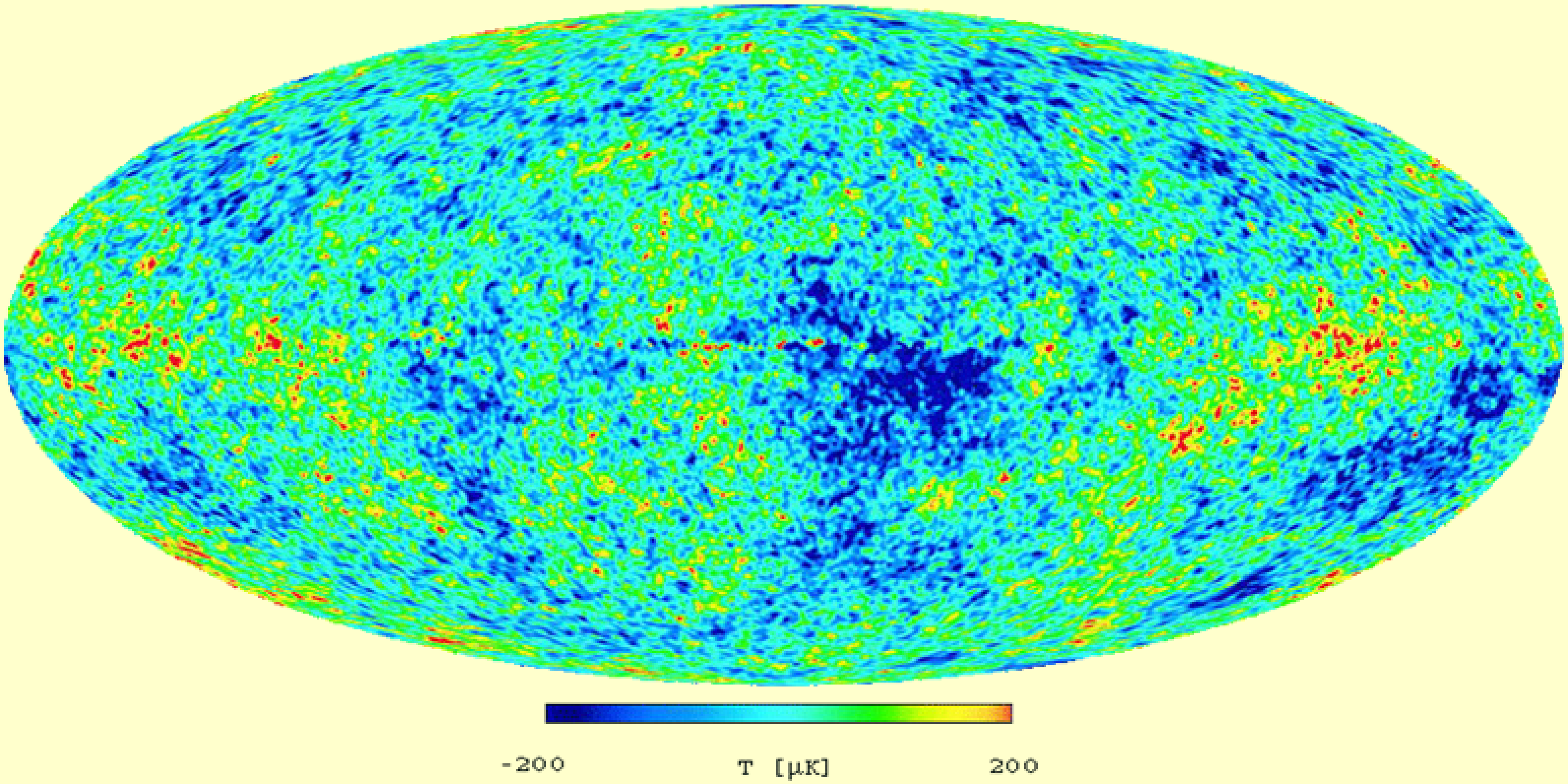
The Physics of the cosmic microwave background

Warsaw, February 2007



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Contents

The homogeneous and isotropic universe

- **the metric**

$$ds^2 = a^2\left(-dt^2 + \frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$$

- **Friedmann's equations**

$$\left(\frac{\dot{a}}{a}\right)^2 + k = \frac{8\pi G}{3}a^2\rho + \frac{a^2}{3}\Lambda$$

$$\left(\frac{\dot{a}}{a}\right)^{\cdot} = -\frac{4\pi G}{3}a^2(\rho + 3p) + \frac{a^2}{3}\Lambda$$

- **cosmological parameters**

$$H_0 = \frac{\dot{a}}{a}(t_0) = h100\mathrm{km/sMpc} \text{ Hubble parameter}$$

$$\rho_c = \frac{3}{8\pi G}H_0^2 \quad \text{critical density}$$

$$\Omega_m = \frac{\rho_m(t_0)}{\rho_c} \quad \text{matter density parameter}$$

$$\Omega_b = \frac{\rho_b(t_0)}{\rho_c} \quad \text{baryon density parameter}$$

$$\Omega_r = \frac{\rho_r(t_0)}{\rho_c} \quad \text{radiation density parameter}$$

$$\Omega_k = \frac{-k}{a_0^2 H_0^2} \quad \text{curvature parameter}$$

$$\Omega_\Lambda = \frac{\Lambda}{3H_0^2} \quad \text{cosmological constant parameter}$$

- reionisation

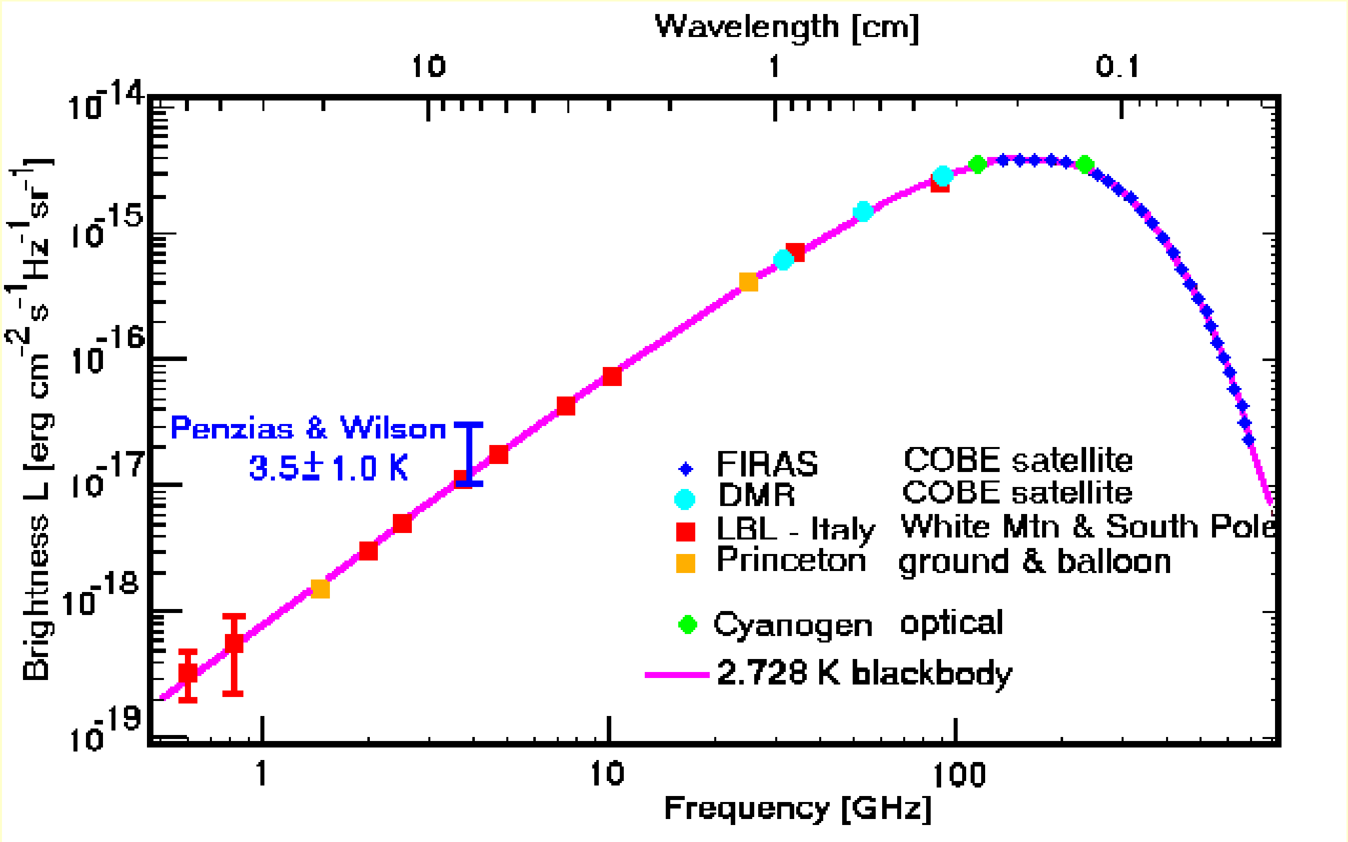
τ	optical depth to the last scattering surface
z_{rei}	redshift of reionisation

- Description of perturbations

A_s	amplitude of scalar perturbations
n_s	spectral index of scalar perturbations
$R = A_T/A_s$	amplitude of tensor perturbations
n_T	spectral index of tensor perturbations
(σ_8	amplitude of perturbations at $8h^{-1}\mathrm{Mpc}$)

The CMB

- After recombination (T ~ 3000K, t~3.8x10^5 years) the photons propagate freely, simply redshifted due to the expansion of the universe
- The spectrum of the CMB is a 'perfect' Planck spectrum:



$|\mu| < 10^{-4}$

$y < 10^{-5}$

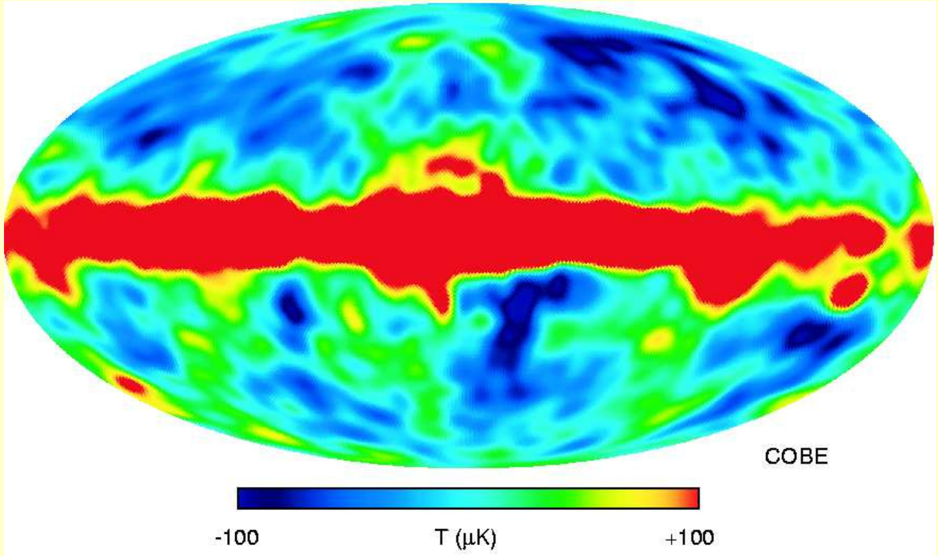
$Y_{ff} < 2\times 10^{-5}$

$\Rightarrow$ ARCADE

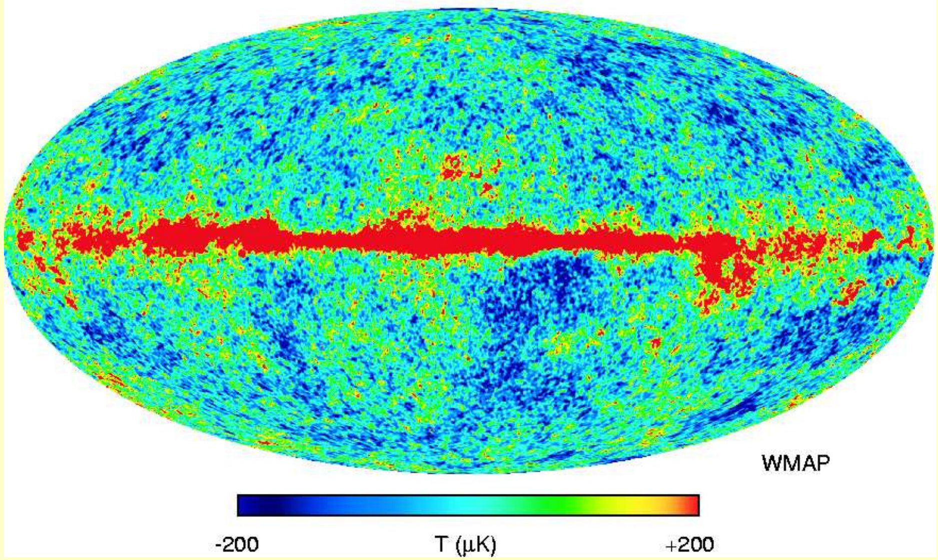
$\Rightarrow$ DIMES

CMB anisotropies

COBE (1992)



WMAP (2003)



The CMB has small fluctuations,

$$\Delta T/T \sim \text{a few} \times 10^{-5}.$$

As we shall see they reflect roughly the amplitude of the gravitational potential.

=> CMB anisotropies can be treated with linear perturbation theory.

The basic idea is, that structure grew out of small initial fluctuations by gravitational instability.

=> At least the beginning of their evolution can be treated with linear perturbation theory.

As we shall see, the gravitational potential does not grow within linear perturbation theory. Hence initial fluctuations with an amplitude of $\sim$ a few $\times 10^{-5}$ are needed.

During a phase of inflationary expansion of the universe such fluctuations emerge out of the quantum fluctuations of the inflation and the gravitational field.

Linear cosmological perturbation theory

- **metric perturbations**

$$g_{\mu\nu} = a^2(\eta_{\mu\nu} + h_{\mu\nu})$$

$$h_{\mu\nu}dx^\mu dx^\nu = -2A dt^2 - 2B_i dx^i dt + 2H_{ij}dx^i dx^j$$

- **Decomposition into scalar, vector and tensor components**

$$B_i = \nabla_i B^{(S)} + B_i^{(V)}$$

$$H_{ij} = -H_L + \left(\nabla_i \nabla_j - \frac{1}{3}\gamma_{ij}\right) H_T + \frac{1}{2}\left(H_{i|j}^{(V)} + H_{j|i}^{(V)}\right) + H_{ij}^{(T)}$$

$$\nabla_i B^{(V)i} = \nabla_i H^{(V)i} = \nabla_i H^{(T)ij} = 0$$

Perturbations of the energy momentum tensor

Density and velocity

$$T^{\mu}_{\nu} u^{\nu} = -\rho u^{\mu}, \quad u^2 = -1$$

$$\rho = \bar{\rho}\,(1+\delta)\,, \quad u = u^0 \partial_t + u^i \partial_i$$

$$u^0 = \frac{1}{a}(1-A) \qquad u^i = \frac{1}{a} v^i$$

stress tensor

$$\tau^{\mu\nu} = P^{\mu}_{\alpha} P^{\nu}_{\beta} T^{\alpha\beta} \qquad P^{\mu}_{\nu} \equiv u^{\mu} u_{\nu} + \delta^{\mu}_{\nu}$$

$$\tau^i_j = \bar{p}\left[(1+\pi_L)\,\delta^i_j + \Pi^i_j\right]$$

Gauge invariance

Linear perturbations change under linearized coordinate transformations, but physical effects are independent of them. It is thus useful to express the equations in terms of gauge-invariant combinations. These usually also have a simple physical meaning.

Gauge invariant metric fluctuations (the Bardeen potentials)

$$\Psi = A - \frac{\dot{a}}{a}(k^{-2}\dot{H}_T - k^{-1}B) - k^{-2}\ddot{H}_T + k^{-1}\dot{B}$$

$$\Phi = H_L + \frac{1}{3}H_T - \frac{\dot{a}}{a}(k^{-2}\dot{H}_T - k^{-1}B)$$

Ψ is the analog of the Newtonian potential. In simple cases $\Phi=\Psi$.

$$h_{\mu\nu}^{(long)} = -2\Psi dt^2 - 2\Phi\gamma_{ij}dx^i dx^j$$

(longitudinal gauge, $B= H_T=0$)

Gauge invariant variables for perturbations of the energy momentum tensor

The anisotropic stress potential **P**

The entropy perturbation

$$\Gamma = \pi_L - \frac{c_s^2}{w}\delta$$

$w=p/\rho$
$c^2_s=p'/\rho'$

Velocity and density perturbations

$$V \equiv v - \frac{1}{k}\dot{H}_T = v^{(\mathrm{long})}$$

$$D_g \equiv \delta + 3(1+w)\left(H_L + \frac{1}{3}H_T\right) = \delta^{(\mathrm{long})} + 3(1+w)\Phi$$

$$D \equiv \delta^{(\mathrm{long})} + 3(1+w)\left(\frac{\dot{a}}{a}\right)\frac{V}{k}$$

The Weyl tensor

The Weyl tensor of a Friedman universe vanishes. Its perturbation it therefore a gauge invariant quantity. For scalar perturbations, its 'magnetic part' vanishes and the electric part is given by

$$E_{ij} = C^{\mu}{}_{ij\nu}u_{\mu}\, u^{\nu} = \tfrac{1}{2}[\partial_i\, \partial_j(\Phi + \Psi) - 1/3\Delta(\Phi + \Psi)\gamma_{ij}]$$

- **Einstein equations**

constraints

$$4\pi G a^2 \rho D = (k^2 - 3\kappa)\Phi$$

$$4\pi G a^2 (\rho + p) V = k\left(\left(\frac{\dot{a}}{a}\right)\Psi - \dot{\Phi}\right)$$

dynamical

$$k^2(\Phi - \Psi) = 4\pi G a^2 p \Pi$$

- **Conservation equations**

$$\dot{D}_g + 3\left(c_s^2 - w\right)\left(\frac{\dot{a}}{a}\right) D_g + (1+w)kV + 3w\left(\frac{\dot{a}}{a}\right)\Gamma = 0$$

$$\dot{V} + \left(\frac{\dot{a}}{a}\right)\left(1 - 3c_s^2\right)V = k\left(\Psi + 3c_s^2\Phi\right) + \frac{c_s^2 k}{1+w} D_g + \frac{wk}{1+w}\left[\Gamma - \frac{2}{3}\left(1 - \frac{3\kappa}{k^2}\right)\Pi\right]$$

Simple solutions and consequences

matter $D \propto a \qquad D_g \propto \text{const.} + (kt)^2\ , \qquad V \propto kt\ , \qquad \Psi = \text{const.}$

radiation

$$D_g = D_2\left[\cos(x) - \frac{2}{x}\sin(x)\right] + D_1\left[\sin(x) + \frac{2}{x}\cos(x)\right]$$

$x=c_skt$

$$V = -\frac{\sqrt{3}}{4}D_g' \qquad \Psi = -\frac{D_g + \frac{4}{\sqrt{3}x}V}{4+2x^2}$$

- The D_1-mode is singular, the D_2-mode is the adiabatic mode
- In a mixed matter/radiation model there is a second regular mode, the isocurvature mode
- On super horizon scales, $x<1$, Ψ is constant
- On sub horizon scales, D_g and V oscillate while Ψ oscillates and decays like $1/x^2$ in a radiation universe.

lightlike geodesics

From the surface of last scattering into our antennas the CMB photons travel along geodesics. By integrating the geodesic equation, we obtain the change of energy in a given direction n:

$E_f/E_i = (n{\cdot}u)_f/(n{\cdot}u)_i = [T_f/T_i](1+ \Delta T_f/T_f - \Delta T_i/T_i)$

This corresponds to a temperature variation. In first order perturbation theory one finds for scalar perturbations

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)} n^j + \Psi + \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0} (\dot{\Psi} + \dot{\Phi})(\eta, \mathbf{x}(\eta))\, d\eta$$

acoustic oscillations

Doppler term

gravitat. potentiel
(Sachs Wolfe)

integrated Sachs Wolfe
ISW

Boltzmann eqn. I

Integrating the 1-particle distribution function of photons over energy, one arrives at the brightness,

$$\iota(t,\mathbf{x},\mathbf{n}) = \rho(t)\left[1 + 4\left(\frac{\Delta T}{T}(t,\mathbf{x},\mathbf{n}) - \Phi(t,\mathbf{x})\right)\right]$$

Taking into acount elastic Thomson scattering before decoupling, one obtains a the following Boltzmann eqn. (in k-space, $M \equiv \Delta T/T$)

$$\dot{\mathcal{M}} + ik\mu\mathcal{M} = -ik\mu(\Psi+\Phi) + a\sigma_T n_e\left[-\mathcal{M} + \mathcal{M}_0 + i\mu V_b + \frac{1}{2}P_2(\mu)\mathcal{M}_2\right]$$

with

$$\mathcal{M} = \sum_\ell (2\ell+1)(-i)^\ell \mathcal{M}_\ell(t,k)P_\ell(\mu)$$

we find the Boltzmann hierarchy

$$\dot{\mathcal{M}}_\ell + \frac{k(\ell+1)}{2\ell+1}\mathcal{M}_{\ell+1} - \frac{k\ell}{2\ell+1}\mathcal{M}_{\ell-1} + a\sigma_T n_e \mathcal{M}_\ell = \frac{k}{3}(\Psi+\Phi)\delta_{\ell 1} + a\sigma_T n_e\left[\mathcal{M}_0\delta_{\ell 0} + \frac{1}{3}V_b\delta_{\ell 1} + \frac{1}{10}\mathcal{M}_2\delta_{\ell 2}\right]$$

Boltzmann eqn. II

Integral 'solution', $\kappa = \int a\sigma_T\, n_e\, dt$

$$\mathcal{M}(t_0) = \int_{t_{\rm in}}^{t_0} dt e^{ik\mu(t-t_0)} e^{-\kappa}\left[-ik\mu(\Phi+\Psi) + \underbrace{a\sigma_T n_e e^{-\kappa}}_{\text{g}}\left(\mathcal{M}_0 + i\mu V_B + \frac{1}{2}P_2(\mu)\mathcal{M}_2\right)\right]$$

Via integrations by part we can move all μ dependence in the exponential,

$$\begin{aligned} \mathcal{M}(k,\mu) &= \int_{t_{\rm in}}^{t_0} dt e^{ik\mu(t-t_0)} S(k,t) \\ S(k,t) &= e^{-\kappa}(\dot\Phi+\dot\Psi) + g\left(\mathcal{M}_0 - \dot V_B/k - \frac{1}{4}\mathcal{M}_2 - \frac{3}{4k^2}\ddot{\mathcal{M}}_2\right) - \dot g\left(V_B/k + \frac{3}{4k^2}\dot{\mathcal{M}}_2\right) - \frac{3\ddot g}{4k^2}\mathcal{M}_2 \end{aligned}$$

$$\mathcal{M}_\ell(k) = \int_{t_{\rm in}}^{t_0} dt j_\ell(k(t_0-t)) S(k,t)$$

$$C_\ell = (4\pi)^2 \int dk k^2 |\mathcal{M}_\ell(k)|^2$$

The power spectrum of CMB anisotropies

$\Delta T(\mathbf{n})$ is a function on the sphere, we can expand it in spherical harmonics

$$\frac{\Delta T}{T}(\mathbf{x}_0, \mathbf{n}, \quad) = \sum_{\ell,m} a_{\ell m}(\mathbf{x}_0) Y_{\ell m}(\mathbf{n}) \qquad \langle a_{\ell m} \cdot a^*_{\ell' m'} \rangle = \delta_{\ell\ell'} \delta_{mm'} C_\ell$$

consequence of statistical isotropy

$$\langle \frac{\Delta T}{T}(\mathbf{n}) \frac{\Delta T}{T}(\mathbf{n}') \rangle = \frac{1}{4\pi} \sum_\ell (2\ell+1) C_\ell P_\ell(\mathbf{n} \cdot \mathbf{n}')$$

observed mean

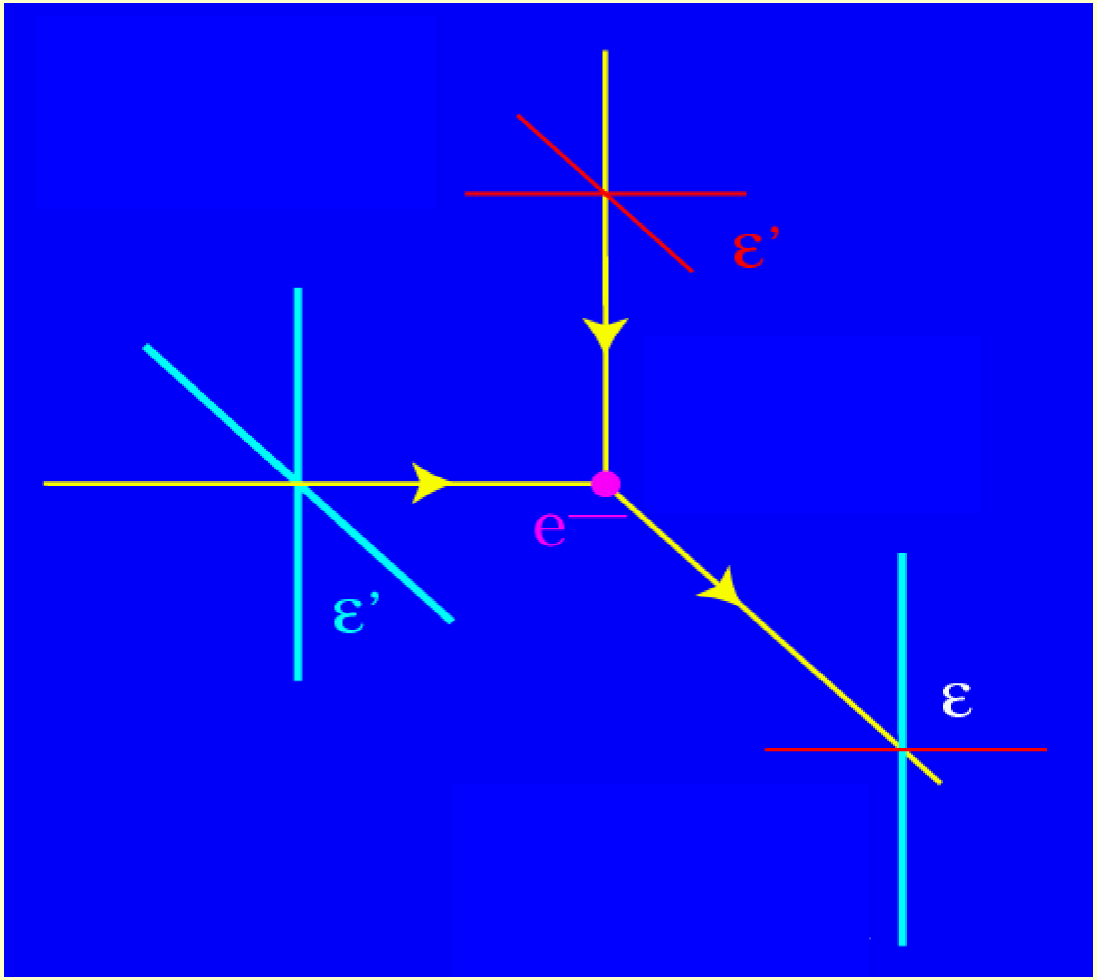
$$\frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2 \equiv C_\ell^{\rm obs}$$

cosmic variance (if the a_{lm} 's are Gaussian)

$$\frac{\sqrt{\left\langle (C_\ell^{(obs)} - C_\ell)^2 \right\rangle}}{C_\ell} = \sqrt{\frac{2}{2\ell+1}}$$

Polarisation

- Thomson scattering depends on polarisation: a quadrupole anisotropy of the incoming wave generates linear polarisation of the outgoing wave.



Polarisation can be described by the Stokes parameters, but they depend on the choice of the coordinate system. The (complex) amplitude

$\varepsilon_i e^i$ of the 2-component electric field defines the spin 2 intensity $A_{ij} = \varepsilon_i^* \varepsilon_j(n)$ which can be written in terms of Pauli matrices as

$$A = \frac{1}{2}[I\sigma_0 + U\sigma_1 + V\sigma_2 + Q\sigma_3] = \frac{1}{2}[I\sigma_0 + V\sigma_2 + (Q+iU)\sigma_+ + (Q-iU)\sigma_-]$$

$Q \pm iU$ are the $m = \pm 2$ spin eigenstates, which are expanded in spin 2 spherical harmonics. Their real and imaginary parts are called the 'electric' and 'magnetic' polarisations.

$$[Q(\mathbf{n}) \pm iU(\mathbf{n})]\sigma_\pm(\mathbf{n})_{ab} = \sum_{\ell m} a_{\ell m}^{(\pm)} [{}_{\pm 2}Y_{\ell m}(\mathbf{n})]_{ab}$$

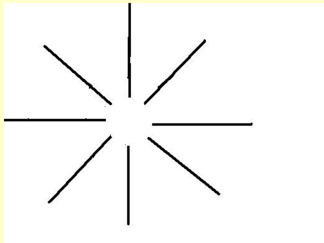
$$a_{\ell m}^E = \frac{1}{2}\left(a_{\ell m}^{(+)} + a_{\ell m}^{(-)}\right), \qquad a_{\ell m}^B = \frac{-i}{2}\left(a_{\ell m}^{(+)} - a_{\ell m}^{(-)}\right)$$

$$\langle a_{\ell m}^X a_{\ell' m'}^{*Y} \rangle = \delta_{\ell\ell'}\delta_{mm'}C_\ell^{XY}$$

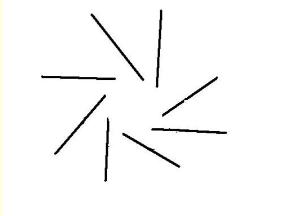
(Seljak & Zaldarriaga, 97, Kamionkowski et al. '97, Hu & White '97)

Under parity operation ${}_{\pm 2}Y_{l\,m} \rightarrow (-1)^l {}_{\mp 2}\,Y_{l\,m}$
Hence E has the same parity as ΔT while B has parity $(-1)^{l+1}$. E describes gradient fields on the sphere (generated by scalar as well as tensor modes), while B describes the rotational component of the polarisation field (generated only by tensor or vector modes).

E-polarisation
(generated by scalar and tensor modes)



B-polarisation
(generated only by the tensor mode)



Due to their parity, T and B and E and B are not correlated while T and E are.

An additional effect on CMB fluctuations is Silk damping: on small scales, of the order of the size of the mean free path of CMB photons, fluctuations are damped due to free streaming: photons stream out of over-densities into under-densities.

To compute the effects of Silk damping and polarisation we have to solve the Boltzmann equation for M, E and B of the CMB radiation. This is usually done with the 'line of sight method' in standard, publicly available codes like CMBfast (Seljak & Zaldarriaga) , CAMBcode (Bridle & Lewis) or CMBeasy (Doran).

The physics of CMB fluctuations

- Large scales : The gravitational potential on the surface of last scattering, time dependence of the gravitational potential $\Psi \sim 10^{-5}$.

$\theta > 1^{o}$

$\ell < 100$

- Intermediate scales : Acoustic oscillations of the baryon/photon fluid before recombination.

$6' < \theta < 1^{o}$

$100 < \ell < 800$

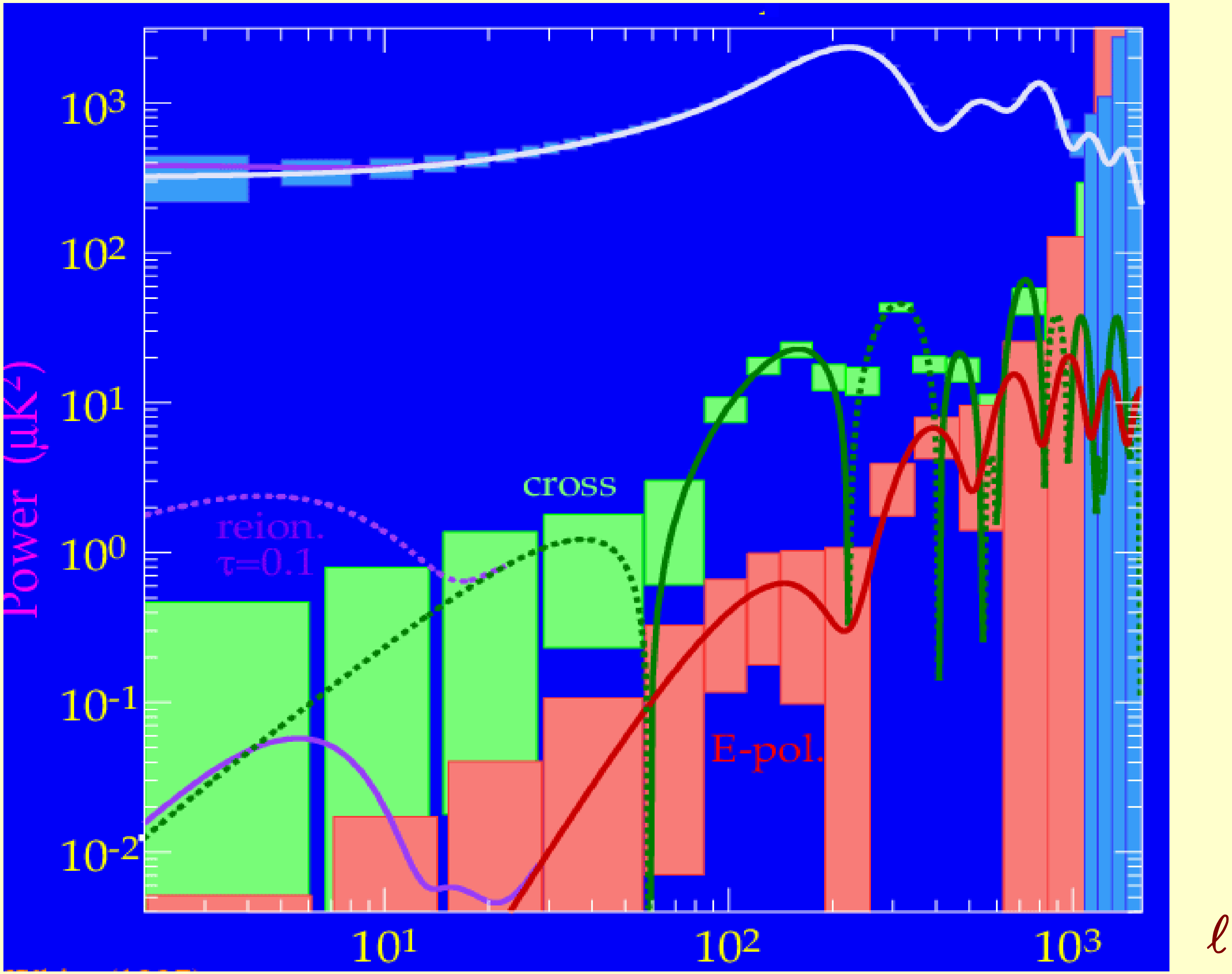
- Small scales : Damping of fluctuations due to the imperfect coupling of photons and electrons during recombination

(Silk damping).

$\theta < 6'$

$800 > \ell$

Power spectra of scalar fluctuations



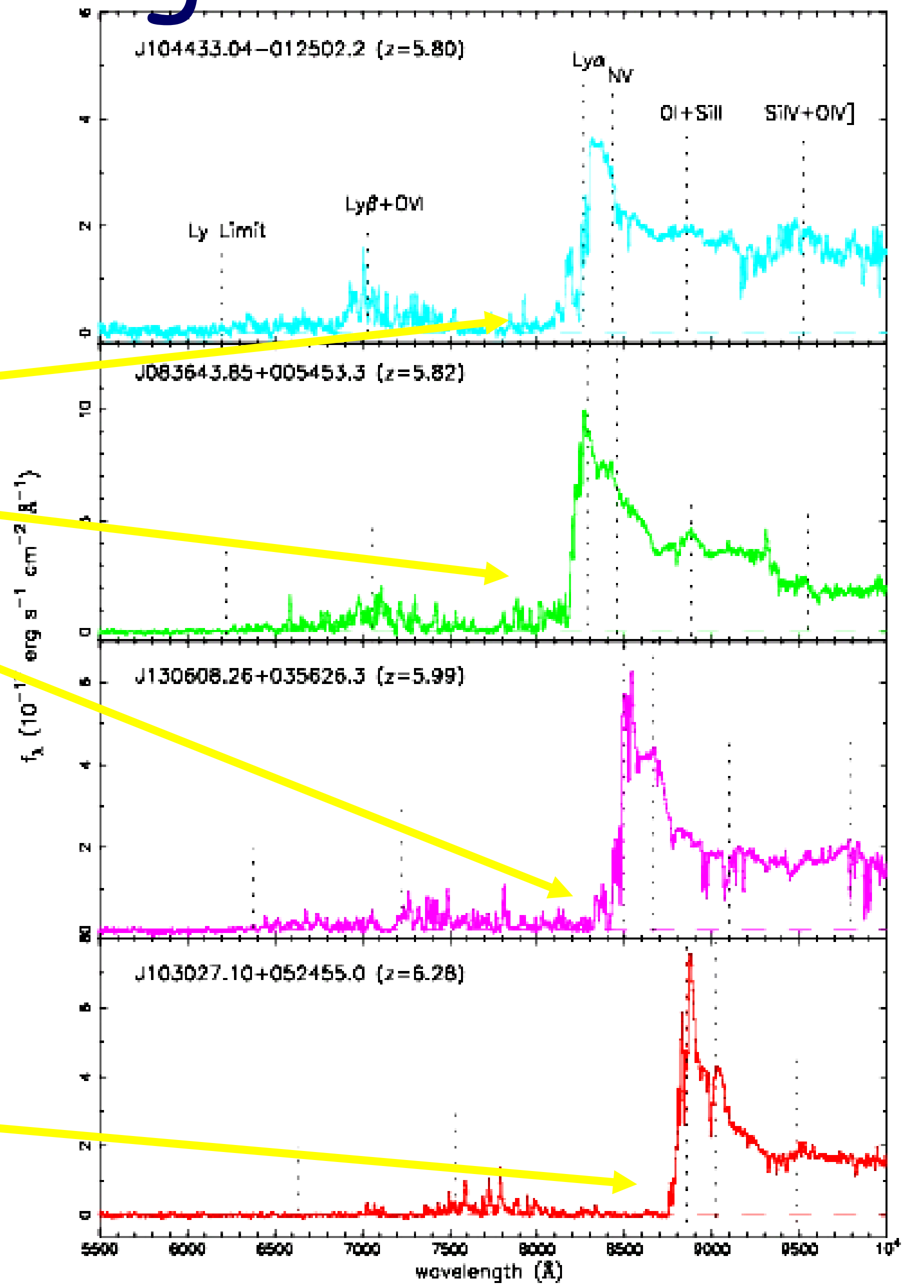
Reionization

The absence of the so called Gunn-Peterson trough in quasar spectra tells us that the universe is reionised since, at least, $z \sim 6$.
Reionisation leads to a certain degree of re-scattering of CMB photons. This induces additional damping of anisotropies and additional polarisation on large scales (up to the horizon scale at reionisation). It enters the CMB spectrum mainly through one parameter, the optical depth τ to the last scattering surface or the redshift of reionisation z_{re} .

Gunn Peterson trough

In quasars with z<6.1 the photons with wavelength shorter that Ly-a are not absorbed.

normal emission



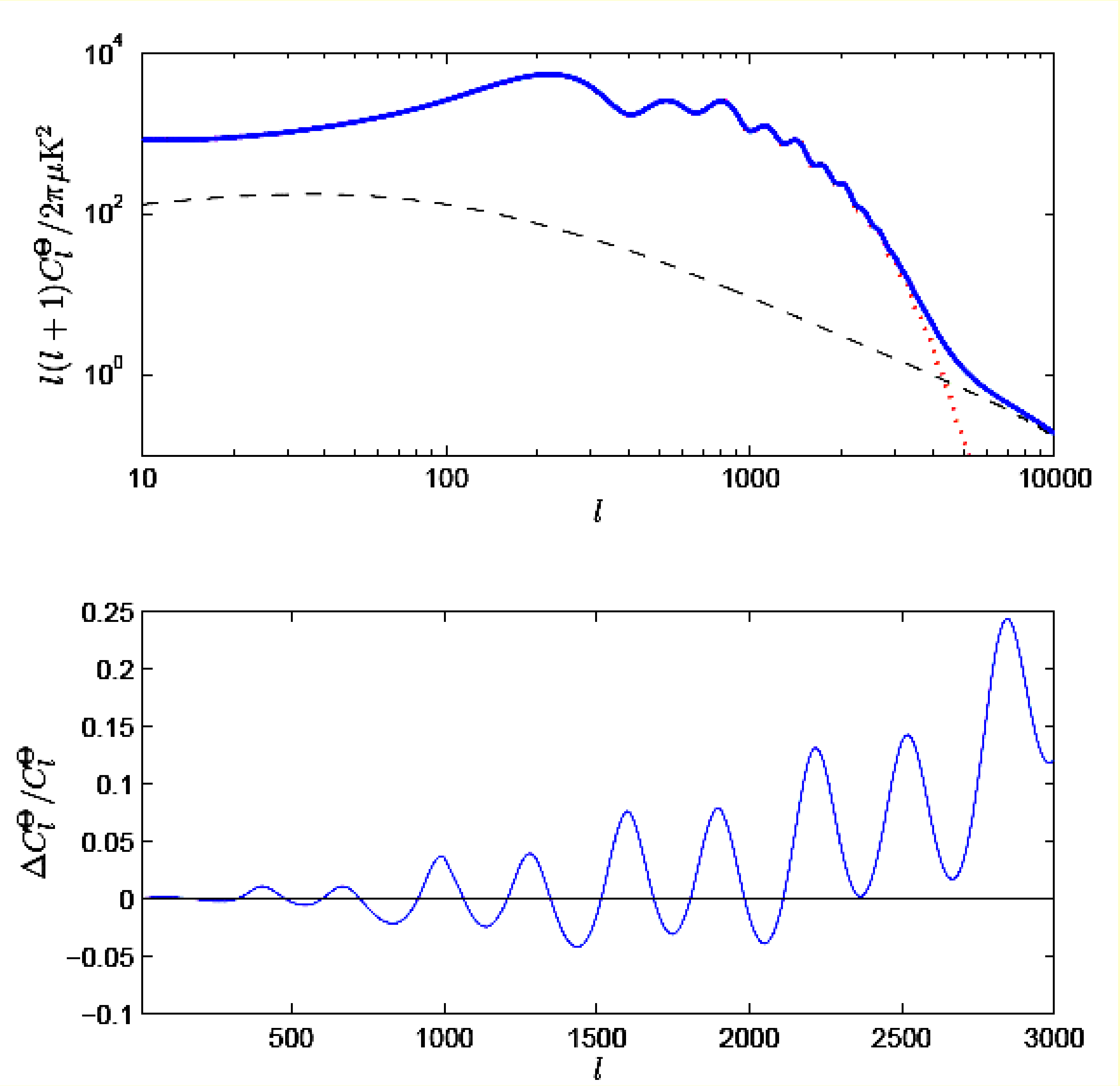
(Becker et al. 2001)

no emission

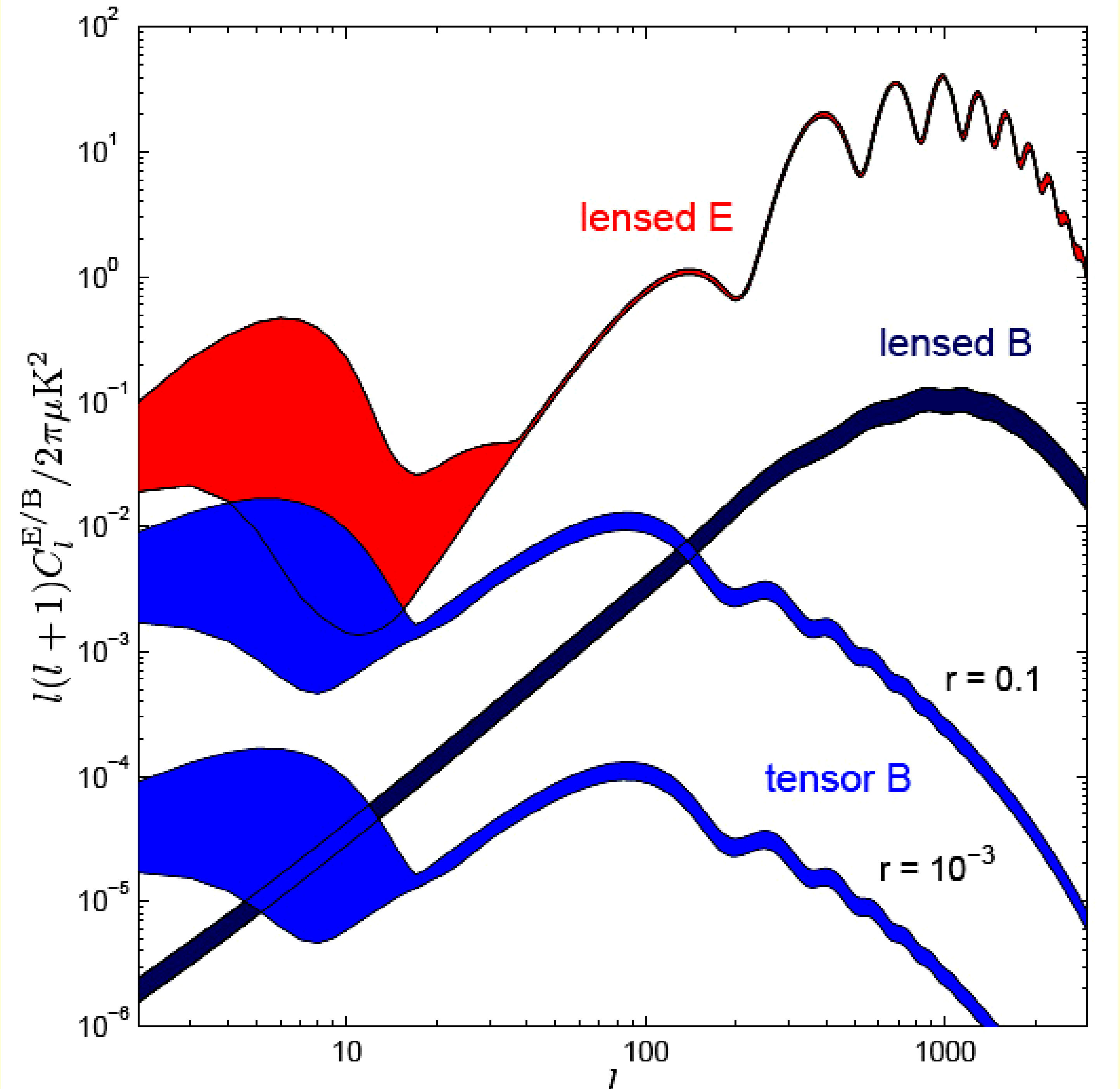
Due

phot

$\alpha =$

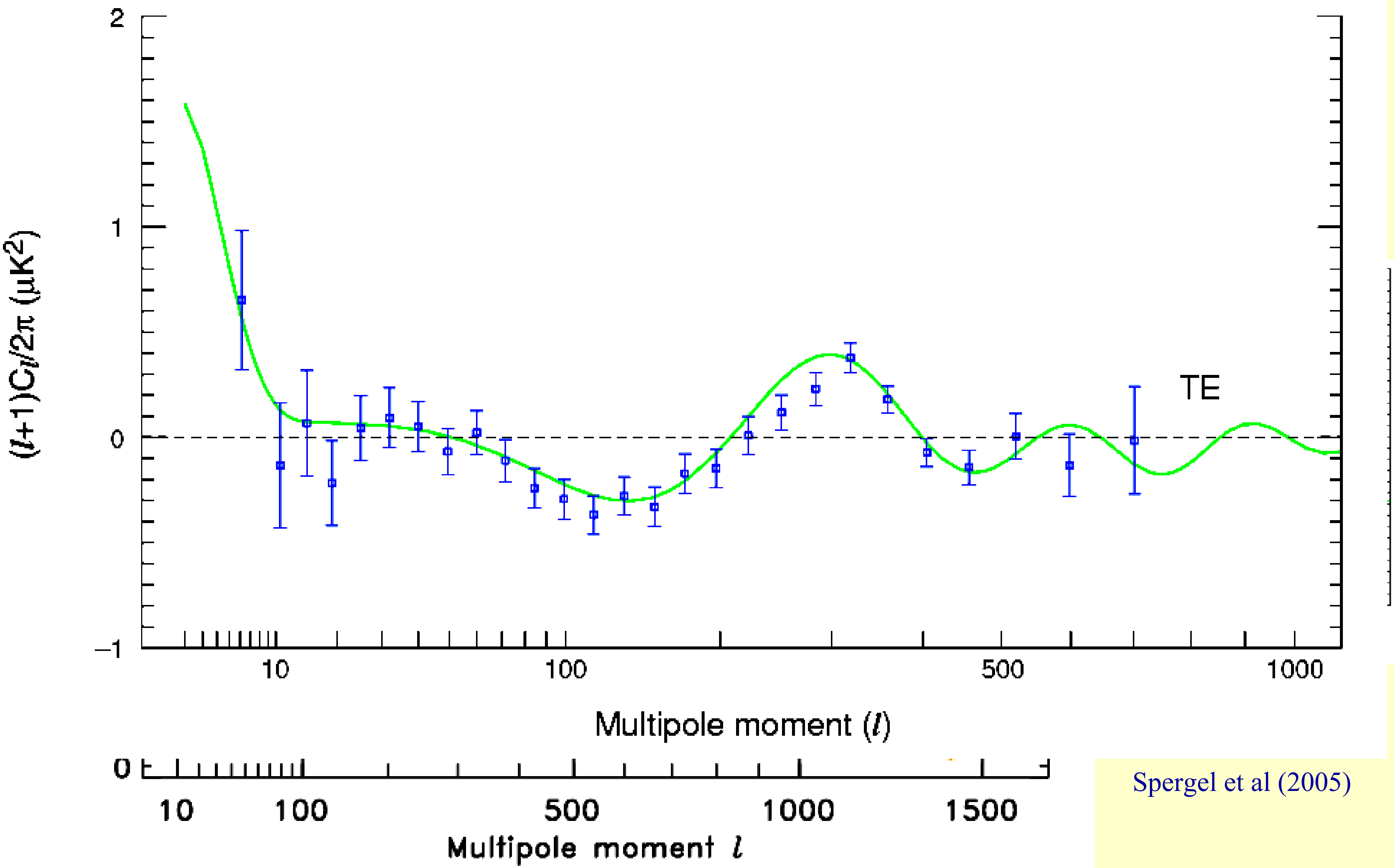


Challinor &
Lewis '06



Challinor & Lewis '06

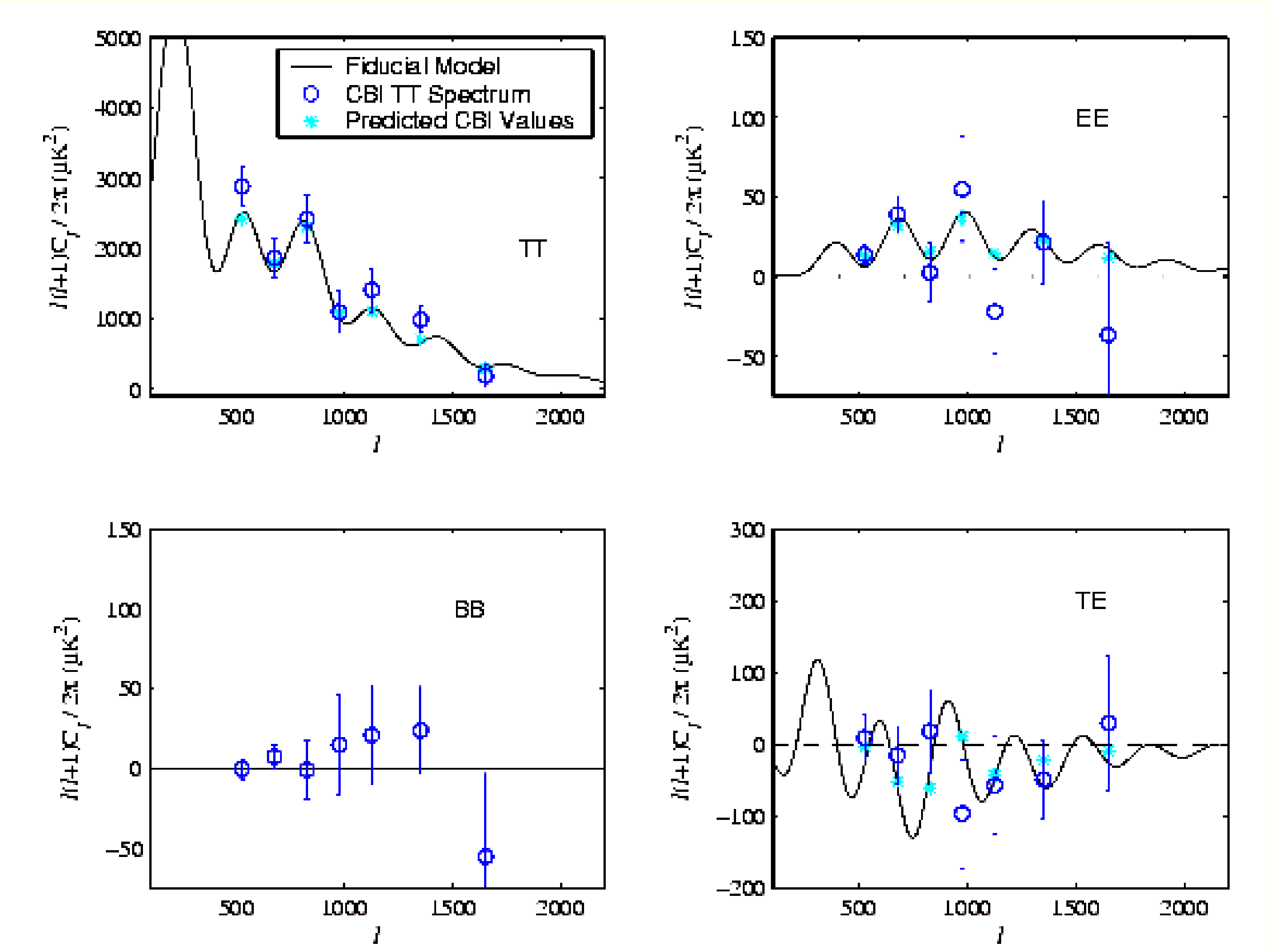
WMAP data



Spergel et al (2005)

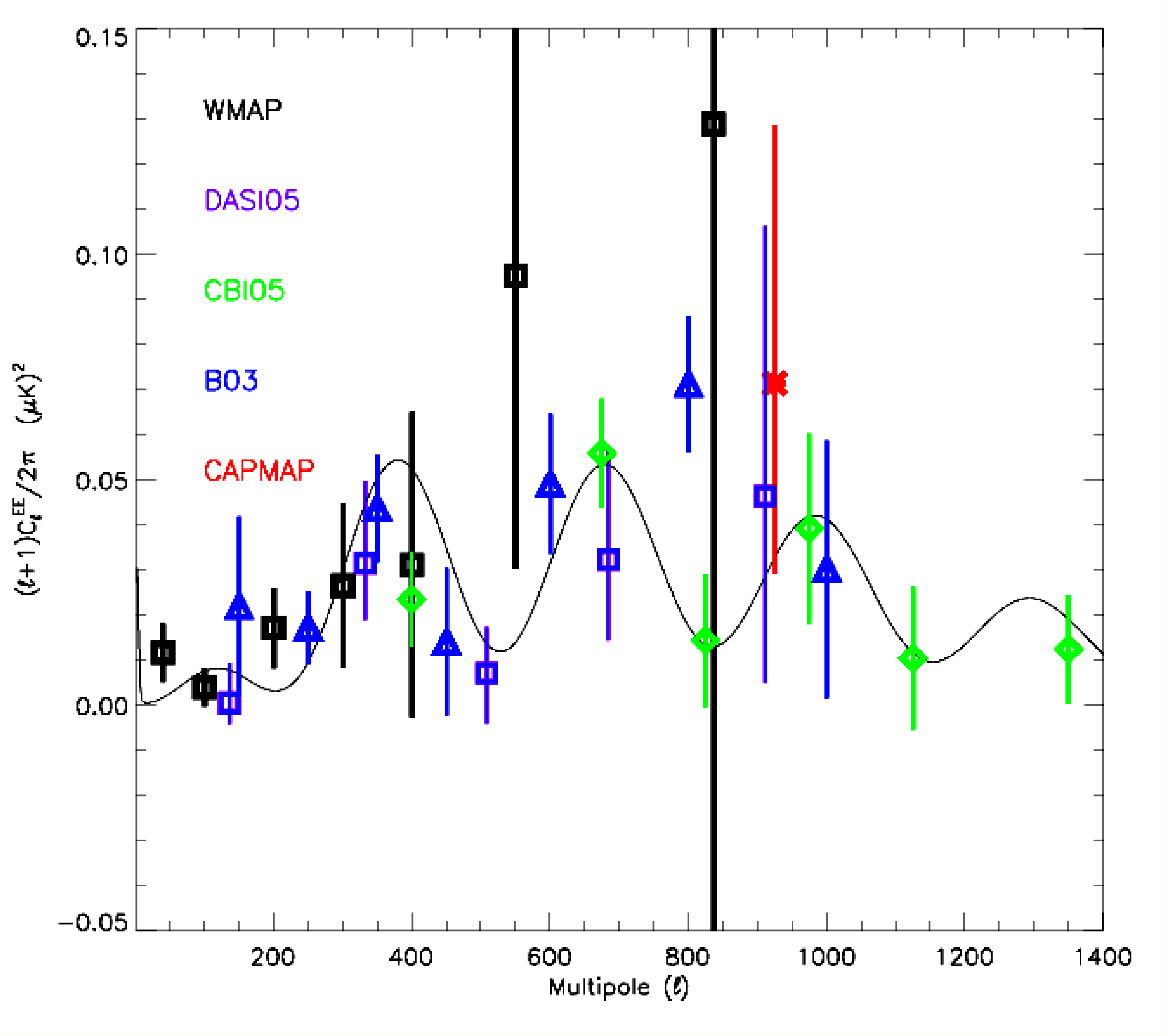
Other polarization data I

CBI



From Readhead et al. 2004

WMAP and other polarisation data



From Page et al. 2006

Acoustic oscillations

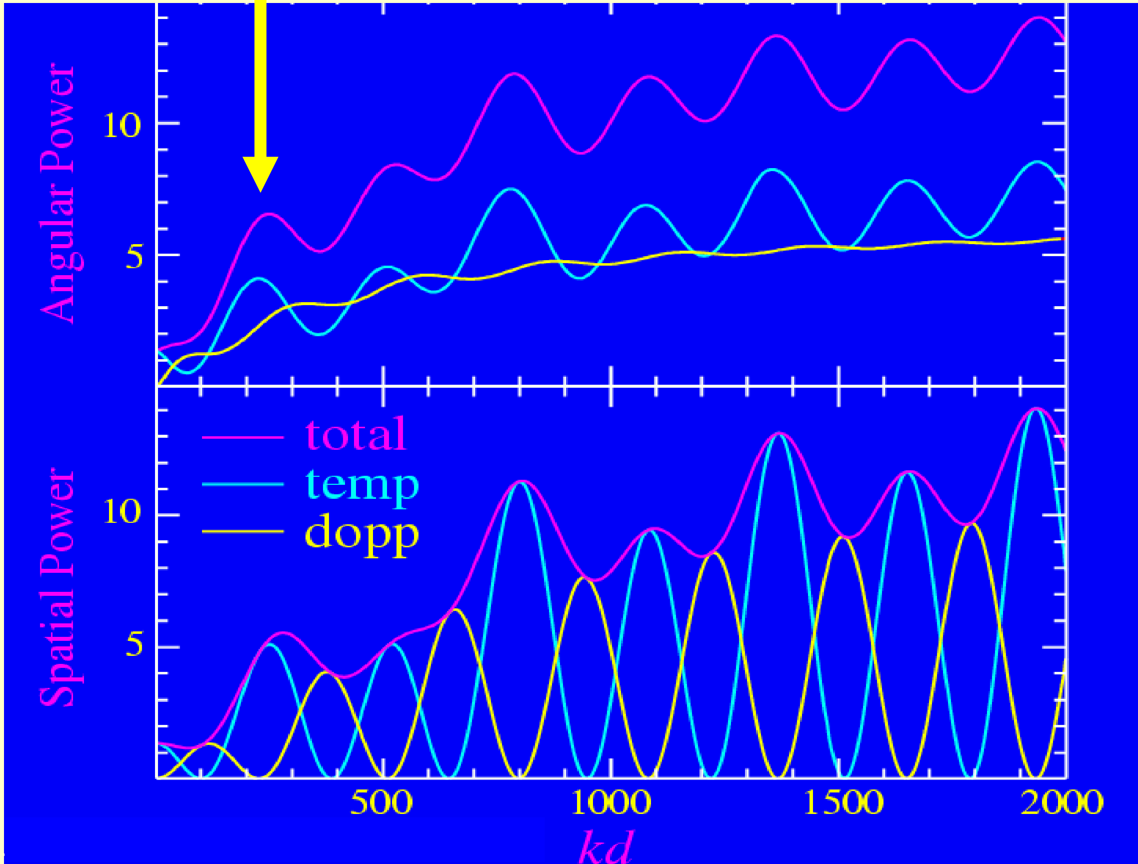
Determine the angular distance to the last scattering surface, z_1

$$\eta_0 - \eta_1 = \frac{1}{H_0 a_0}\int_0^{z_1} \frac{dz}{[\Omega_{\rm rad}(z+1)^4 + \Omega_m(z+1)^3 + \Omega_\Lambda + \Omega_\kappa(z+1)^2]^{\frac{1}{2}}}$$

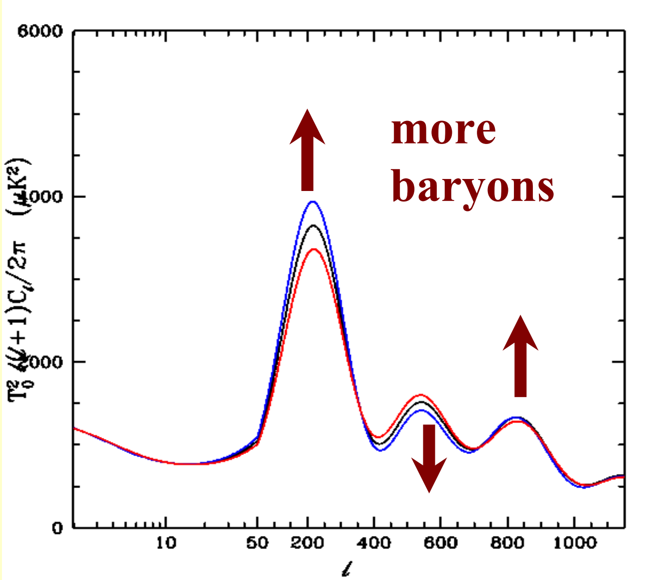
$$\eta_1 = \frac{1}{H_0 a_0}\int_{z_1}^{\infty} \frac{dz}{\left[\Omega_{\rm rad}(z+1)^4 + \Omega_m(z+1)^3 + \Omega_\Lambda + \Omega_\kappa(z+1)^2\right]^{\frac{1}{2}}}.$$

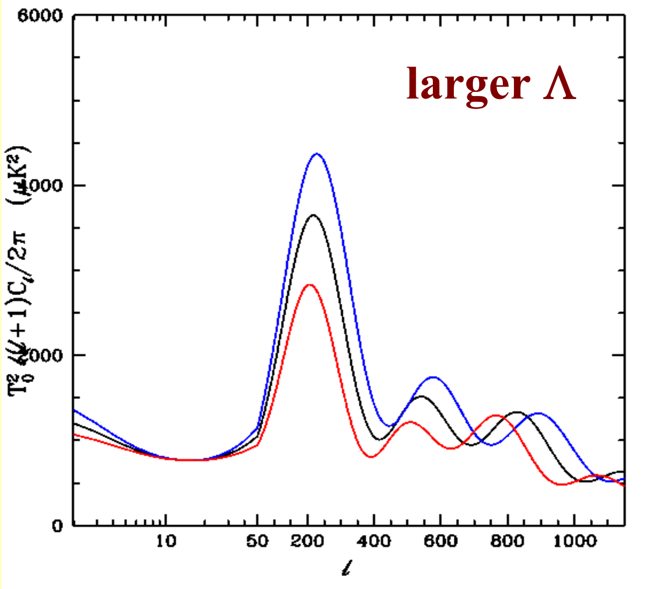
$$\vartheta_A = \frac{c_s \eta_1}{\chi(\eta_0 - \eta_1)}$$

Is known with 1.7% accuracy from WMAP data

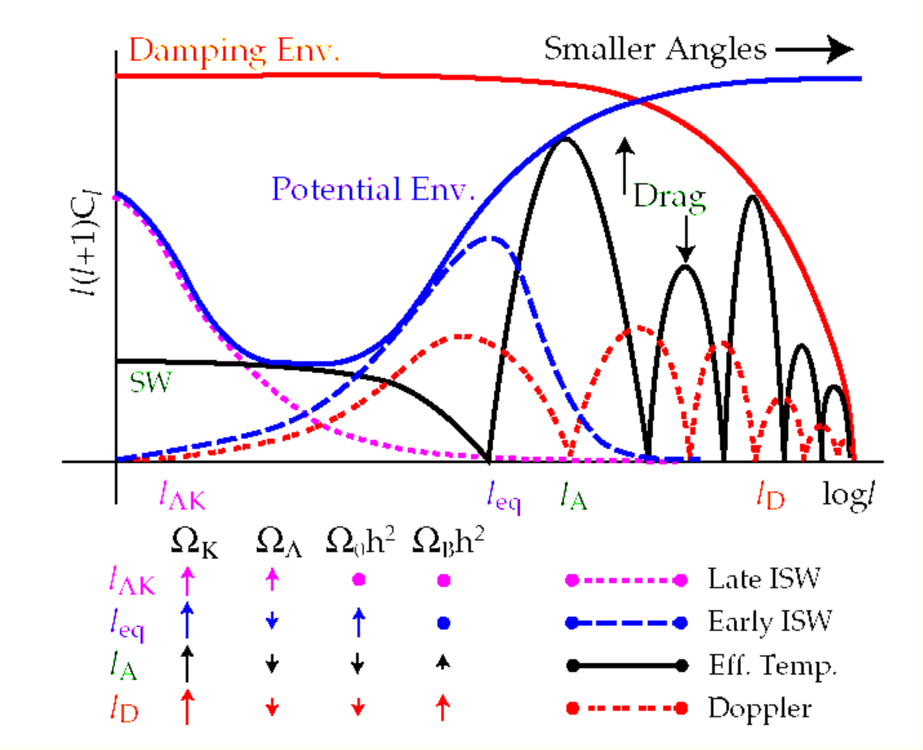


Dependence on cosmological parameters





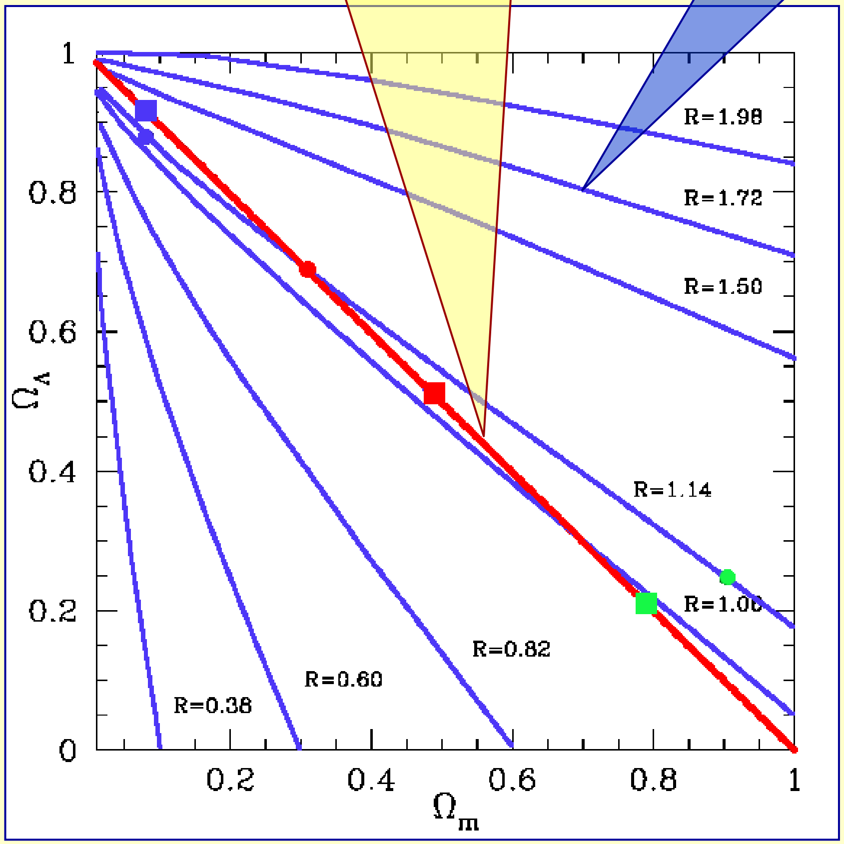
Most cosmological parameters have complicated effects on the CMB spectrum



Geometrical degeneracy

Flat Universe (ligne of constant curvature Ω_K=0)

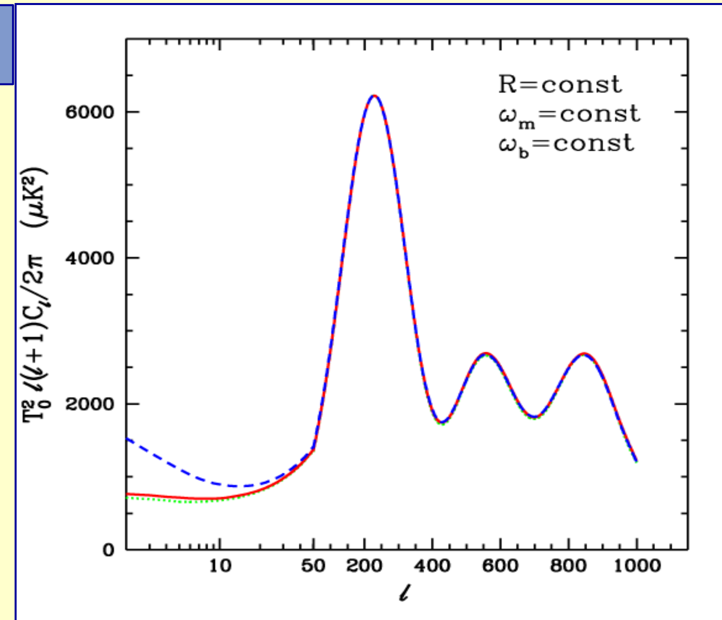
degeneracy lines:

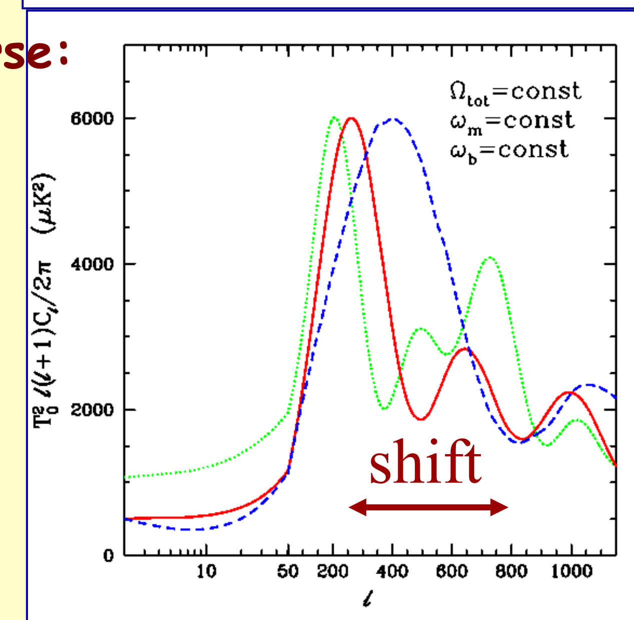


Degeneracy:

$\omega = \Omega\, h^2$

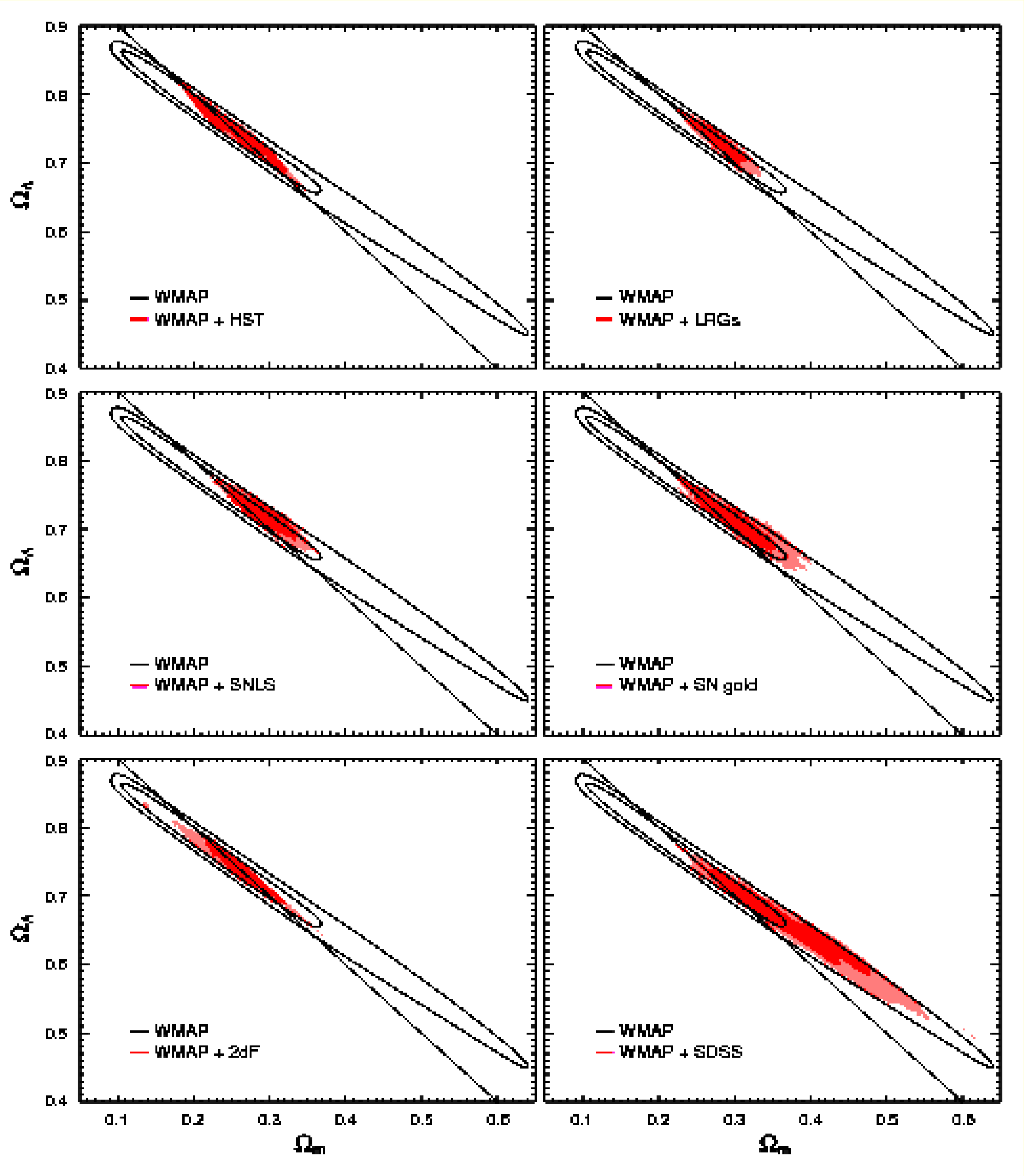
Flat Universe:





Shift parameter: $R = R(\Omega_\Lambda, \Omega_m)$

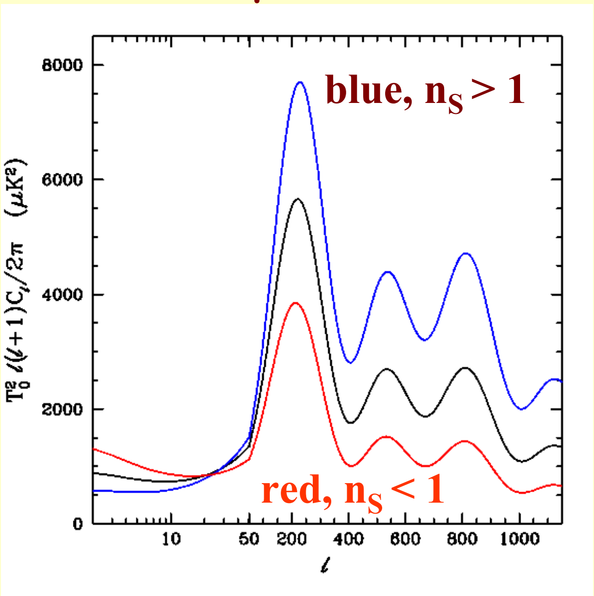
geometrical degeneracy II



Spergel et al. 2006

Primordial parameters

Scalar spectum:



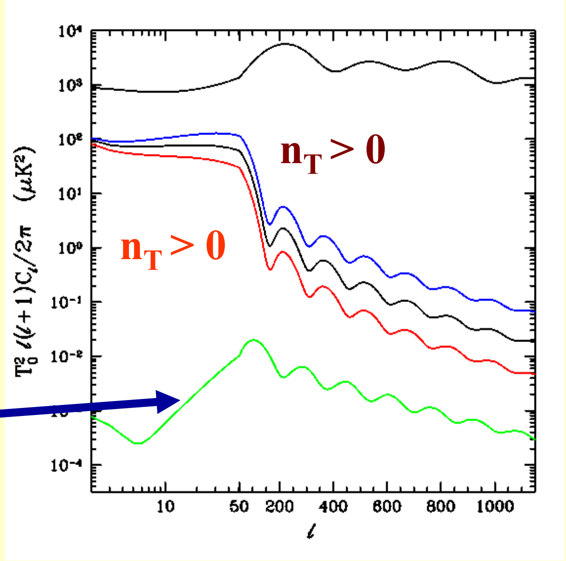
scalar spectral index n_S and amplitude A

$$\langle \Psi^2 \rangle = A k^{n_S - 1}$$

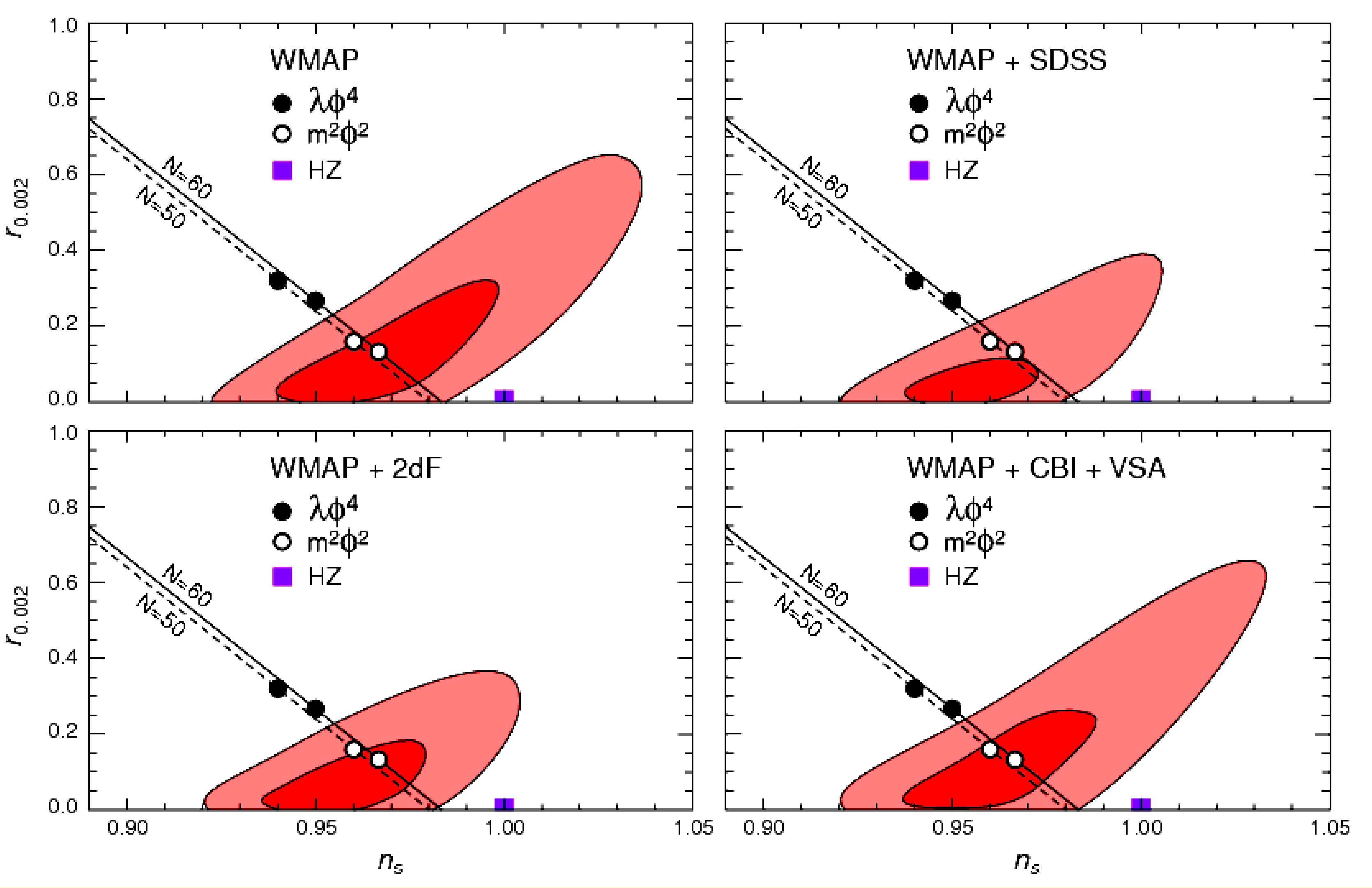
$n_S = 1$: scale invariant spectrum (Harrison-Zel'dovich)

Tensor spectum: (gravity waves)

The 'smoking gun' of inflation, has not yet been detected: B modes of the polarisation (Bpol, ...).



Primordial parameters



Spergel et al. 2006

Measured cosmological parameters

(With CMB + flatness or CMB + Hubble)

Parameter	First Year Mean	WMAPext Mean	Three Year Mean	First Year ML	WMAPext ML	Three Year ML
$100\Omega_b h^2$	$2.38^{+0.13}_{-0.12}$	$2.32^{+0.12}_{-0.11}$	2.23 ± 0.08	2.30	2.21	2.23
$\Omega_m h^2$	$0.144^{+0.016}_{-0.016}$	$0.134^{+0.006}_{-0.006}$	0.126 ± 0.009	0.145	[illegible]	28
H_0	72^{+5}_{-5}	73^{+3}_{-3}	74^{+3}_{-3}	68	[illegible]	3
τ	$0.17^{+0.08}_{-0.07}$	$0.15^{+0.07}_{-0.07}$	0.093 ± 0.029	0.10	0.10	0.092
n_s	$0.99^{+0.04}_{-0.04}$	$0.98^{+0.03}_{-0.03}$	0.961 ± 0.017	0.97	0.96	0.958
Ω_m	$0.29^{+0.07}_{-0.07}$	$0.25^{+0.03}_{-0.03}$	0.234 ± 0.035	0.32	0.27	0.24
σ_8	$0.92^{+0.1}_{-0.1}$	$0.84^{+0.06}_{-0.06}$	0.76 ± 0.05	0.88	0.82	0.77

$\Omega_\Lambda = 0.75 \pm 0.07$

(Spergel et al. 2006)

Attention: **FLATNESS** imposed!!!

On the other hand: $\Omega_{tot} = 1.02$ +/- 0.02 with the HST prior on *h*...

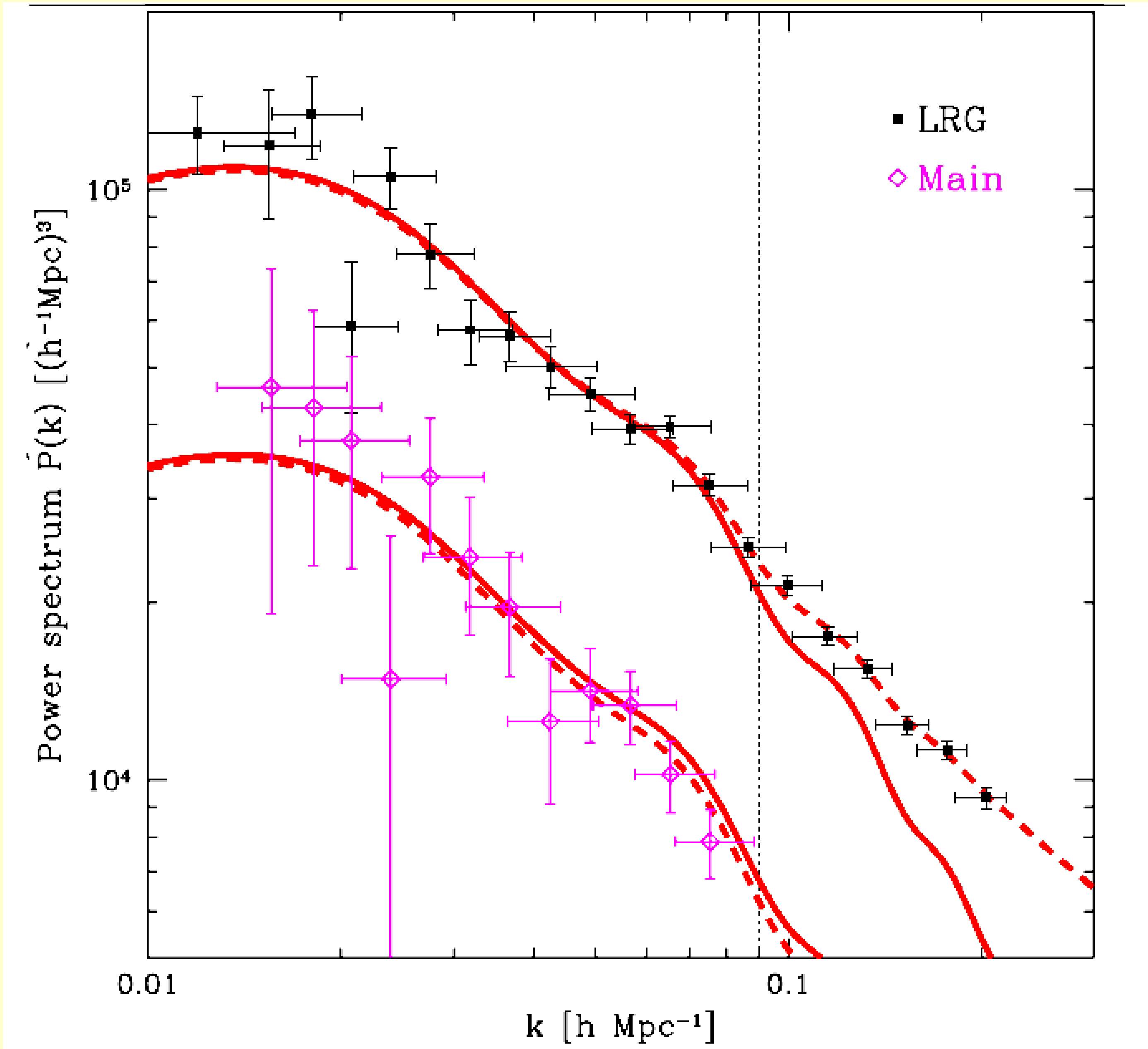
Measured cosmological parameters

Table 6: ΛCDM Model

Parameter	WMAP+ SDSS	WMAP+ LRG	WMAP+ SNLS	WMAP + SN Gold	WMAP+ CFHTLS
$100\Omega_b h^2$	$2.233^{+0.062}_{-0.086}$	$2.242^{+0.062}_{-0.084}$	$2.233^{+0.069}_{-0.088}$	$2.227^{+0.065}_{-0.082}$	$2.255^{+0.062}_{-0.083}$
$\Omega_m h^2$	$0.1329^{+0.0056}_{-0.0075}$	$0.1337^{+0.0044}_{-0.0061}$	$0.1295^{+0.0056}_{-0.0072}$	$0.1349^{+0.0056}_{-0.0071}$	$0.1408^{+0.0034}_{-0.0050}$
h	$0.709^{+0.024}_{-0.032}$	$0.709^{+0.016}_{-0.023}$	$0.723^{+0.021}_{-0.030}$	$0.701^{+0.020}_{-0.026}$	$0.687^{+0.016}_{-0.024}$
A	$0.813^{+0.042}_{-0.052}$	$0.816^{+0.042}_{-0.049}$	$0.808^{+0.044}_{-0.051}$	$0.827^{+0.045}_{-0.053}$	$0.846^{+0.037}_{-0.047}$
τ	$0.079^{+0.029}_{-0.032}$	$0.082^{+0.028}_{-0.033}$	$0.085^{+0.028}_{-0.032}$	$0.079^{+0.028}_{-0.034}$	$0.088^{+0.026}_{-0.032}$
n_s	$0.948^{+0.015}_{-0.018}$	$0.951^{+0.014}_{-0.018}$	$0.950^{+0.015}_{-0.019}$	$0.946^{+0.015}_{-0.019}$	$0.953^{+0.015}_{-0.019}$
σ_8	$0.772^{+0.036}_{-0.048}$	$0.781^{+0.032}_{-0.045}$	$0.758^{+0.038}_{-0.052}$	$0.784^{+0.035}_{-0.049}$	$0.826^{+0.022}_{-0.035}$
Ω_m	$0.266^{+0.026}_{-0.036}$	$0.267^{+0.018}_{-0.025}$	$0.249^{+0.024}_{-0.031}$	$0.276^{+0.023}_{-0.031}$	$0.299^{+0.019}_{-0.025}$

Spergel et al. 2006

Galaxy distribution (LSS)



Tegmark et al. 2006

Sloan LRG combined with WMAP 3

(Tegmark et al. 2006)

e 2: Cosmological parameters measured from WMAP and SDSS LRG data with the Occam's razor approach d marginalized over all other parameters in the vanilla set ($\omega_b, \omega_c, \Omega_\Lambda, A_s, n_s, \tau, b,$

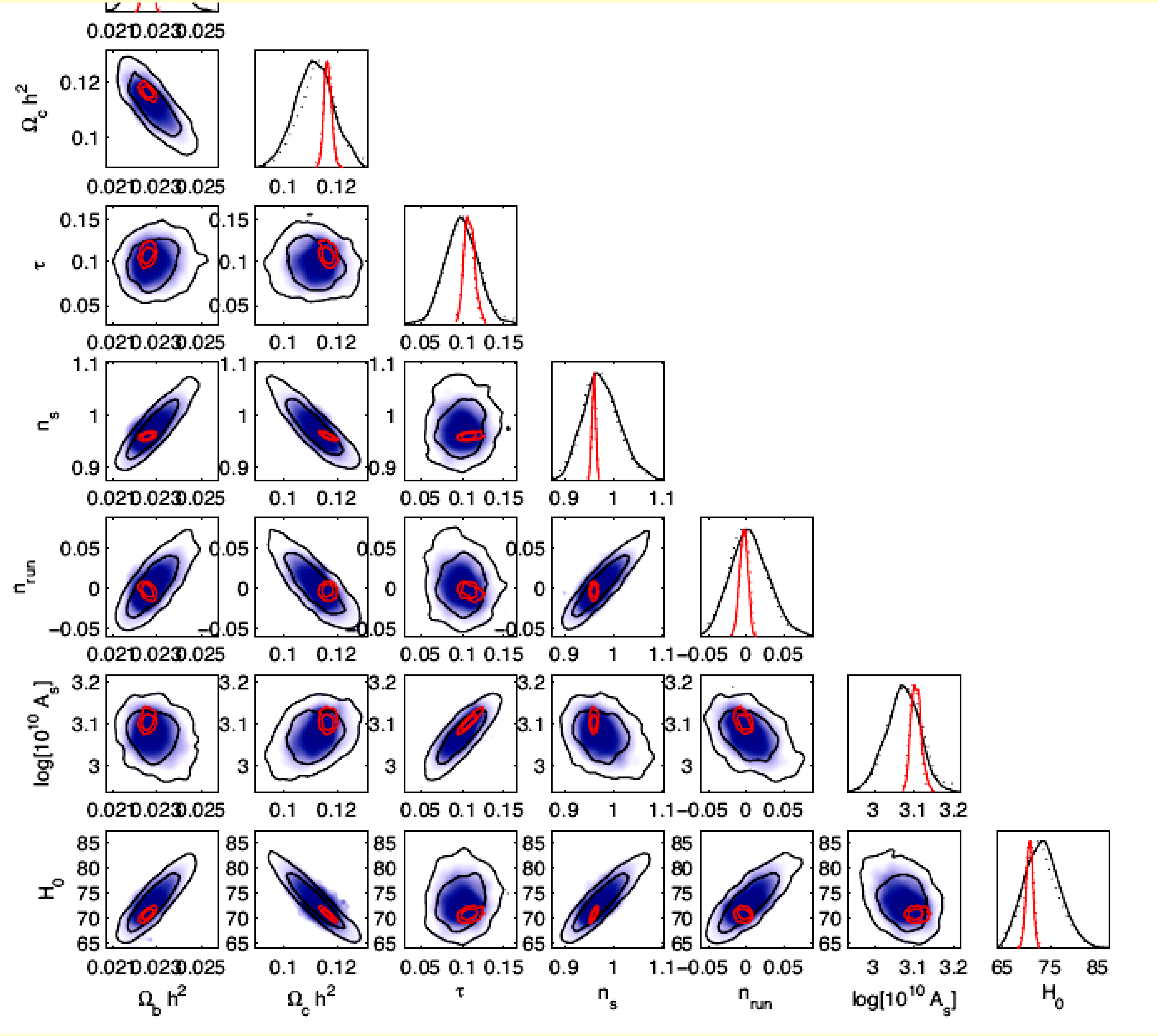
Parameter	Value	Meaning	Definition
Matter budget parameters:			
Ω_{tot}	$1.003^{+0.010}_{-0.009}$	Total density/critical density	$\Omega_{tot} = \Omega_m + \Omega_\Lambda = 1 -$
Ω_Λ	$0.761^{+0.017}_{-0.018}$	Dark energy density parameter	$\Omega_\Lambda \approx h^{-2}\rho_\Lambda(1.88 \times 10^{-}$
ω_b	$0.0222^{+0.0007}_{-0.0007}$	Baryon density	$\omega_b = \Omega_b h^2 \approx \rho_b/(1.88 \times$
ω_c	$0.1050^{+0.0041}_{-0.0040}$	Cold dark matter density	$\omega_c = \Omega_c h^2 \approx \rho_c/(1.88 \times$
ω_ν	< 0.010 (95%)	Massive neutrino density	$\omega_\nu = \Omega_\nu h^2 \approx \rho_\nu/(1.88$
w	$-0.941^{+0.087}_{-0.101}$	Dark energy equation of state	p_Λ/ρ_Λ (approximated as
Seed fluctuation parameters:			
A_s	$0.690^{+0.045}_{-0.044}$	Scalar fluctuation amplitude	Primordial scalar power
r	< 0.30 (95%)	Tensor-to-scalar ratio	Tensor-to-scalar power ra
n_s	$0.953^{+0.016}_{-0.016}$	Scalar spectral index	Primordial spectral index
$n_t + 1$	$0.9861^{+0.0096}_{-0.0142}$	Tensor spectral index	$n_t = -r/8$ assumed
α	$-0.040^{+0.027}_{-0.027}$	Running of spectral index	$\alpha = dn_s/d\ln k$ (approxin
Nuisance parameters:			
τ	$0.087^{+0.028}_{-0.030}$	Reionization optical depth	
b	$1.896^{+0.074}_{-0.069}$	Galaxy bias factor	$b = [P_{galaxy}(k)/P(k)]^{1/}$
Q_{nl}	$30.3^{+4.4}_{-4.1}$	Nonlinear correction parameter [29]	$P_g(k) = P_{dewiggled}(k)b^2$

Sloan LRG combined with WMAP 3

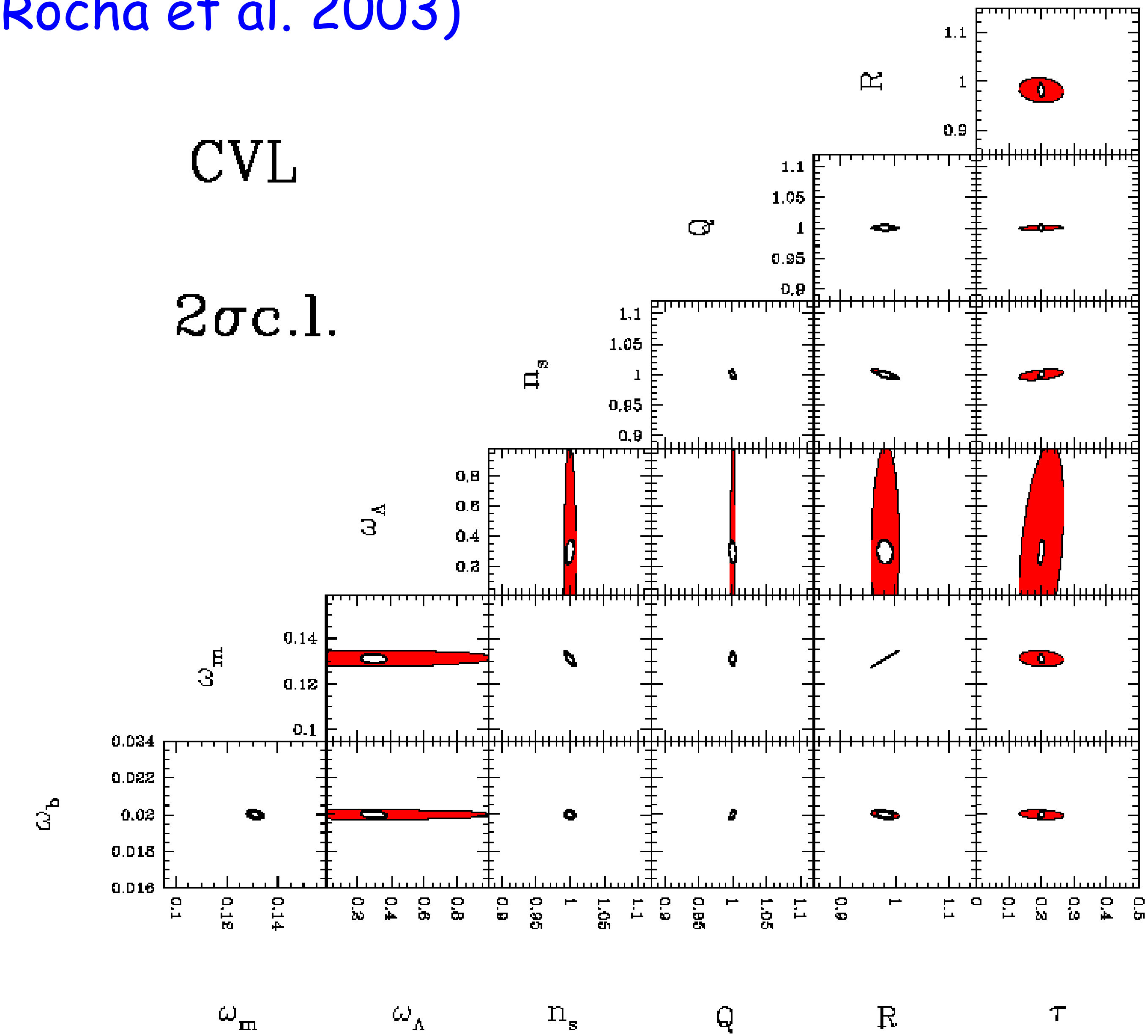
Other popular parameters (determined by those above):			
h	$0.730^{+0.019}_{-0.019}$	Hubble parameter	$h = \sqrt{(\omega_b + \omega_c + \omega_\nu)/(\Omega_{\rm tot} - \Omega_\Lambda)}$
Ω_m	$0.239^{+0.018}_{-0.017}$	Matter density/critical density	$\Omega_m = \Omega_{\rm tot} - \Omega_\Lambda$
Ω_b	$0.0416^{+0.0019}_{-0.0018}$	Baryon density/critical density	$\Omega_b = \omega_b/h^2$
Ω_c	$0.197^{+0.016}_{-0.015}$	CDM density/critical density	$\Omega_c = \omega_c/h^2$
Ω_ν	< 0.024 (95%)	Neutrino density/critical density	$\Omega_\nu = \omega_\nu/h^2$
Ω_k	$-0.0030^{+0.0095}_{-0.0102}$	Spatial curvature	$\Omega_k = 1 - \Omega_{\rm tot}$
ω_m	$0.1272^{+0.0044}_{-0.0043}$	Matter density	$\omega_{\rm m} = \omega_b + \omega_c + \omega_\nu = \Omega_m h^2$
f_ν	< 0.090 (95%)	Dark matter neutrino fraction	$f_\nu = \rho_\nu/\rho_d$
A_t	< 0.21 (95%)	Tensor fluctuation amplitude	$A_t = r A_s$
M_ν	< 0.94 (95%) eV	Sum of neutrino masses	$M_\nu \approx (94.4\ \text{eV}) \times \omega_\nu$ [105]
$A_{.002}$	$0.801^{+0.042}_{-0.043}$	WMAP3 normalization parameter	A_s scaled to $k = 0.002$/Mpc: $A_{.002} = 25^{1-n}$
$r_{.002}$	< 0.33 (95%)	Tensor-to-scalar ratio (WMAP3)	Tensor-to-scalar power ratio at $k = 0.002$/Mpc
σ_8	$0.756^{+0.035}_{-0.035}$	Density fluctuation amplitude	$\sigma_8 = \{4\pi \int_0^\infty [\frac{3}{x^3}(\sin x - x\cos x)]^2 P(k) \frac{k^2 dk}{(2\pi)^3}$
$\sigma_8 \Omega_m^{0.6}$	$0.320^{+0.024}_{-0.023}$	Velocity fluctuation amplitude	
Cosmic history parameters:			
$z_{\rm eq}$	3057^{+105}_{-102}	Matter-radiation Equality redshift	$z_{\rm eq} \approx 24074\omega_{\rm m} - 1$
$z_{\rm rec}$	$1090.25^{+0.93}_{-0.91}$	Recombination redshift	$z_{\rm rec}(\omega_{\rm m}, \omega_b)$ given by eq. (18) of [106]
$z_{\rm ion}$	$11.1^{+2.2}_{-2.7}$	Reionization redshift (abrupt)	$z_{\rm ion} \approx 92(0.03h\tau/\omega_b)^{2/3}\Omega_m^{1/3}$ (assuming abr
$z_{\rm acc}$	$0.855^{+0.059}_{-0.059}$	Acceleration redshift	$z_{\rm acc} = [(-3w-1)\Omega_\Lambda/\Omega_m]^{-1/3w} - 1$ if $w <$
$t_{\rm eq}$	$0.0634^{+0.0045}_{-0.0041}$ Myr	Matter-radiation Equality time	$t_{\rm eq} \approx (9.785\ \text{Gyr}) \times h^{-1} \int_{z_{\rm eq}}^\infty [H_0/H(z)(1+z)]$
$t_{\rm rec}$	$0.3856^{+0.0040}_{-0.0040}$ Myr	Recombination time	$t_{\rm req} \approx (9.785\ \text{Gyr}) \times h^{-1} \int_{z_{\rm rec}}^\infty [H_0/H(z)(1+z$
$t_{\rm ion}$	$0.43^{+0.20}_{-0.10}$ Gyr	Reionization time	$t_{\rm ion} \approx (9.785\ \text{Gyr}) \times h^{-1} \int_{z_{\rm ion}}^\infty [H_0/H(z)(1+z$
$t_{\rm acc}$	$6.74^{+0.25}_{-0.24}$ Gyr	Acceleration time	$t_{\rm acc} \approx (9.785\ \text{Gyr}) \times h^{-1} \int_{z_{\rm acc}}^\infty [H_0/H(z)(1+z$
$t_{\rm now}$	$13.76^{+0.15}_{-0.15}$ Gyr	Age of Universe now	$t_{\rm now} \approx (9.785\ \text{Gyr}) \times h^{-1} \int_0^\infty [H_0/H(z)(1+z)$

Forecast2: Planck 1 year data vs. WMAP 4 year

(Planck consortium 2006)

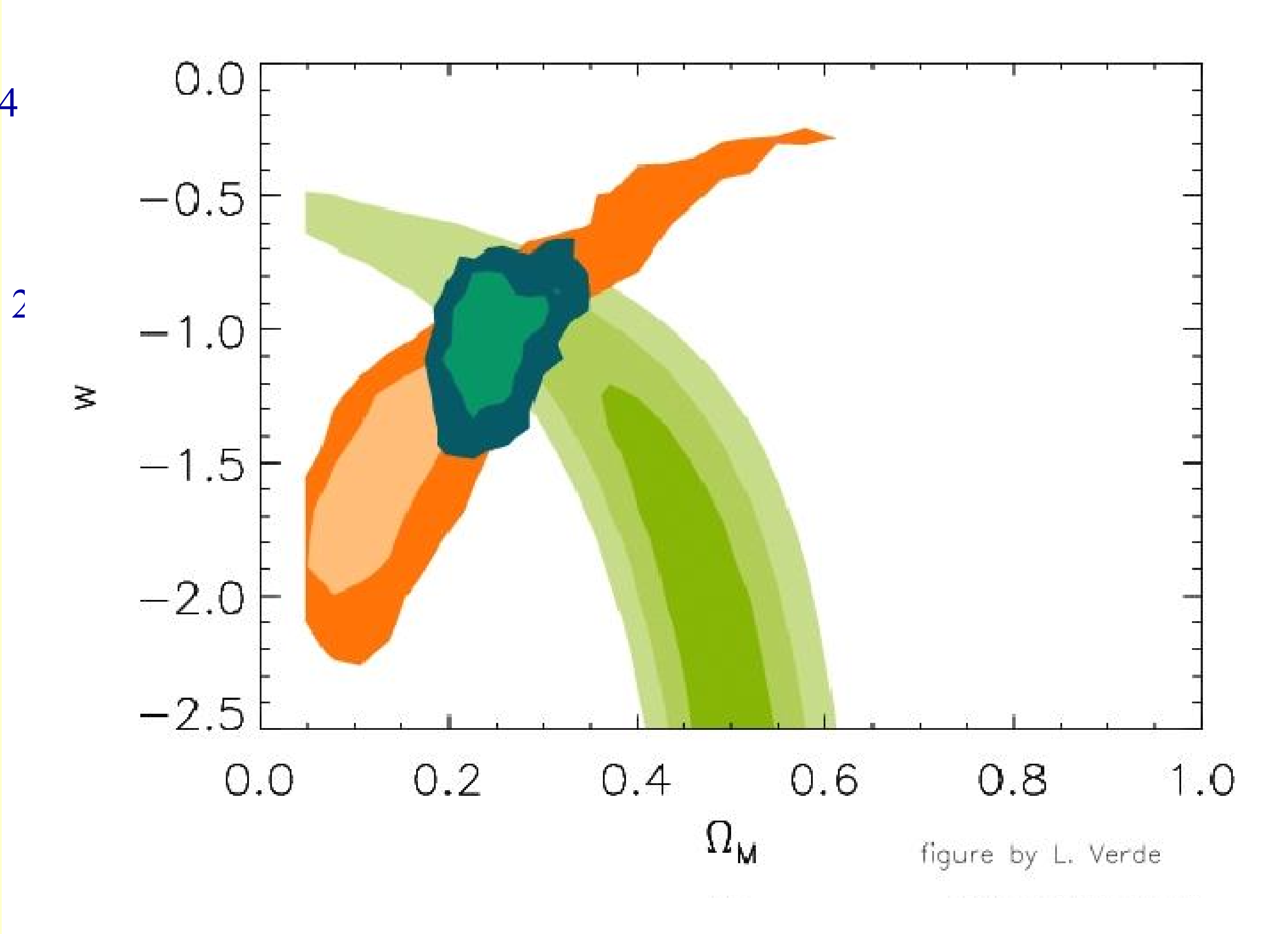


Forecast3: Cosmic variance limited data (Rocha et al. 2003)



Evidence for a cosmological constant

Sn1a, Riess et al. 2004 (green)
CMB + Hubble (orange)
Bi-spectrum β, Verde 2
(blue)



Conclusions

- The CMB with its small perturbations has h[illegible] enormously in determining properties & parameters o[illegible]niverse and it will continue to do so.
- We know the cosmological paramete[illegible] impressive precision which will still improve [illegible]rably during the next years.
- We don't understand at all t[illegible]rre 'mix' of cosmic components: $\Omega_b h^2$ [illegible] $\Omega_m h^2 \sim 0.13$, $\Omega_\Lambda \sim 0.73$
- The simplest model[illegible]ation (a nearly scale invariant spectrum of scal[illegible]urbations, vanishing curvature) is a good fit to the dat[illegible]
- What is [illegible]atter?
- Wh[illegible]ark energy?
- [illegible]hat is the inflaton?

! We have not run out of problems in cosmology!