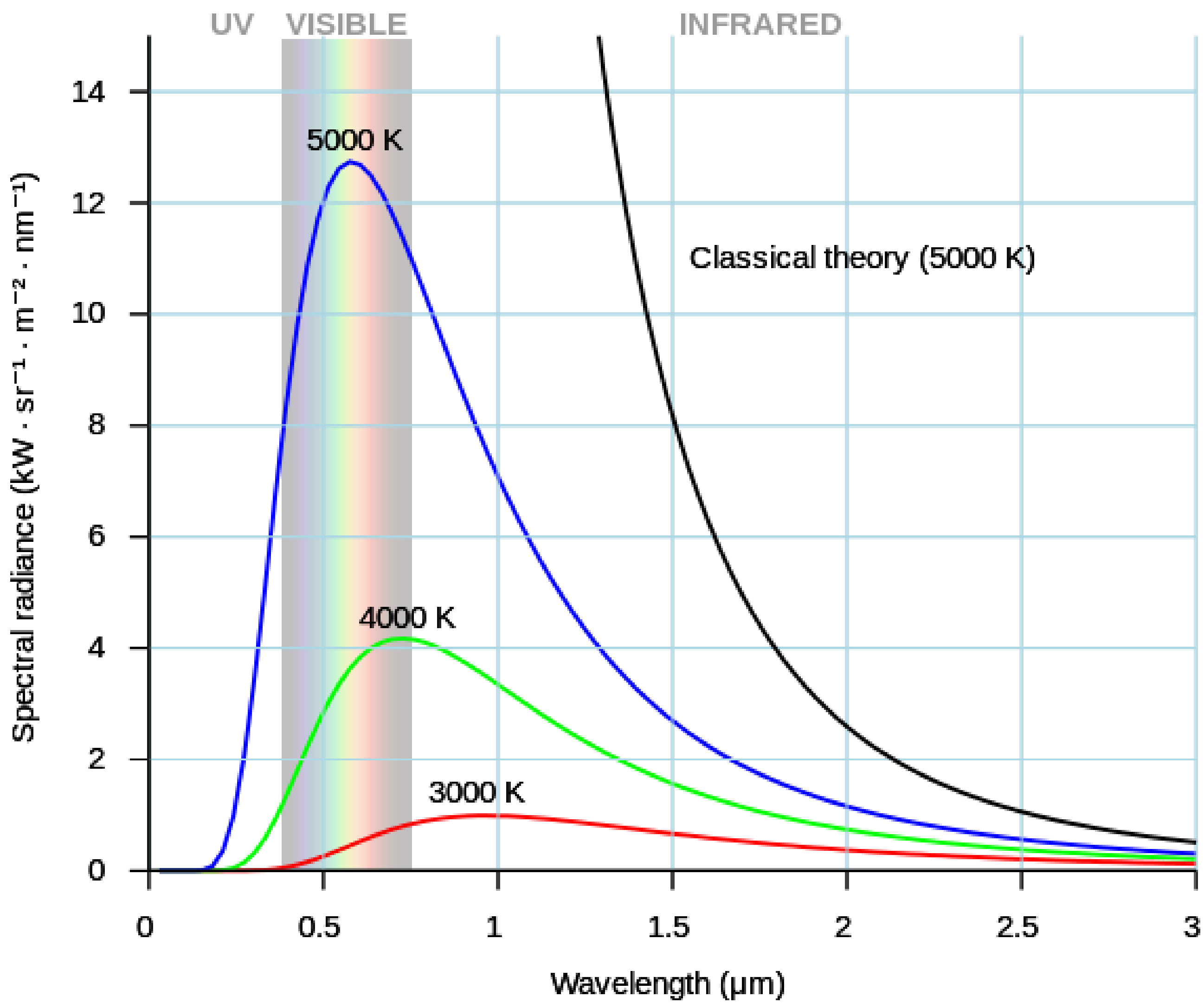




https://en.wikipedia.org/wiki/Ultraviolet_catastrophe#/media/File:Black_body.svg

The history

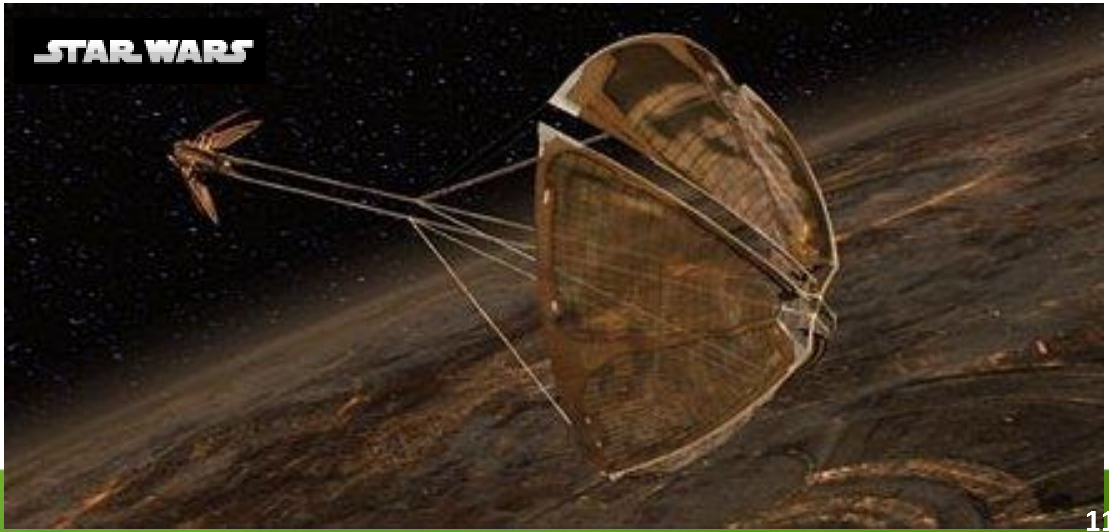
- In the XX century: the matter is (also) a wave and the energy is (also) granular (corpuscular)
- Solved problems:
  - Black body radiation spectrum (Planck 1900, Nobel 1918)
  - Photoelectric effect (Einstein 1905, Nobel 1922)
  - Origin of spectral lines of atoms (Bohr 1913, Nobel 1922)

$$\boxed{p = h / \lambda}$$

- *Photons* – *energy:* $E = h\nu$ (h = 6.626×10^{-32} J s = 4.136 ×10^{-15} eV s)

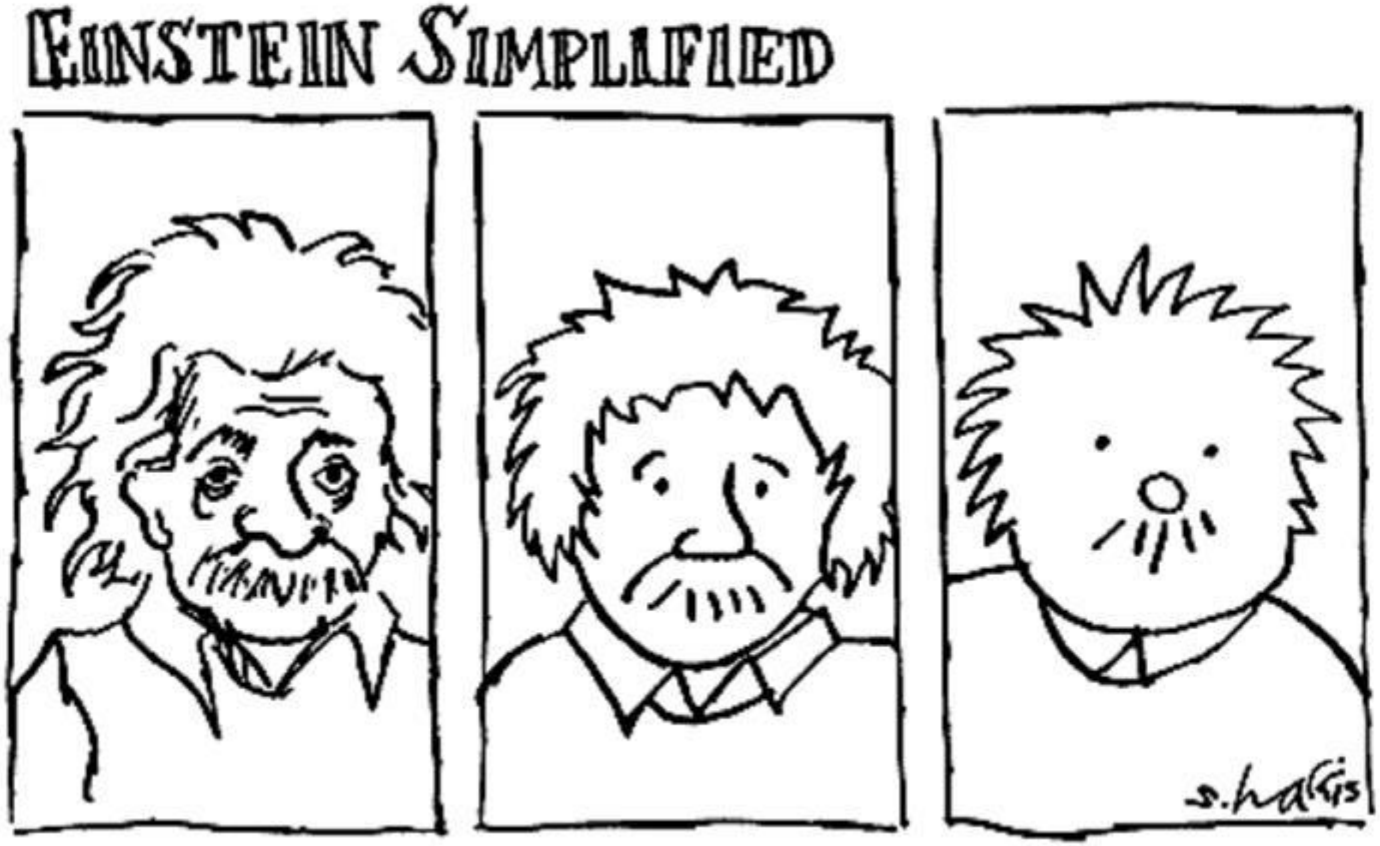
  – momentum: $p = E / c = h / \lambda$

Count Dooku's Geonosian solar sailer



light mill - **Crookes radiometer**

Derivation of Planck's law. Lasers.



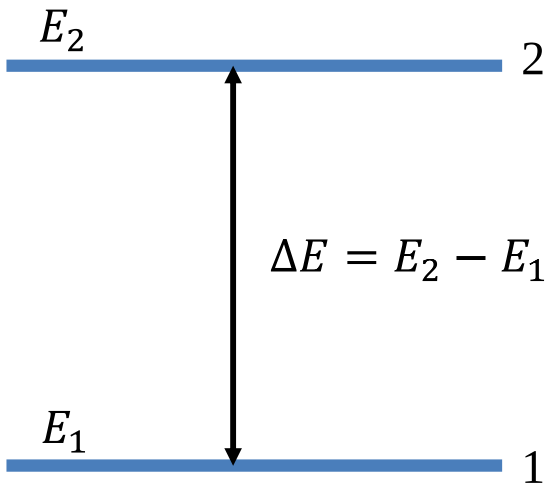
S. Harris

Everything should be made as simple as possible, but not simpler

Albert Einstein

The spontaneous and stimulated emission

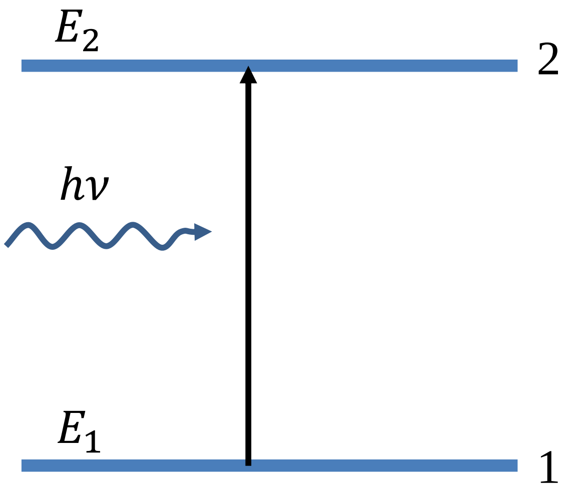
Let’s consider the transition between two states



What are the parameters describing the number of transitions from the state 1 to 2 and vice versa?

The spontaneous and stimulated emission

Let's consider the transition between two states



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$\hbar\omega = h\nu = E_2 - E_1$

1. Absorption

The spontaneous and stimulated emission

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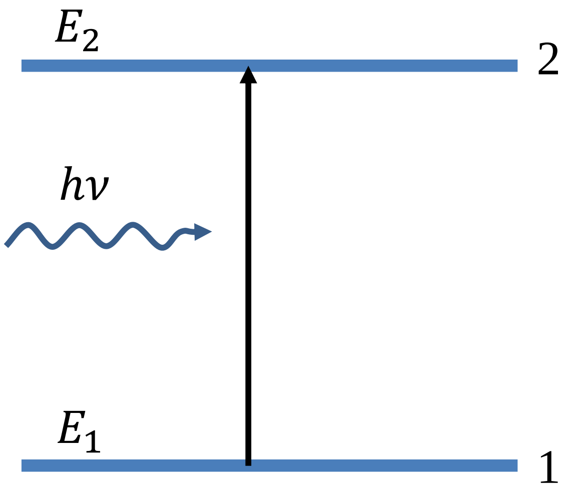

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$$\hbar\omega = h\nu = E_2 - E_1$$

1. Absorption

the number of transitions

$$\left(\frac{dN}{dt}\right)_{abs} = N_1 B_{12} \rho$$

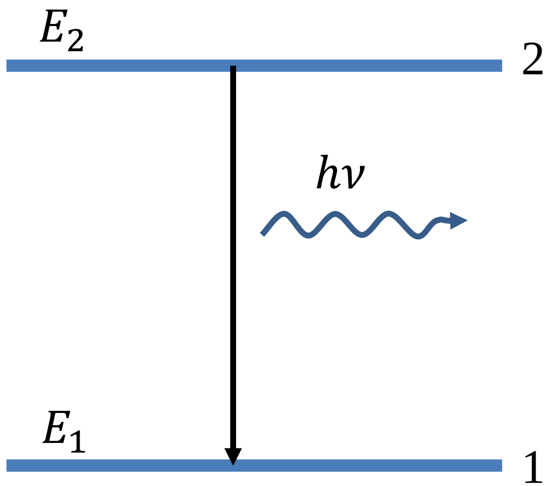
The number of availabale states

The proportionality factor

Radiation energy density

The spontaneous and stimulated emission

Let's consider the transition between two states



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2. Spontaneous emission

The spontaneous and stimulated emission

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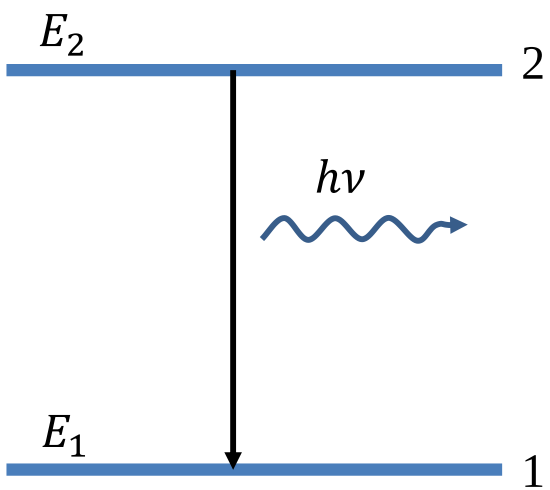

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$$\left(\frac{dN}{dt}\right)_{spon} = A N_2$$

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The number of availabale states

The spontaneous and stimulated emission

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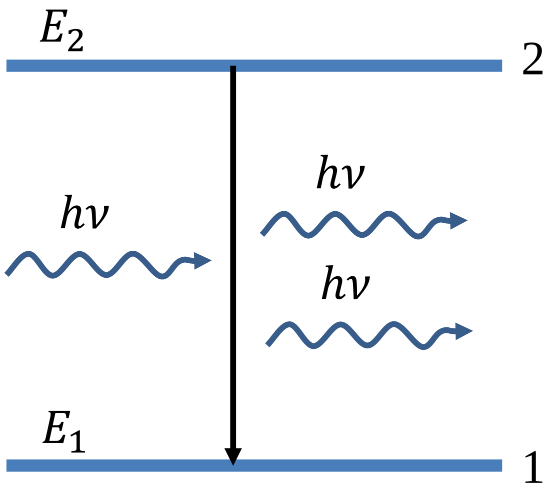

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3. Stimulated emission

The spontaneous and stimulated emission

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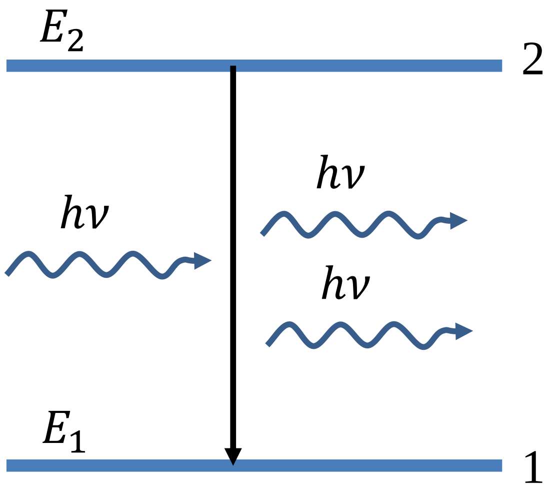

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2. Spontaneous emission

$$\left(\frac{dN}{dt}\right)_{spon} = A N_2$$

3. Stimulated emission

$$\left(\frac{dN}{dt}\right)_{wym} = N_2 B_{21} \rho$$

the number of transitions

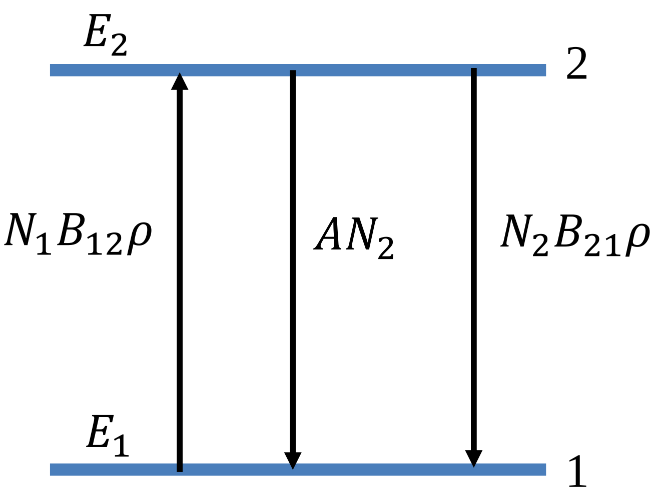
The proportionality factor

Radiation energy density

The number of availabale states

The spontaneous and stimulated emission

Let's consider the transition between two states



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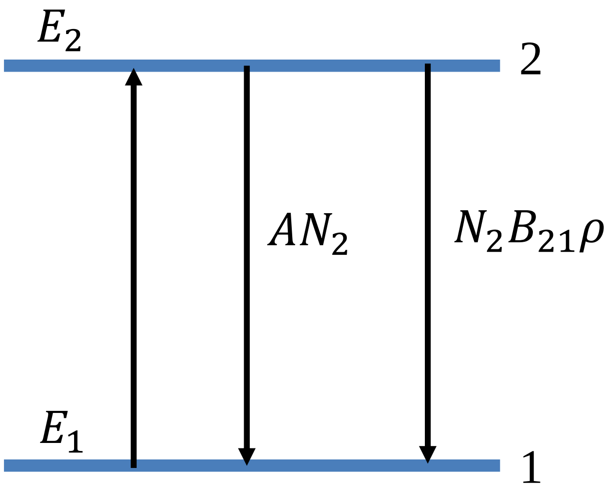
The spontaneous and stimulated emission

Let's consider the transition between two states



$$\left(\frac{dN}{dt}\right)_{abs} = N_1B_{12}\rho$$

$$\left(\frac{dN}{dt}\right)_{spon} = AN_2$$

$$\left(\frac{dN}{dt}\right)_{wym} = N_2B_{21}\rho$$

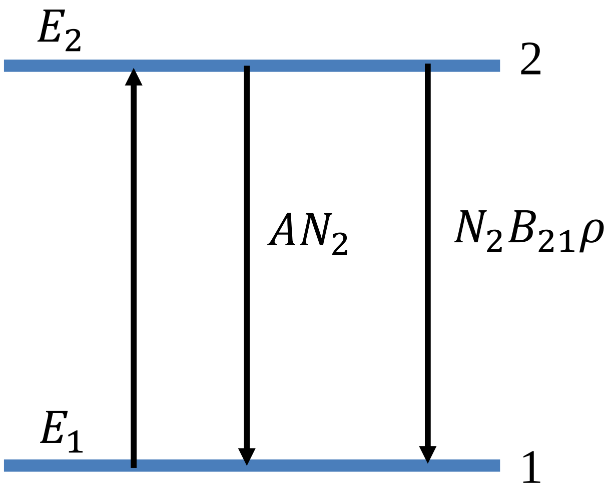
In thermal equilibrium conditions (a necessary condition, but it is also true in states far from equilibrium, eg. in lasers!)

$$\left(\frac{dN}{dt}\right)_{abs} = \left(\frac{dN}{dt}\right)_{spon} + \left(\frac{dN}{dt}\right)_{wym}$$

$$N_1B_{12}\rho = AN_2 + N_2B_{21}\rho$$

The spontaneous and stimulated emission

Let's consider the transition between two states



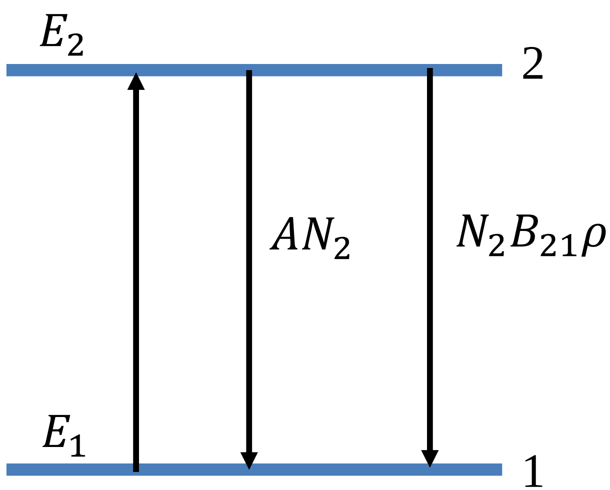
$$N_1B_{12}\rho = AN_2 + N_2B_{21}\rho$$

$$\rho = \frac{A}{B_{21}} \times \frac{1}{\frac{N_1B_{12}}{N_2B_{21}} - 1}$$

The occupations of N_1 and N_2 in thermal equilibrium conditions are given by Boltzman distribution

The spontaneous and stimulated emission

Let's consider the transition between two states



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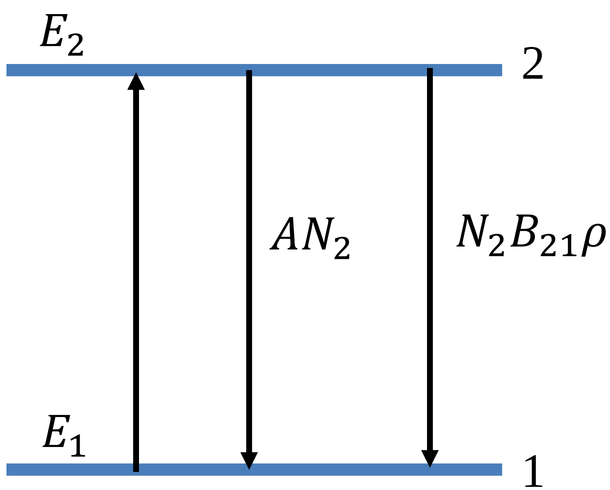
$$N_1 = \text{const}\, e^{-\frac{E_1}{kT}} \qquad N_2 = \text{const}\, e^{-\frac{E_2}{kT}} \qquad \frac{N_1}{N_2} = e^{-\frac{(E_1 - E_2)}{kT}} = e^{\frac{h\nu}{kT}}$$

What happens with ρ when $T \to \infty$?

The spontaneous and stimulated emission

Let's consider the transition between two states



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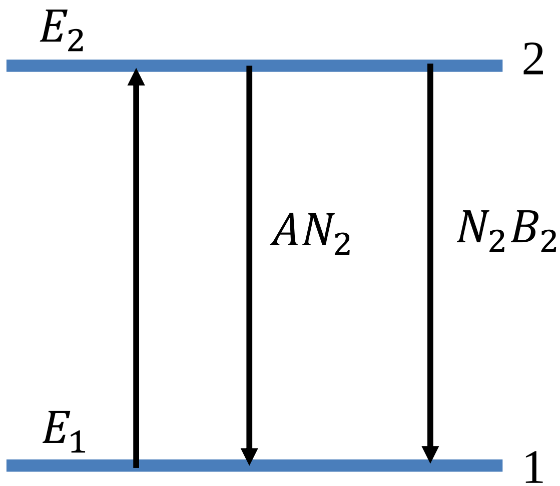
What happens with ρ when $T \to \infty$? $\quad B_{12} = B_{21}$

Considering the degree of the degeneracy of levels $\quad g_{12}B_{12} = g_{21}B_{21}$

The spontaneous and stimulated emission

Let's consider the transition between two states



$$\rho(\nu,T)=\frac{A}{B_{21}}\times\frac{1}{\frac{N_1B_{12}}{N_2B_{21}}-1}=\frac{A}{B}\times\frac{1}{\exp\left(\frac{h\nu}{kT}\right)-1}$$

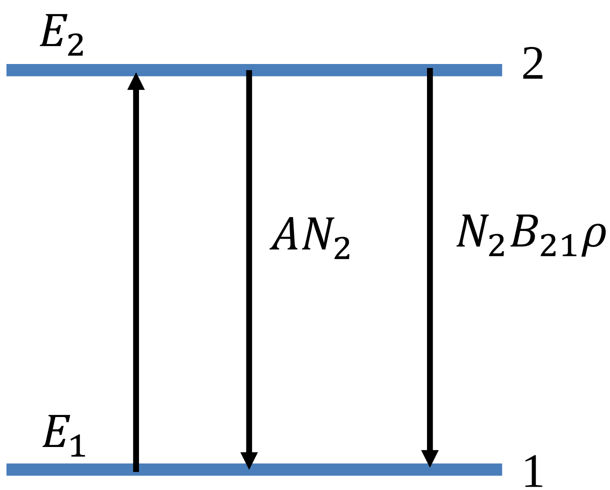
In turn, for $h\nu \ll kT$ we have Reileigh-Jeans law

$$\rho(\nu,T)d\nu=\frac{8\pi\nu^2}{c^3}kTd\nu$$

Expanding the exponential function

The spontaneous and stimulated emission

Let's consider the transition between two states



$$\rho(\nu,T) = \frac{A}{B_{21}} \times \frac{1}{\frac{N_1B_{12}}{N_2B_{21}} - 1} = \frac{A}{B} \times \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

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$$\rho(\nu,T)d\nu = \frac{8\pi\nu^2}{c^3}kTd\nu$$

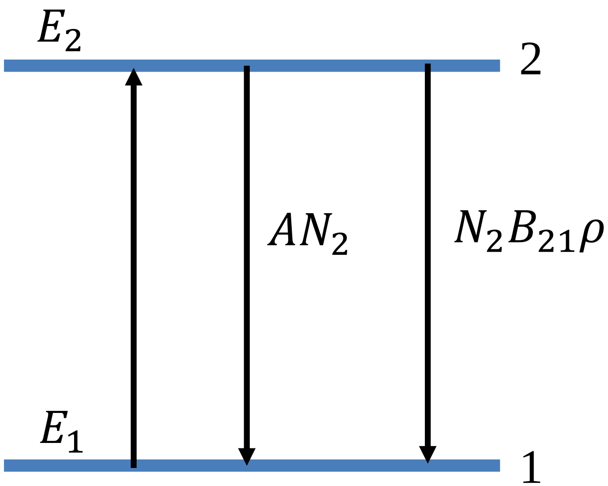
Expanding the exponential function $\rho(\nu,T) \approx \frac{A}{B}kT/h\nu$

Thus: $\frac{A}{B} = \frac{8\pi}{c^3}h\nu^3 = D(\nu)h\nu$

The amount of radiation modes in a given volume

The spontaneous and stimulated emission

Let's consider the transition between two states



$$\rho(\nu, T) = \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1} \frac{8\pi\nu^2}{c^3} h\nu$$

Planck equation

A and B are **Einstein coefficients**. Units of A:

$$\left(\frac{dN}{dt}\right)_{spon} = AN_2 \Rightarrow A = \frac{1}{\tau}$$

and: $B = [\tau D(\nu) h\nu]^{-1}$

Electromagnetic wave

The perturbation in a form of an electromagnetic wave.

$$H \approx \frac{e}{m}\vec{A}\vec{p}$$

$$\boxed{P_{nm} = \frac{w_{nm}}{\tau} = \frac{2\pi}{\hbar}\left|\langle n|w^{\pm}|m\rangle\right|^2 \delta(E_n - E_m \pm \hbar\omega)}$$

$$\vec{A} = \overrightarrow{A_0}\left\{e^{-i(\omega t - \vec{k}\vec{r})} + e^{i(\omega t - \vec{k}\vec{r})}\right\}$$

expanding a series $\quad \vec{p}\, e^{-i(\vec{k}\vec{r})} \approx \vec{p}\left[1 + \left(-i\vec{k}\vec{r}\right) + \frac{\left(-i\vec{k}\vec{r}\right)^2}{2!} + \cdots\right]$

after laborious calculations we get the probability of emission of electromagnetic radiation dipole (described by the operator $e\vec{r}$)

$$\boxed{A_{nm} = \frac{w_{nm}}{\tau} = \frac{{\omega_{nm}}^3 e^2}{3\pi\varepsilon_0\hbar c^3}|\langle n|\vec{r}|m\rangle|^2 = \frac{4\alpha}{3}\frac{{\omega_{nm}}^3}{c^2}|\langle n|\vec{r}|m\rangle|^2} \qquad \alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} \approx \frac{1}{137}$$

It is one of the **Einstein coefficients** (lasers, etc. - next week!) for nondegenerated states.

Fala elektromagnetyczna

The perturbation in a form of an electromagnetic wave.

$$\boxed{A_{nm} = \frac{{\omega_{nm}}^3 e^2}{3\pi\varepsilon_0 \hbar c^3} |\langle m|\vec{r}|n\rangle|^2 = \frac{4\alpha}{3}\frac{{\omega_{nm}}^3}{c^2} |\langle m|\vec{r}|n\rangle|^2}$$

In the case o degenerated states we introduce „oscillator strength”

$$A_{nm} = \frac{4\alpha}{3}\frac{{\omega_{nm}}^3}{c^2}\frac{S_{mn}}{g_m} \qquad S_{nm} = \sum_i \sum_j \left|\langle n_i|\vec{r}|m_j\rangle\right|^2$$

the degeneracy of the initial state

In the case of the hydrogen atom states it is convenient to represent operator $\vec{r}$ in the circular form:

$$\left|\langle n_i|\vec{r}|m_j\rangle\right|^2 = \left|\langle n_i|z|m_j\rangle\right|^2 + \frac{1}{2}\left|\langle n_i|x+iy|m_j\rangle\right|^2 + \frac{1}{2}\left|\langle n_i|x-iy|m_j\rangle\right|^2$$

it is easy to then integrate spherical harmonics, because:

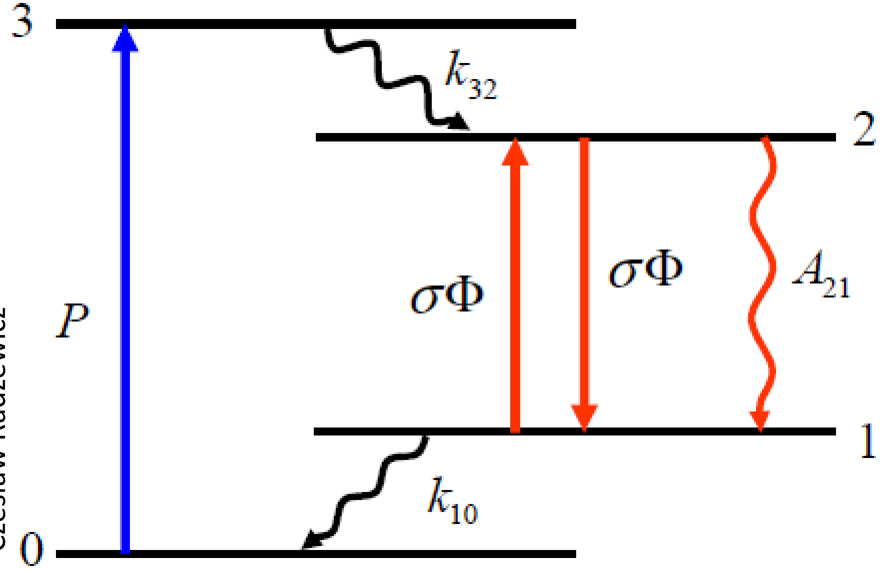
$$z = r\cos\vartheta$$

$$x \pm iy = re^{\pm i\varphi}\sin\vartheta$$

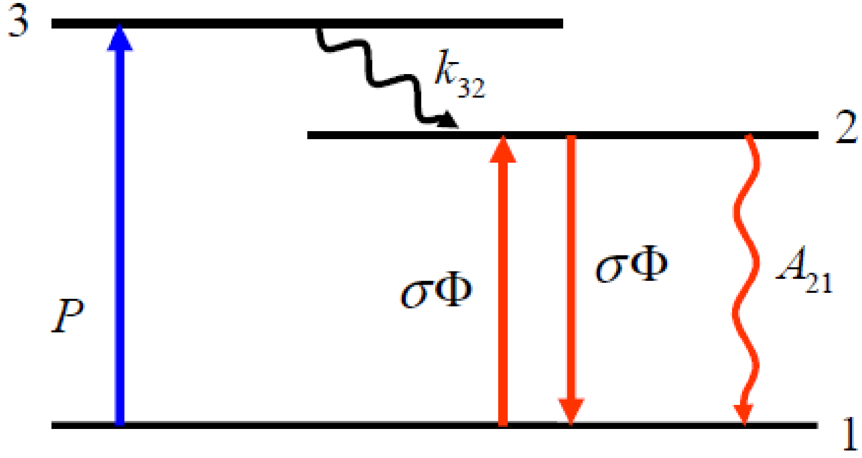
Check it!

Fala elektromagnetyczna

Laser needs minimum 3 states



Rzadko stosowany laser 3-poziomowy



Czesław Radzewicz

The revision of optics

Maxwell equations

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J}_{sw} + \frac{\partial \vec{D}}{\partial t}$$

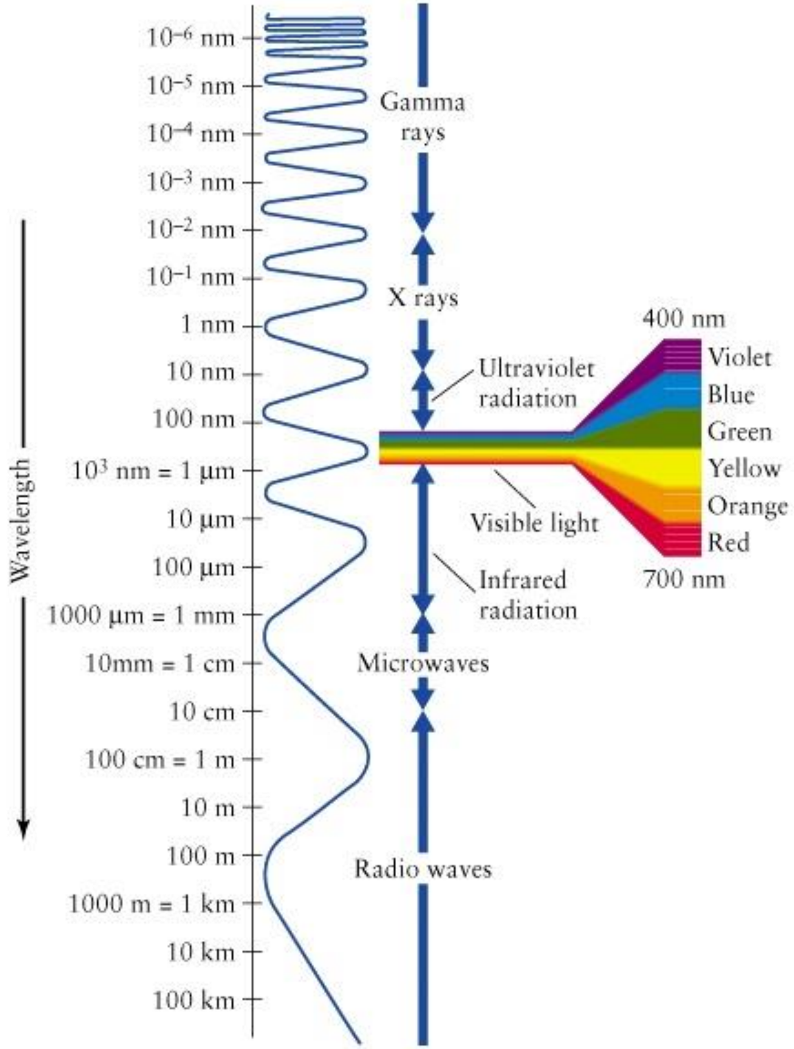
$$\nabla \vec{D} = \rho_{sw}$$

$$\nabla \vec{B} = 0$$

$$\vec{D} = \varepsilon \varepsilon_0 \vec{E}$$

$$\vec{B} = \mu \mu_0 \vec{H}$$

$$\vec{J}_{sw} = \sigma \vec{E}$$



The revision of optics

Electromagnetic wave in vacuum	Electromagnetic wave in dielectric
Maxwell equations: $$\nabla \times \vec{E} = rot\ \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ $$\nabla \times \vec{B} = rot\ \vec{B} = \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$ Wave equation: $$\Delta \vec{E} = \varepsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$ $$\Delta \vec{B} = \varepsilon_0 \mu_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$ The speed of the electromagnetic wave: $$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}, \qquad c \approx 3 \cdot 10^8 \frac{m}{s}$$ Refractive index: $$n = 1 \qquad \boxed{k = \frac{\omega}{c}}$$	Maxwell equations: $$\nabla \times \vec{E} = rot\ \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ $$\nabla \times \vec{B} = rot\ \vec{B} = \varepsilon_0 \mu_0 \varepsilon \mu \frac{\partial \vec{E}}{\partial t}$$ Wave equation: $$\Delta \vec{E} = \varepsilon_0 \mu_0 \varepsilon \mu \frac{\partial^2 \vec{E}}{\partial t^2}$$ $$\Delta \vec{B} = \varepsilon_0 \mu_0 \varepsilon \mu \frac{\partial^2 \vec{B}}{\partial t^2}$$ The speed of the electromagnetic wave: $$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0 \varepsilon \mu}} = \frac{c}{n}$$ Refractive index: $$n = \frac{c}{v} = \sqrt{\varepsilon \mu} \qquad \boxed{k = \frac{n\omega}{c}}$$

The revision of optics

Electromagnetic wave in vacuum	Electromagnetic wave in dielectric
Maxwell equations: $\nabla \times \vec{E} = rot\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $\nabla \times \vec{B} = rot\,\vec{B} = \varepsilon_0\mu_0 \frac{\partial \vec{E}}{\partial t}$ Wave $\Delta \vec{B} = \varepsilon_0\mu_0 \frac{\partial^2 \vec{B}}{\partial t^2}$ The speed of the electromagnetic wave: $c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}, \qquad c \approx 3 \cdot 10^8 \frac{m}{s}$ Refractive index: $n = 1$ $\qquad k = \frac{\omega}{c}$	Maxwell equations: $\nabla \times \vec{E} = rot\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $\nabla \times \vec{B} = rot\,\vec{B} = \varepsilon_0\mu_0\varepsilon\mu \frac{\partial \vec{E}}{\partial t}$ $\Delta \vec{B} = \varepsilon_0\mu_0\varepsilon\mu \frac{\partial^2 \vec{B}}{\partial t^2}$ The speed of the electromagnetic wave: $c = \frac{1}{\sqrt{\varepsilon_0\mu_0\varepsilon\mu}} = \frac{c}{n}$ Refractive index: $n = \frac{c}{v} = \sqrt{\varepsilon\mu}$ $\qquad k = \frac{n\omega}{c}$

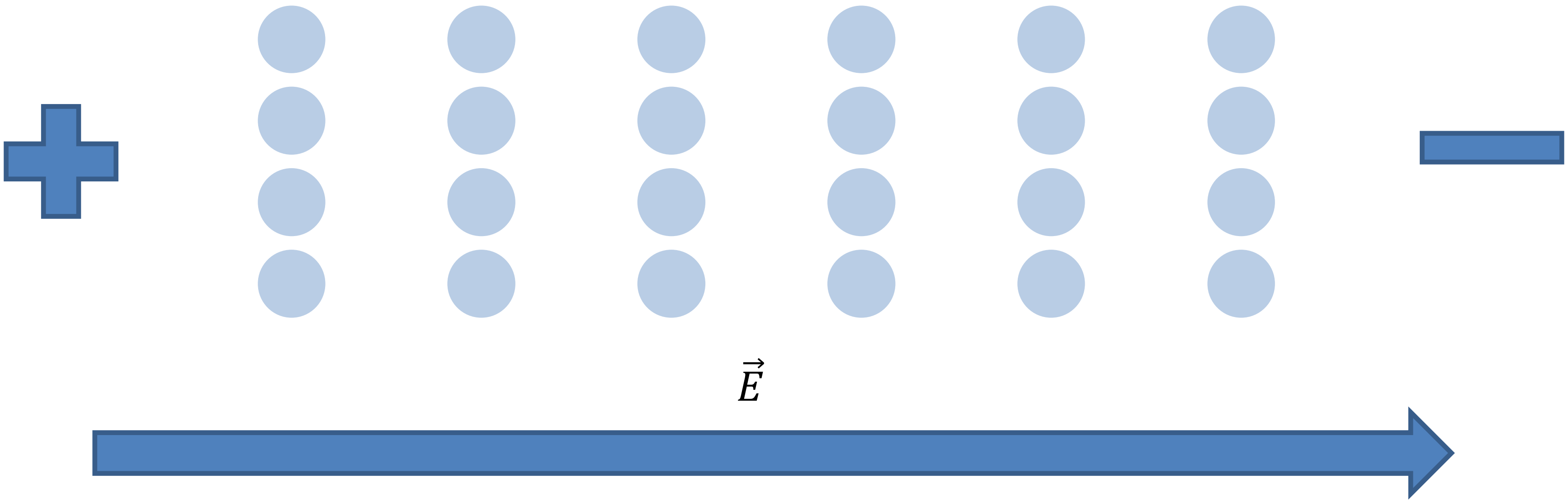
But how the medium interacts with an electromagnetic wave? Is ε (so n) constant?

Classical theory for the index of refraction

Classical theory for the index of refraction

The Lorentz Oscillator model

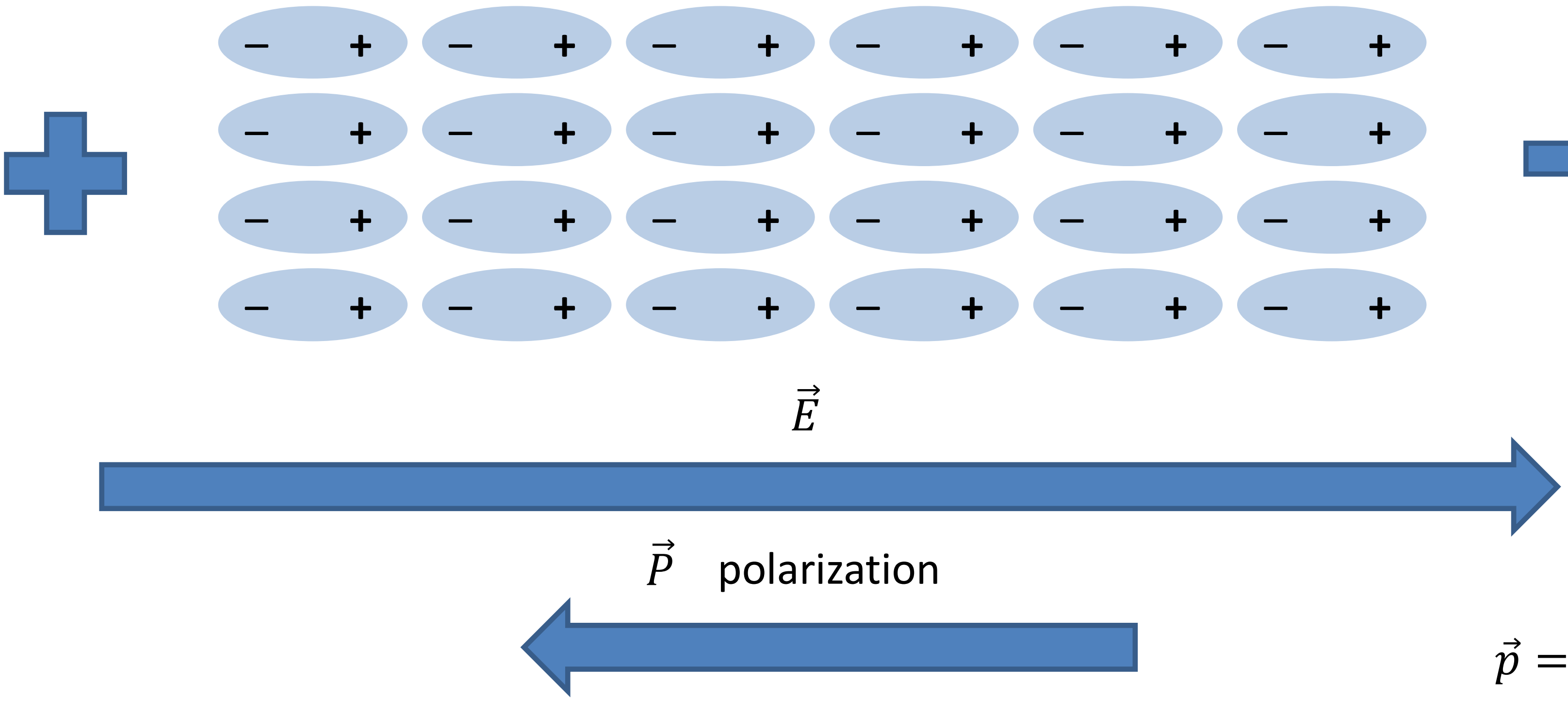
A polarizable medium - dielectric:



Classical theory for the index of refraction

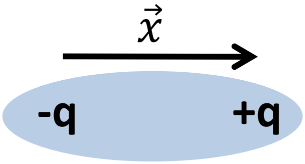
The Lorentz Oscillator model

A polarizable medium - dielectric:



$$\vec{p} = q\vec{x}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$



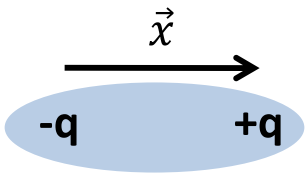
The dipole moment of an atom (molecule)

Classical theory for the index of refraction

The Lorentz Oscillator model

We consider

- A space filled with oscillators of the resonant frequency ω_0 and damping factor γ;
- Oscillators have a mass m, the charge q
- They are moved by the oscillating electric field $\vec{E}$.



$\vec{p} = q\vec{x}$ the dipole moment of an atom (molecule)

polarization $\vec{P} = N\vec{p} = N\left(\varepsilon_0 \alpha \vec{E}\right) = \varepsilon_0 \chi \vec{E}$

polarisability
polaryzowalność

dielectric susceptibility

Classical theory for the index of refraction

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We consider

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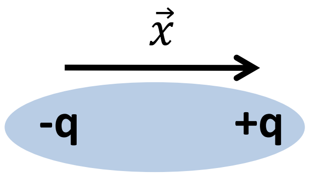
We are determining
$n^2 = \varepsilon = 1 + \chi$



$\vec{p} = q\vec{x}$

Thus $\vec{D} = \varepsilon_0\vec{E} + \vec{P} = \varepsilon_0(1+\chi)\vec{E} = \varepsilon_0\varepsilon\vec{E}$

polarization $\vec{P}(t) = N\vec{p}(t) = Nq\vec{x}(t) = \varepsilon_0\chi\vec{E}(t)$

We have to determine $\vec{x}(t)$

Classical theory for the index of refraction

The Lorentz Oscillator model

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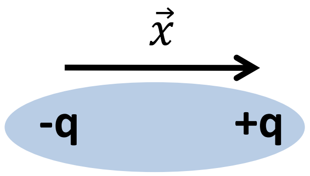

$$\vec{p} = q\vec{x}$$

$$\frac{d^2\vec{x}}{dt^2} + \gamma\frac{d\vec{x}}{dt} + \omega_0^2\vec{x} = \frac{q}{m}\vec{E}e^{i\omega t}$$

damping

the elastic force

driving force

the steady state solution:
$\vec{x}(t) = \vec{x}_0 e^{i\omega t}$

The Lorentz Oscillator model

We consider

- A space filled with oscillators of the resonant frequency ω_0 and damping factor γ;
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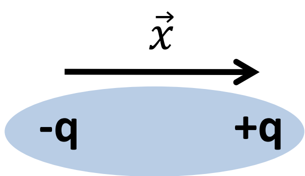
We are determining
$n^2 = \varepsilon = 1 + \chi$



$\vec{p} = q\vec{x}$

on exercises

$$\frac{d\ldots}{d\ldots} \ldots \frac{\ldots}{dt} + \omega_0^2\vec{x} = \frac{q}{m}\vec{E}e^{i\omega t}$$

damping

the elastic force

driving force

the steady state solution:
$\vec{x}(t) = \vec{x}_0 e^{i\omega t}$

Classical theory for the index of refraction

The wave in the media (different):

$$\frac{d^2\vec{x}}{dt^2} + \gamma\frac{d\vec{x}}{dt} + \omega_0^2\vec{x} = \frac{q}{m}\vec{E}e^{i\omega t}$$

Lorentz model

$$\frac{d^2\vec{x}}{dt^2} + \gamma\frac{d\vec{x}}{dt} + \omega_0^2\vec{x} = 0$$

Emission spectrum

$$\frac{d^2\vec{x}}{dt^2} + 0 + 0 = \frac{q}{m}\vec{E}e^{i\omega t}$$

Plasma waves

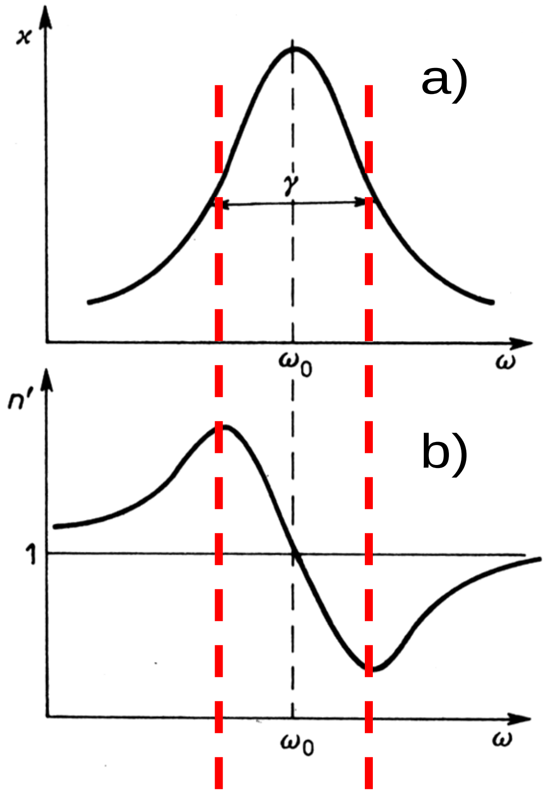
the steady state solution:

$$\vec{x}(t) = \vec{x}_0 e^{i\omega t}$$

Classical theory for the index of refraction

The Lorentz Oscillator model

We obtain:



Kramers – Kronig dispersion relations

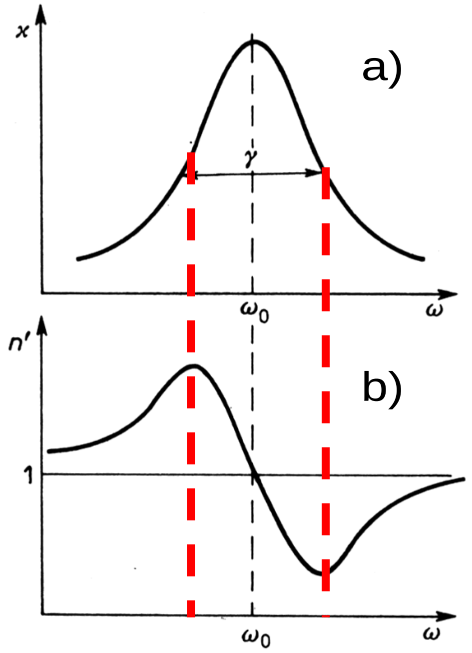
anomalous dispersion

$$\kappa = \frac{Nq^2}{2\varepsilon_0 m}\frac{\gamma\omega}{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2}$$

$$n' = 1 + \frac{Nq^2}{2\varepsilon_0 m}\frac{\omega_0^2-\omega^2}{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2}$$

Classical theory for the index of refraction

The Lorentz Oscillator model (dispersion medium)



- n is the refractive index and indicates the phase velocity – real refractive index.
- Without absorption $n' = n$.
- the imaginary part κ is called the "extinction coefficient and indicates absorption of the wave propagating through the material
- The quantity $\frac{dn'}{d\omega}$ is called the *dispersion of the medium*

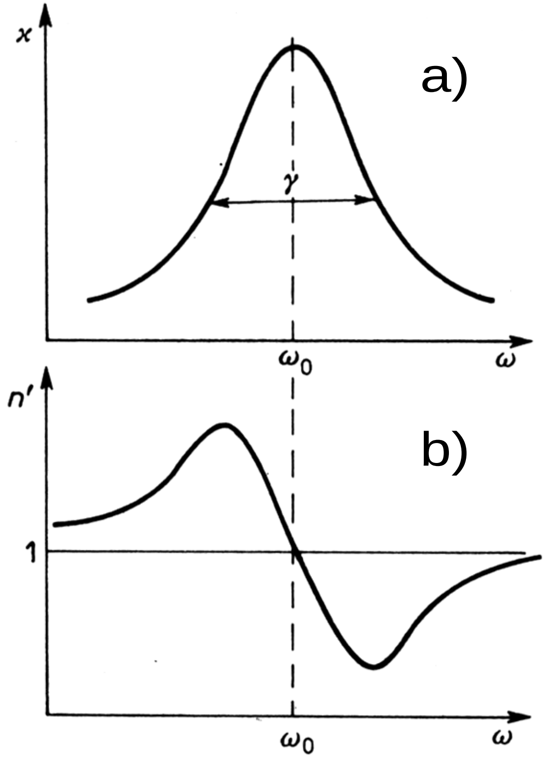
$\frac{dn'}{d\omega}$

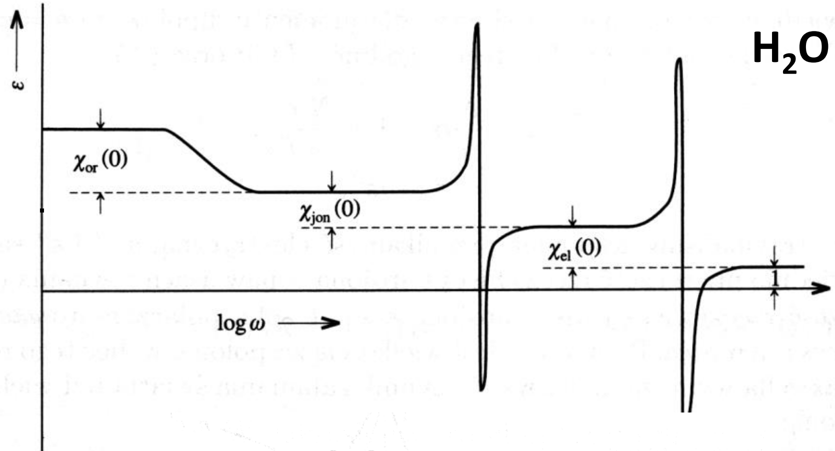
- Far from the resonance this is positive function – **normal dispersion**
- At the frequencies close to the resonance it is negative – **anomalous dispersion**

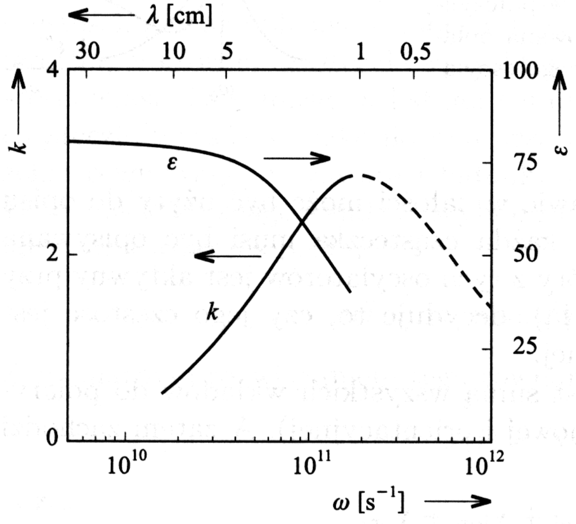
Classical theory for the index of refraction

The Lorentz Oscillator model

Several resonances in the medium



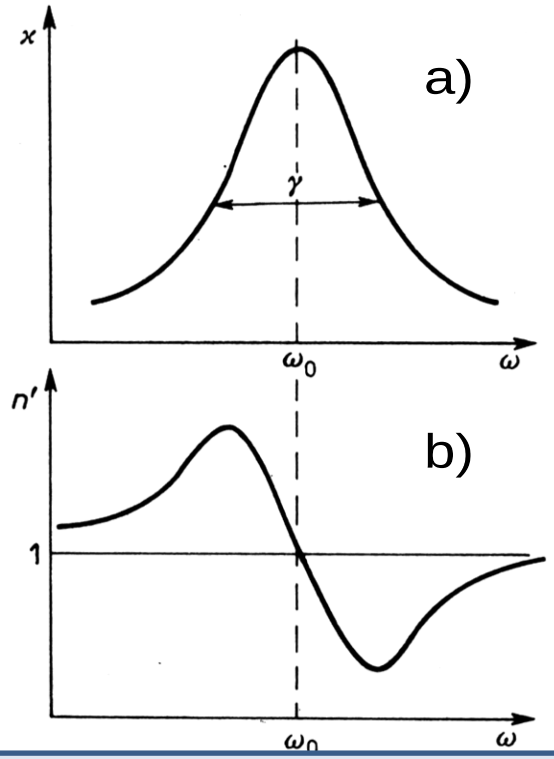


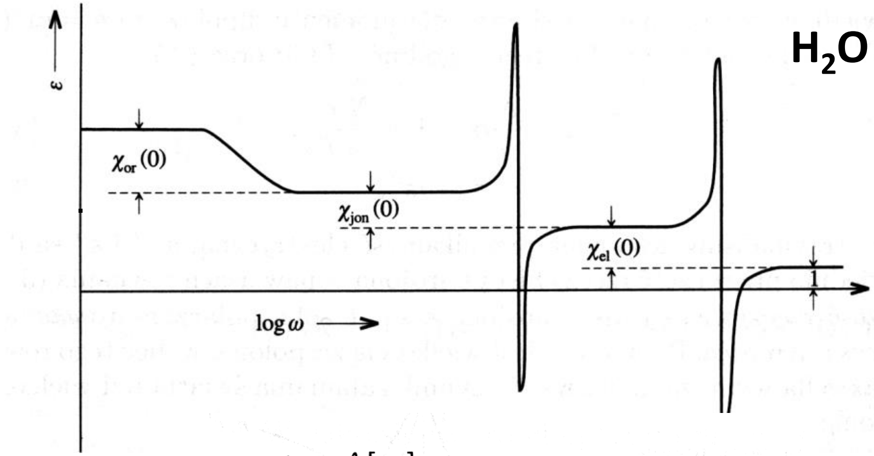


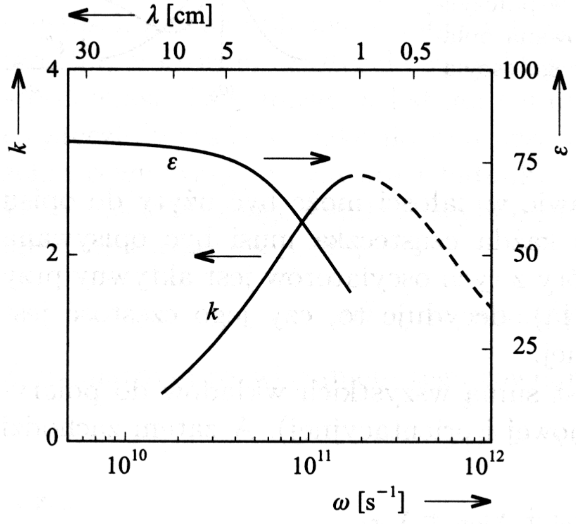
Classical theory for the index of refraction

The Lorentz Oscillator model

Several resonances in the medium





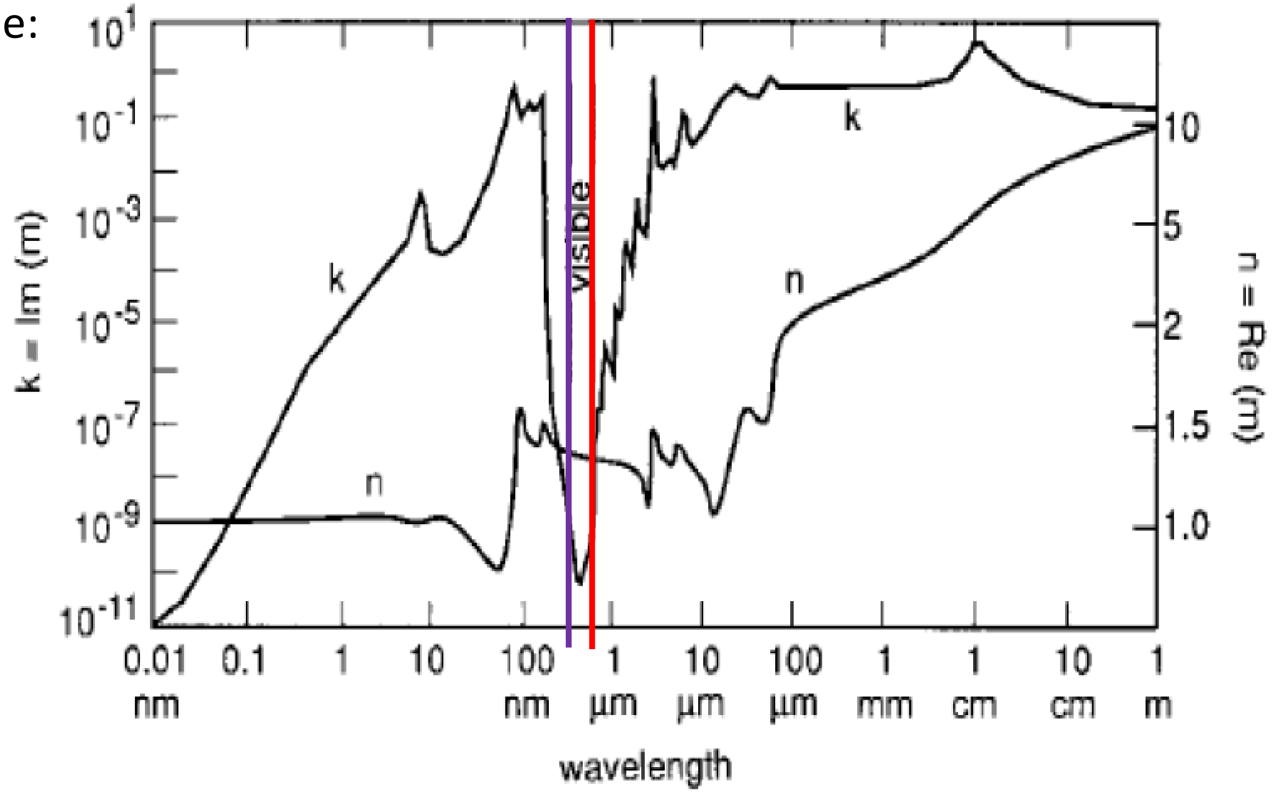


For one oscillator frequency ω_0 $\varepsilon_L = 1$, but for many frequencies it is approximately constant sum of the contributions from all the other.

Classical theory for the index of refraction

The Lorentz Oscillator model

Water example:



V. M. Zoloratev and A. V. Demin, "Optical Constants of Water over a Broad Range of Wavelengths, 0.1 Å–1 m," *Opt. Spectrosc.* (U.S.S.R.) 43(2):157 (Aug. 1977).

Classical theory for the index of refraction

Lambert-Beer law：

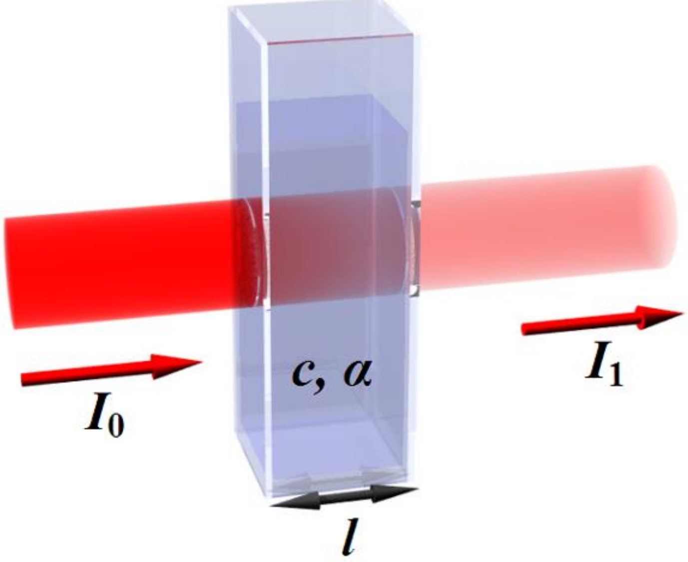
The electromagnetic field of the wave passing through the medium:

$$\vec{E}(z,t) = \vec{E}_0 \exp[i(\omega_0 t - k_0 n' z + i k_0 \kappa z)] = \vec{E}_0 \exp\left(-\frac{2\pi}{\lambda}\kappa z\right) \exp[i(\omega_0 t - k_0 n' z)]$$

ω_0, k_0– of the wave in vacuum

The intensity $$I(z) \propto \left|\vec{E}(z)\right|^2 = E_0^2 \exp\left(-\frac{4\pi}{\lambda}\kappa z\right)$$

$$\boxed{I(z) = I_0 e^{-\alpha z}} \qquad \alpha = 2\kappa k_0$$



Absorption coefficuent $\alpha = 2\kappa k_0$

Derivation of Planck's law. Lasers.

S. Harris

"BUT THIS IS THE SIMPLIFIED VERSION FOR THE GENERAL PUBLIC."