Dark Energy, MOND and sub-millimiter tests of gravity

Ignacio Navarro

DAMTP, University of Cambridge

First Workshop in Particle Physics and Cosmology

Warsaw - April 2006

I.N. and K. Van Acoleyen, gr-qc/0506096 [PLB622:05]
I.N. and K. Van Acoleyen, gr-qc/0511045 [JCAP:06]
I.N. and K. Van Acoleyen, gr-qc/0512109
I.N. and K. Van Acoleyen, to appear

Modified gravity: motivation

- **Dark Energy**: cosmological constant problem, equation of state?

  If $\omega_{DE} \equiv \frac{p_{DE}}{\rho_{DE}} \neq -1$, what is the Dark Energy?

  In particular if $\omega_{DE} < -1$ we have to accept phantom matter or modified gravity!

- **Dark Matter**: problems with CDM, success of MOND fitting rotation curves from visible matter

  CDM: cuspy halos, small satellites $\longrightarrow$ Warm, Self-interacting Dark Matter?

  MOND: modification of Newton's potential, very succesful at the galactic level [Milgrom'84]

$$a > a_0 \longrightarrow \text{no modification}$$
$$a < a_0 \longrightarrow \text{modification of Newton's potential}$$
$$a_0 \sim 10^{-8} cm/s^2 \sim cH_0$$
$$\downarrow$$
$$\text{suggests link with Dark Energy}$$

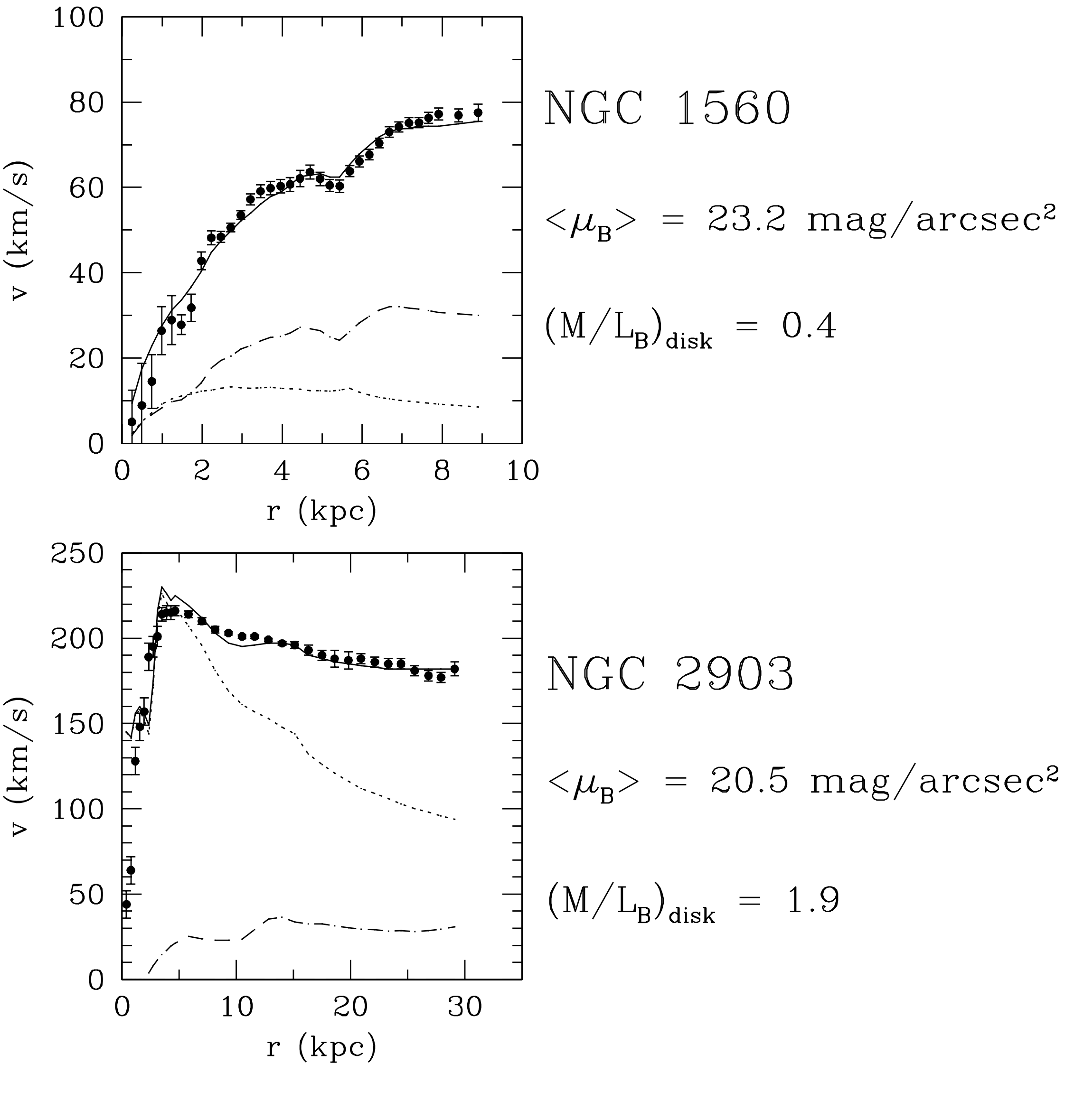


Figure 1: R. H. Sanders and S. S. McGaugh, Ann. Rev. Astron. Astrophys. **40**, 263 (2002)

Modifying gravity below a fixed acceleration scale

We assume we have a correction to the EH action such that:

$$S = \int d^4x \sqrt{-g} \frac{1}{16\pi G_N} \left\{ R - \mu^2 \mathrm{Log}\left[f(R, P - 4Q) \right] \right\}$$

$$P \equiv R_{\mu\nu} R^{\mu\nu} \quad \text{and} \quad Q \equiv R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho}.$$

$$f \to 0 \quad \text{for} \quad R^{\sigma}_{\mu\nu\lambda} \to 0$$

$$f \simeq Q/Q_0 \quad \text{when} \quad Q \gg R^2, P$$

- The extra term will dominate at low curvatures but it will be negligible at large curvatures.
- Minkowski spacetime is not a solution, de Sitter is in general.
- If the "crossover" scale is when $R \sim H_0^2$ the models have de Sitter solutions with $R \sim H_0^2$, but the extra term would be negligible when $R >> H_0^2$
- To explain current acceleration we need $\mu \sim H_0$

But these models raise other questions:

- Are there stability issues (negative energy particles, ghosts)?
- Is it possible to reproduce a correct weak field limit (in the Solar System)?
- Can these theories be used as models of Dark Energy/Dark Matter?
- If so, what would be their experimental signatures?

Some answers:

- The model is ghost free, no negative energy modes. The vacuum is stable
- We have an extra scalar field, besides the massless spin two graviton
- The mass of this extra field grows in regions of large curvature. It modifies gravity only at large distances below a fixed ACCELERATION SCALE

Particle content of modified gravity

$$\mathcal{L} = R + F(R, P, Q)$$
$$\downarrow$$
fourth order EOM
$$\downarrow$$
eight propagating d.o f. :

[Hindawi et al.,PRD'96]

- two in a massless spin two graviton
- one in a massive scalar
- five in a massive spin two ghost

We would like $F(R, P, Q)$ such that

- The spin-two ghost is absent
- The extra massive scalar gets a large mass and effectively decouples in regions of large curvature such as the Solar System

Expand the action up to second order in $h_{\mu\nu}$ over the vacuum (a constant curvature maximally symmetric spacetime)

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$$

We are asking what is the "kinetic and mass terms" for $h_{\mu\nu}$.

When $\mathcal{L} = R + F(R, P, Q)$, the linearisation of the action over vacuum will be the same as the linearisation of [Chiba,JCAP'05]

$$S = \int d^4x \sqrt{-g} \frac{1}{16\pi G_N} \left[-\Lambda + \delta R + \frac{1}{6m_0^2} R^2 - \frac{1}{2m_2^2} C^{\mu\nu\lambda\sigma} C_{\mu\nu\lambda\sigma} \right]$$

the mass of the ghost is m_2 and the scalar mass is m_0 where

$$\begin{aligned}
\Lambda &\equiv \big\langle F - RF_R + R^2 \left(F_{RR}/2 - F_P/4 - F_Q/6\right) + R^3 \left(F_{RP}/2 + F_{RQ}/3\right) \\
&\qquad + R^4 \left(F_{PP}/8 + F_{QQ}/18 + F_{PQ}/6\right) \big\rangle_0 \\
\delta &\equiv \big\langle 1 + F_R - RF_{RR} - R^2 \left(F_{RP} + 2F_{RQ}/3\right) \\
&\qquad - R^3 \left(F_{PP}/4 + F_{QQ}/9 + F_{PQ}/3\right) \big\rangle_0 \\
m_0^{-2} &\equiv \big\langle (3F_{RR} + 2F_P + 2F_Q) + R\left(3F_{RP} + 2F_{RQ}\right) \\
&\qquad + R^2 \left(3F_{PP}/4 + F_{QQ}/3 + F_{PQ}\right) \big\rangle_0 \\
m_2^{-2} &\equiv - \langle F_P + 4F_Q \rangle_0 .
\end{aligned}$$

- So when $F(R, P, Q) = F(R, Q - 4P)$, $m_2 = \infty$. There is no ghost!
- In this case the excitations over vacuum are the usual spin two massless graviton plus a scalar.
- Notice that the (effective) Planck mass and the mass of the extra scalar depend on the (local) background curvature.

For our logarithmic action in vacuum $R \sim F = \mu^2 \text{Log}[f]$. The mass of the extra scalar in vacuum is

$$m_0^2 \sim H_0^6/\mu^4 \sim \mu^2$$

$$ds^2 = -\left(1 - \frac{8\hat{G}M}{3r} - H_0^2 r^2\right) dt^2 + \left(1 - \frac{4\hat{G}M}{3r} - H_0^2 r^2\right)^{-1} dr^2 + r^2 d\Omega^2$$

But the linearisation breaks down at high energies or short distances:

$$S \simeq S^{(2)} + M_p^2 \int d^4x \left(\left\langle \tilde{F}_{UR} \right\rangle_0 (\partial^2 h)^2 R^{(1)} + \left\langle \tilde{F}_{UU} \right\rangle_0 (\partial^2 h)^4 \right.$$
$$\left. + \left\langle \tilde{F}_{UUR} \right\rangle_0 (\partial^2 h)^4 R^{(1)} + \left\langle \tilde{F}_{UUU} \right\rangle_0 (\partial^2 h)^6 + \ldots \right)$$

where $\tilde{F} \equiv \text{Log}[f]$, $U \equiv 5R^2/6 + Q - 4P$

We can not trust the linearised action over vacuum when

$$E > \Lambda_s \equiv \left(M_p \mu^3\right)^{1/4}$$

or

$$r < r_V \equiv \left(\frac{GM}{\mu^3}\right)^{1/4}$$

Expectations based on the linearisation over vacuum

A massive scalar yields a contribution to the potential like

$$V_s \sim \frac{e^{-m_0 r}}{r}$$

so we expect a modification in the weak field limit when

$$r < m_0^{-1}$$

We evaluate m_0 in the Schwarzschild solution where:

$$R = 0, \quad P = R_{\mu\nu}R^{\mu\nu} = 0, \quad Q = R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho} = \frac{48(GM)^2}{r^6}$$

In our model m_0 depends on r and we can expect a modification when

$$r < m_0^{-1} \sim \frac{\mu}{Q^{\frac{1}{2}}} \sim \mu\frac{r^3}{GM}$$

$$\Downarrow$$

$$r > \sqrt{\frac{GM}{\mu}} \equiv r_c$$

A LONG DISTANCE MODIFICATION OF NEWTONIAN GRAVITY BELOW THE MOND CRITICAL ACCELERATION!

The modification becomes important for
$a < a_0 \sim H_0$

$$G_{\mu\nu} + \mu^2 H_{\mu\nu} = 0.$$

Expanding the solution to the EoM in powers of μ^2 the zero-th order background is the Schwarzschild solution and the first correction is

$$ds^2 \simeq -A(r)dt^2 + B^{-1}(r)dr^2 + r^2 d\Omega_2^2$$

$$A(r) = 1 - \frac{2G_N M}{r}\left(1 + \frac{4}{3}\left(\frac{r}{r_c}\right)^4 + \mathcal{O}\left(\left(\frac{r}{r_c}\right)^8\right)\right)$$
$$B(r) = 1 - \frac{2G_N M}{r}\left(1 - 2\left(\frac{r}{r_c}\right)^4 + \mathcal{O}\left(\left(\frac{r}{r_c}\right)^8\right)\right)$$

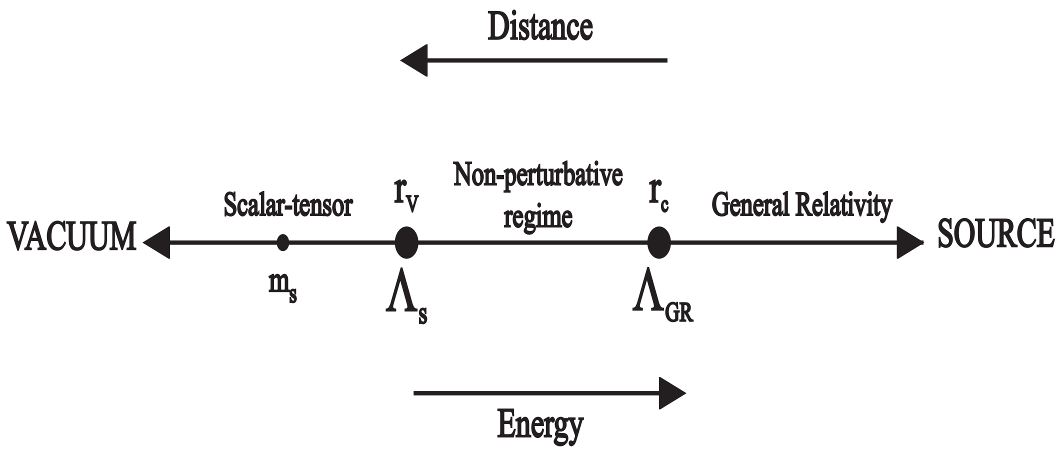
Gravity is modified beyond the distance r_c given now by

$$\frac{2G_N M}{r_c^2} = \mu \sim H_0$$

the relation

$$a_0 \sim 10^{-8} cm/s^2 \sim cH_0$$

has a natural explanation in these theories



- So at small energies or large distances we have a scalar-tensor theory, with a scalar of mass
$$m_0^2 \sim R^{4n+4}/\mu^{4n+2} \sim \mu^2$$
- At larger energies ($E > \Lambda_s$) or smaller distances ($r < r_V$) we enter a non perturbative regime. It can, perhaps, be understood in terms of a theory with "running couplings"
$$m_0^2 \sim Q/\mu^2 \sim \frac{(GM)^2}{r^6\mu^2}$$
- At even larger energies ($E > \Lambda_{GR}$) or smaller distances ($r < r_c$) the extra degree of freedom decouples and we recover GR dynamics.

Some numbers:

for $M_{sun} \rightarrow r_c \simeq 3000\ AU$, $r_V \simeq 7\ kpc$

for $10^{11} M_{sun} \rightarrow r_c \simeq 4\ kpc$, $r_V \simeq 4\ Mpc$

for $10^{15} M_{sun} \rightarrow r_c \simeq 400\ kpc$, $r_V \simeq 40\ Mpc$

Is this theory compatible with high precision measurements of orbits at the solar system level?

Most stringest test: Lunar Laser Ranging. The anomalous precession per revolution is bound to be

$$\Delta\phi < 2.4 \times 10^{-11}$$

Our theory predicts

$$\Delta\phi \simeq \pi r \frac{d}{dr}\left(r^2 \frac{d}{dr}\left(\frac{\delta V}{r V_N}\right)\right) \simeq 16\pi \left(\frac{r_{(Moon-Earth)}}{r_{c(Earth)}}\right)^4 \sim 10^{-12}$$

PASSES PRECESSION OF PERIHELION TESTS!

We expect modifications of Newton's law whenever $r < m_0^{-1}$. The inverse square law for gravity has been tested on the lab down to scales of order $1mm$ [Adelberger et al.,ARPS53'03]

What is the local mass of the scalar on the Earth surface?

The gravitational field here is dominated by the one produced by the Earth:

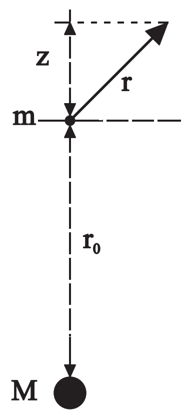
$$m_{0(Earth)} \sim \frac{GM_{(Earth)}}{r^3_{(Earth)} H_0} \sim (0.1mm)^{-1}$$

PASSES TESTS OF SHORT DISTANCE CORRECTIONS!

Predicts: both anomalous perihelion shifts for the Moon and short distance corrections to Newton's law one order of magnitude below current bounds

Anisotropic corrections to Newton's law at the sub-mm scale: a smoking gun for MOND-like modified gravity

Consider the situation:



$$ds^2 = -(1+2\Phi)dt^2 + w_i(dt dx^i + dx^i dt) + [(1-2\Psi)\delta_{ij} + 2s_{ij}]dx^i dx^j$$

When $r_0 \ll r_c(M)$, $r \ll r_0$, $Gm \ll r$, $GM \ll r_0$

$$\Phi^{(0)} \approx \Psi^{(0)} \approx -\frac{Gm}{r} - \frac{GM}{r_0}\left(1 + \frac{z}{r_0} + \frac{3}{2}\frac{z^2}{r_0^2} - \frac{r^2}{2r_0^2}\right)$$

To get the first order corrections $\Psi^{(1)}$, $\Phi^{(1)}$ we have to solve

$$G^{(1)}_{\mu\nu} = -\mu^2 H^{(0)}_{\mu\nu}$$

We get

$$G_{00}^{(1)} \approx 2\nabla^2\Psi^{(1)} \ = \mu^2 H_{00}^{(0)} \approx -\frac{3}{4}\frac{Gm}{r^3}\frac{1}{m_s^2 r^2}(3 - 30\frac{z^2}{r^2} + 35\frac{z^4}{r^4})$$

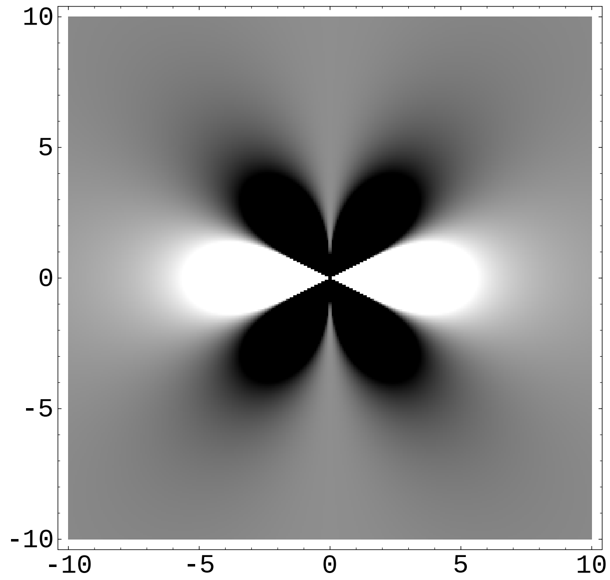
$$G_{\mu}^{\mu(1)} \approx 2\nabla^2\Phi^{(1)} - 4\nabla^2\Psi^{(1)} = \mu^2 H_{\mu}^{\mu(0)} \approx 0$$

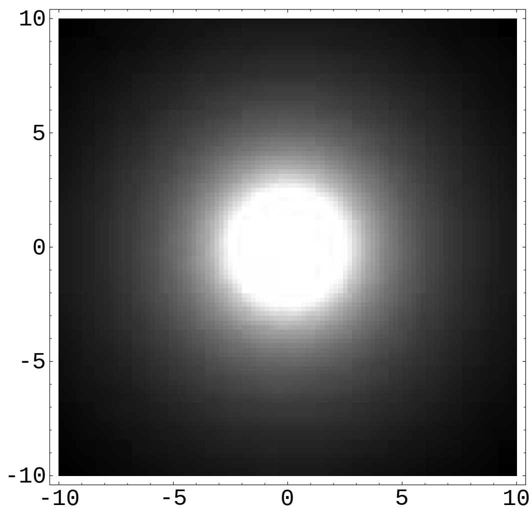
where $m_s \equiv \frac{GM}{r_0^3\mu}$ is the "local mass" of the scalar around $r = 0$

We get therefore the short-distance correction of Newton's potential

$$\Phi \simeq \Phi^{(0)} + \Phi^{(1)} = -\frac{Gm}{r}\left[1 + \frac{3}{8m_s^2 r^2}\left(1 - 6\frac{z^2}{r^2} + 5\frac{z^4}{r^4}\right) + \mathcal{O}\left(1/m_s^3 r^3\right)\right]$$

The correction to the potential





Summary and conclusions

- Actions of the type $\mathcal{L} = R - \mu^2 \text{Log}[f(R, Q - 4P)]$ can explain the acceleration of the universe when $\mu \sim H_0$

- In these models Newtonian gravity is modified below a characteristic acceleration scale given by $a_0 = \mu \sim H_0$

- The Planck mass is rescaled at large distances to a value

$$M_{p(eff)}^2 = \left\langle 1 - \mu^2 \frac{f_R - 2Rf_Q}{f} \right\rangle_0 M_p^2$$

- They also predict short distance deviations from Newton's law, where the short distance scale depends on the local background curvature. On the Earth surface the correction is anisotropic and appears at a scale

$$m_s^{-1} \sim \mu/Q^{1/2} \sim H_0 r_e^3/(G_N M_e) \sim 0.1mm$$

Will be tested soon. Smoking gun for MOND-like modified gravity!