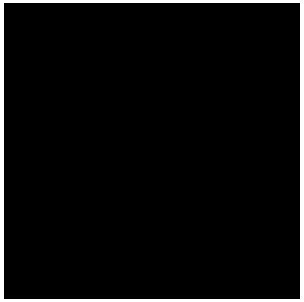
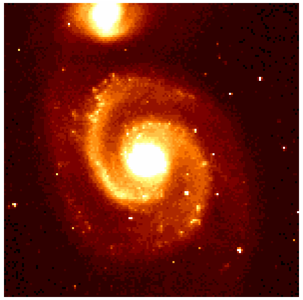
Cold dark matter in brane cosmology scenario

Shaaban Khalil

Center for theoretical Physics at BUE
& Ain Shams University



Dark matter



Not dark matter

Outline

- Introduction:
  - Why do we need Dark Matter ?
  - What is the Dark Matter made of ?
  - Dark Matter candidates: Neutrinos, Axions, WIMPs.
- Supersymmetry and Neutralino Dark Matter.
- Brane Cosmology in 5D Space-time
- Modified Friedmann equation in 5D
- DM relic density in non-conventional brane cosmology
- Conclusions.

Why do we need Dark Matter

- Most astronomers, cosmologists and particle physicists are convinced that 90% of the mass of the Universe is due to some non-luminous matter, called `dark matter'.

- Although the existence of dark matter was suggested 73 years ago, still we do not know its composition.

- In 1933, Fritz Zwicky provided evidence that the mass of the luminous matter in the Coma cluster was much smaller than its total mass implied by the motion of cluster member galaxies.

- Only in the 1970's the existence of dark matter began to be considered seriously.

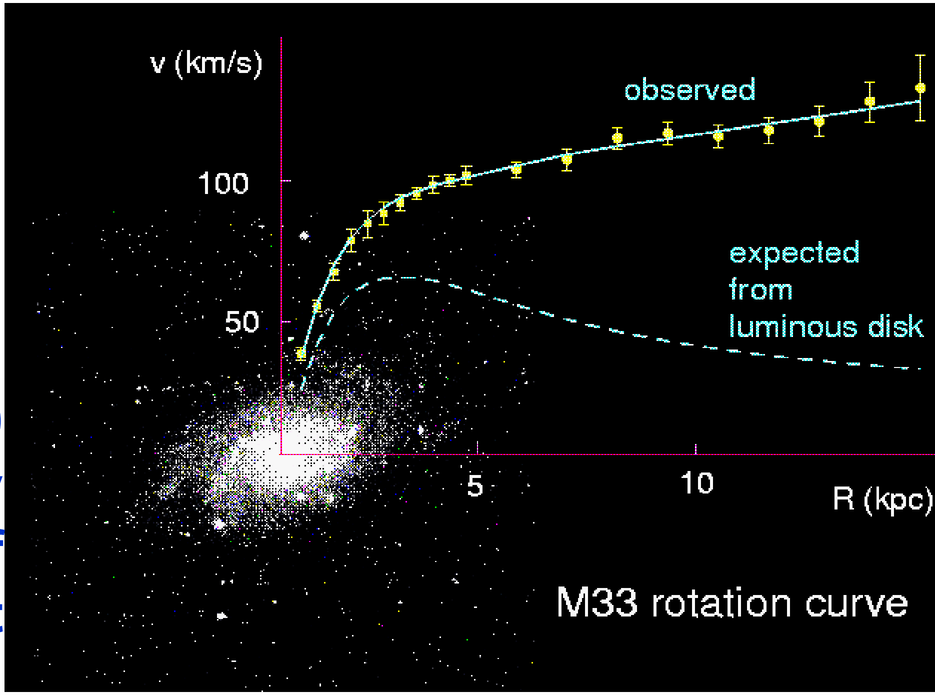
Dark matter in galaxies

- Important evidence for the existence of DM comes from the study of rotation velocity of stars or hydrogen clouds located far away from galactic centres.

- From Newton's laws, the velocity of rotating objects

$$v(r) = \sqrt{\frac{GM(r)}{r}}$$

- The observation of about 1000 spiral galaxies has consistently shown, that away from the centre of galaxies the rotation velocities do not drop off with distance.



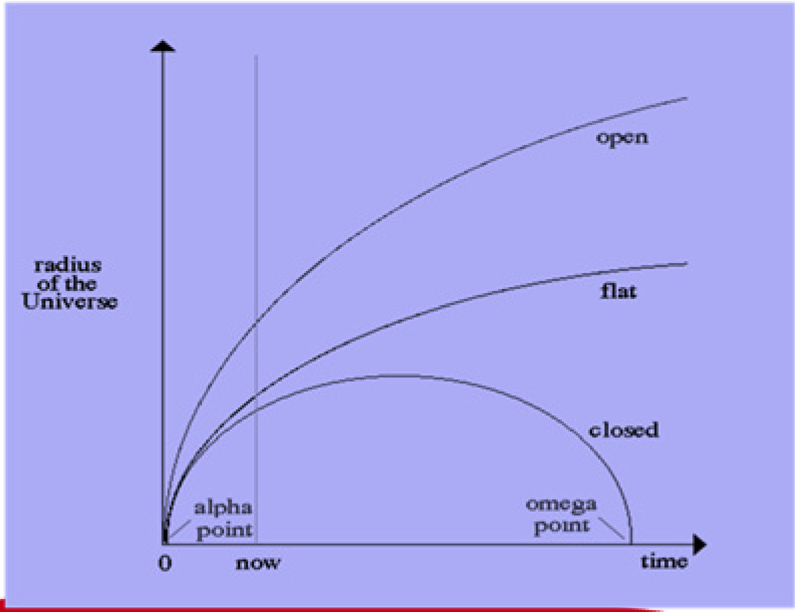
- The explanation for these flat rotation curves is to assume that disk galaxies are immersed in extended dark matter halos.

- At small distances this dark matter is only a small fraction of the galaxy mass inside those distances, it becomes a very large amount at larger distances.

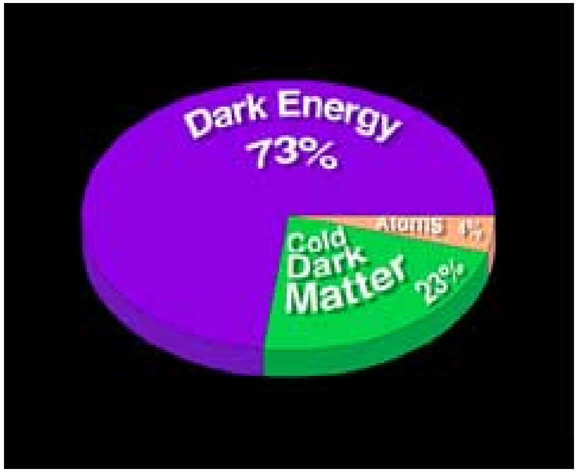
- The mass density averaged over the Universe, ρ, in units of the critical density, $\rho_c \sim 10^{-29}$ g cm^{-3} is defined as $\Omega = \rho / \rho_c$.

- If $\rho > \rho_c$ ($\Omega>1$) the Universe expands to a maximum then contracts leading to an inverse Big Bang (closed Universe).

- If $\rho > \rho_c$ ($\Omega<1$) the Universe expands forever (open Universe). The same for $\Omega=1$, where the geometry of the Universe is flat.



- Current observations of luminous matter in galaxies determine $\Omega \sim 0.01$. Analyses of rotation curves imply $\Omega > 0.3$



- Theoretical arguments prefer a $\Omega \sim 1$ flat Universe.

- Thus, larger amounts of dark matter Or non-vanishing vacuum energy density $\rho_{c.c}$ contribution to the density of the Universe.

$$\Omega = \Omega_{lum} + \Omega_{dark} + \Omega_{c.c}$$

- It is fair to say that a small number of authors suggest that dark matter is not really necessary to explain rotation curves.
- Their approach consists of modifying the Newton's law at galactic scales.

What is DM made of ?

- The Big-Bang nucleosynthesis, which explains the origin of the elements, sets a limit to the number of baryons that exists in the Universe: $\Omega_{baryon} < 0.04$
- Thus, baryonic objects are likely components of the dark matter but more non-baryonic candidates are needed.
- Particle physics provides this type of candidate for dark matter.
- The three most promising are: axions, neutrinos and neutralinos

Dark Matter Candidates

- Weakly interacting massive particles (WIMPs) are very interesting candidates for dark matter in the Universe.

- They were in thermal equilibrium with the SM particles in the early Universe, and decoupled when they were non-relativistic.

- The relic density of WIMPs can be computed with the result

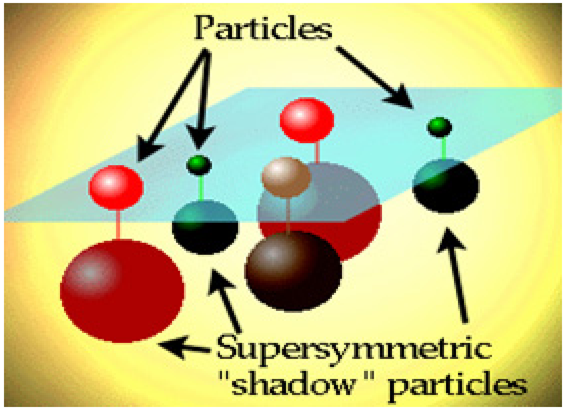
$$\Omega_{WIMP} \approx \frac{7 \times 10^{-27}\, cm^{3} s^{-1}}{\langle \sigma_{ann} v \rangle}$$

- For weakly coupled particle with $\sigma \sim \alpha^2/m^2_{weak} \Longrightarrow \sigma \sim 10^{-9}\ GeV^{-2}$

One obtains, $< \sigma_{ann} v > \sim 10^{-26}\ cm^3\ s^{-1}$. This number is close to the value that we need to obtain the observed density of the Universe.

- This is a possible hint that new physics at the weak scale provides us with a reliable solution to the dark matter problem.

Supersymmetry



- SUSY is a new type of symmetry relates bosons and fermions.

- SUSY introduces a new unification between particles of different spin.

- Higgs is no longer a mysterious particle. SUSY introduce fundamental scalars (squarks, sleptons).

- SUSY ensures the stability of the hierarchy between the weak and the Planck scales.

- Within SUSY the three gauge coupling constants of the SM join at a single unification scale.

- Local SUSY leads to a partial unification of the SM with gravity: Supergravity, which is the low-energy limit of superstrings.

- **In simplest SUSY models, there is ONLY interactions between one SM particle and two SUSY particles.**

- **SUSY particles are produced or destroyed only in pairs. The LSP is absolutely stable.**

- **LSP may be a candidate for DM, *Goldberg 1983*.**

- **SUSY fulfils the two crucial requirements: new physics at the electroweak scale with a stable particle.**

- **In MSSM, the LSP is an electrically neutral with no strong Interactions particle, called neutralino.**

Lightest SUSY particle

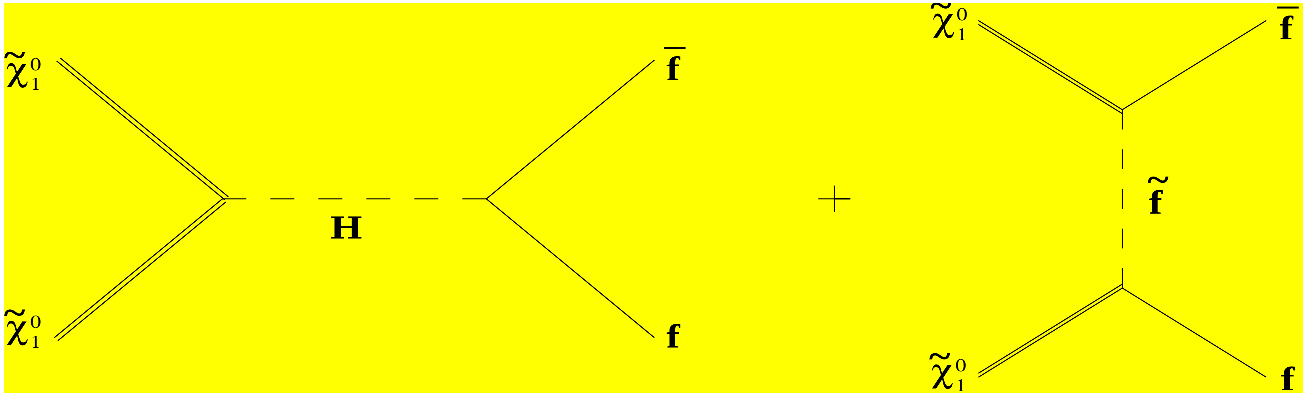
- Neutralinos are superpositions of the fermionic partners of the neutral electroweak gauge bosons, bino and wino, and the fermionic partners of the two neutral Higgs bosons, Higgsinos.

$$\begin{pmatrix} M1 & 0 & -M_Z c\beta s\theta_w & M_Z s\beta s\theta_w \\ 0 & M2 & M_Z c\beta c\theta_w & -M_Z s\beta c\theta_w \\ -M_Z c\beta s\theta_w & M_Z c\beta c\theta_w & 0 & -\mu \\ M_Z s\beta s\theta_w & -M_Z s\beta c\theta_w & -\mu & 0 \end{pmatrix}$$

$$\tilde{\chi}_1^0 = N_{11} B^0 + N_{12} W^0 + N_{13} \tilde{H}_u^0 + N_{14} \tilde{H}_d^0$$

- Exp. limit on LSP mass is m_{χ_1} > 37 GeV.

- There are numerous final states into which the LSP can annihilate. The most important ones occur at the tree level.



- **fermion-antifermion pairs give the dominant contribution to LSP annhiliation.**

- **Many regions of the parameter space of the MSSM produce values of the annihilation cross sectionin the interesting range ($\sigma \sim 10^{-9}$ GeV^{-2}).**

- **Therefore, the neutralino is a very good candidate to account for the dark matter in the Universe.**

- **In the usual early-Universe model, thermal production of neutralinos gives rise to** $\Omega_{\tilde{\chi}_1^0} h^2 \propto 1/\langle \sigma_{\tilde{\chi}_1^0}^{ann} v \rangle$
- **The relic density is inversely proportional to the annihilation cross section.**
- **Large cross section leads to very small relic density, which contradicts the observational results:**

$$0.1 \lesssim \Omega_{\tilde{\chi}_1^0} h^2 \lesssim 0.3.$$

- **Different cosmological scenarios might give rise to different results.**
- **The relic density in the context of some non-standard cosmological scenarios implies different results.**
- **We study the relic density in the context of brane comsology.**

Brane Cosmology in 5D Space-time

- The theory we consider has the following action:

$$S_5 = -\frac{1}{2k_5^2}\int d^5x\sqrt{-g}\,R + \int d^5x\sqrt{-g}\,L_m$$

k_5 is given by:

$k_5^2 = 8\pi G_5 = M_5^{-3}$

- The 5D metric is of the form

$$ds^2 = -q^2(t,y)dt^2 + a^2(t,y)\delta_{ij}dx^i dx^j + b^2(t,y)dy^2$$

- The energy-momentum tensor:

$$T_B^A = T_B^A\,|_{bulk} + T_B^A\,|_{brane}$$

- Assuming an empty bulk & the matter content in the brane is

$$T_B^A = \frac{\delta(y)}{b}\,diag\left(-\rho, p, p, p, 0\right)$$

❑ **In order to have well defined geometry, the metric is required to be continuous across the brane at y=0. However its derivative w.r.t y may be discontinuous in y=0**

Thus, one has

$$a'' = \hat{a}'' + [a']\delta(y)$$

$[a']$ Is the jump in 1st derivative across y=0, $[a'] = a'(0^+) - a'(0^-)$

Matching the δ(y) in G_{00}&G_{ii}, we get

$$\frac{[a']}{a_0 b_0} = -\frac{k_5^2}{3}\rho,$$

$$\frac{[q']}{q_0 b_0} = \frac{k_5^2}{3}(3p + 2\rho).$$

Modified Friedmann equation in 5D

❑ **The new Friedmann equation is 5D:**

$$H^2 = \frac{8\pi G}{3}\rho_M\left[1 + \frac{\rho_M}{2\sigma}\right] + \frac{\Lambda_4}{3} + \frac{\mu}{a^4}$$

$$\frac{8\pi G}{3} = \frac{\sigma}{18}$$

$$\frac{\Lambda_4}{3} = \frac{\sigma^2}{36} + \frac{\Lambda_5}{6}$$

- σ is the brane tension.
- Tuning between Λ_5 and σ establishes $\Lambda_4 = 0$
- gravitational constant depends on tension σ
- μ is constant of integration (may be +ve or –ve)

The presence of the energy density σ allows for $H^2 \sim \rho_M$ as in conventional cosmology.

DM relic density in non-conventional brane cosmology

- In the standard computation for relic density, χ is assumed to be in thermal equilibrium and decoupled when the annihilation rate $\Gamma_\chi = \langle \sigma_\chi^{ann} v \rangle$ dropped below the expansion rate of the universe: $\Gamma_\chi \leq H$
- The relic density is determined by Boltzmann equation:

$$\frac{dY}{dx} = \frac{1}{3H}\frac{ds}{dx}\langle\sigma_\chi^{ann}\ v\rangle\left(Y^2 - Y_{eq}^2\right).$$

Where $x=m_\chi/T$, $Y=n_\chi/s$ and $Y^{eq}=n_\chi^{eq}/s$.

- In radiation domination era, the entropy is given by:

$$s=k_1\ x^{-3}$$

with $k_1=2\pi^2/45\ g_*\ \chi^3$, is the same in both standard and Brane comsology.

- The Hubble parameter (as function of T) is given by:

$$H_s = \sqrt{\frac{4\pi^3 g_* m_\chi^4}{45 M_{Pl}^2}} x^{-2} = \sqrt{k_2}\, x^{-2}$$

And

$$H_b = (k_2 x^{-4} + k_3 x^{-8})^{1/2}$$

- In the standard case:

$$\left(\frac{dY}{dx}\right)_s = -\sqrt{\frac{\pi g_*}{45}} M_{pl} m_\chi \frac{\langle \sigma_\chi^{ann} v\rangle}{x^2} (Y^2 - Y_{eq}^2)$$

- In brane comsology

$$\left(\frac{dY}{dx}\right)_b = -\sqrt{\frac{\pi g_*}{45}} M_{pl} m_\chi \left(x^4 + \frac{k_3}{k_2}\right)^{-1/2} \langle \sigma_\chi^{ann} v\rangle (Y^2 - Y_{eq}^2)$$

- In the limit of $k_3 \rightarrow 0$ (i.e. $\sigma \rightarrow \infty$), the brane equation tends to the standard Boltzmann equation.

- The transition temperature is defined as

$$\rho(T_t) = 2\sigma \Longrightarrow T_t = 0.51 \times 10^{-9} M_5^{\frac{3}{2}} GeV$$

- To analyze the brane cosmology effect on the WIMP relic density, the freeze out temperature of the WIMP should be higher than the transition temperature , i.e., $T_F \geq T_t$. Thus

$$M_5 \leq 1.57 \times 10^6 \left(\frac{m_\chi}{x_F}\right)^{2/3}.$$

- To obtain the present WIMP abundance Y_∞, we should integrate the Boltzmann equation for the WIMP number density from x_F (the decoupling temperature) to x_∞ (present temperature)

$$Y_{\infty b}^{-1} = \sqrt{\frac{\pi g_*}{45}} M_{pl}\, m_\chi \left[3\sqrt{\frac{k_2}{k_3}} b \left(\sinh^{-1}\left(\sqrt{\frac{k_3}{k_2}} x_F^{-2}\right) - \sinh^{-1}\left(\sqrt{\frac{k_3}{k_2}} x_t^{-2}\right)\right) + a \left(\frac{1}{x}\, {}_2F_1\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{-k_3}{k_2 x^4}\right]\right)_{x_F}^{x_t} + \left(\frac{a}{x_t} + \frac{3b}{x_t^2}\right)\right],$$

❑ For $x_t = x_F$ the expression of $Y^{-1}_{\infty b}$ coincides with the standard $Y^{-1}_{\infty s}$:

$$Y^{-1}_{\infty s} = \sqrt{\frac{\pi g_*}{45}}\, M_{pl}\, m_\chi \left(\frac{a}{x_F} + \frac{3b}{x_F^2}\right).$$

❑ The relic abundance of the WIMP is given by

$$\Omega_\chi h^2 = \frac{\rho_\chi}{\rho_c/h^2} = 2.9 \times 10^8\, Y_\infty \left(\frac{m_\chi}{\mathrm{GeV}}\right),$$

❑ The critical density ρ_c is given by $\rho_c \sim 10^{-5}\, h^2$ GeV cm^{-3} and h is the Hubble constant, $h \sim 0.7$.

❑ The relic density is inversely proportional to its annihilation cross section as in the standard case.

❑ Unlike the standard case, it depends explicitly on WIMP mass since

$k_3/k_2 \propto m_\chi^4$

$$R = (\Omega_\chi h^2)_b / (\Omega_\chi h^2)_s$$

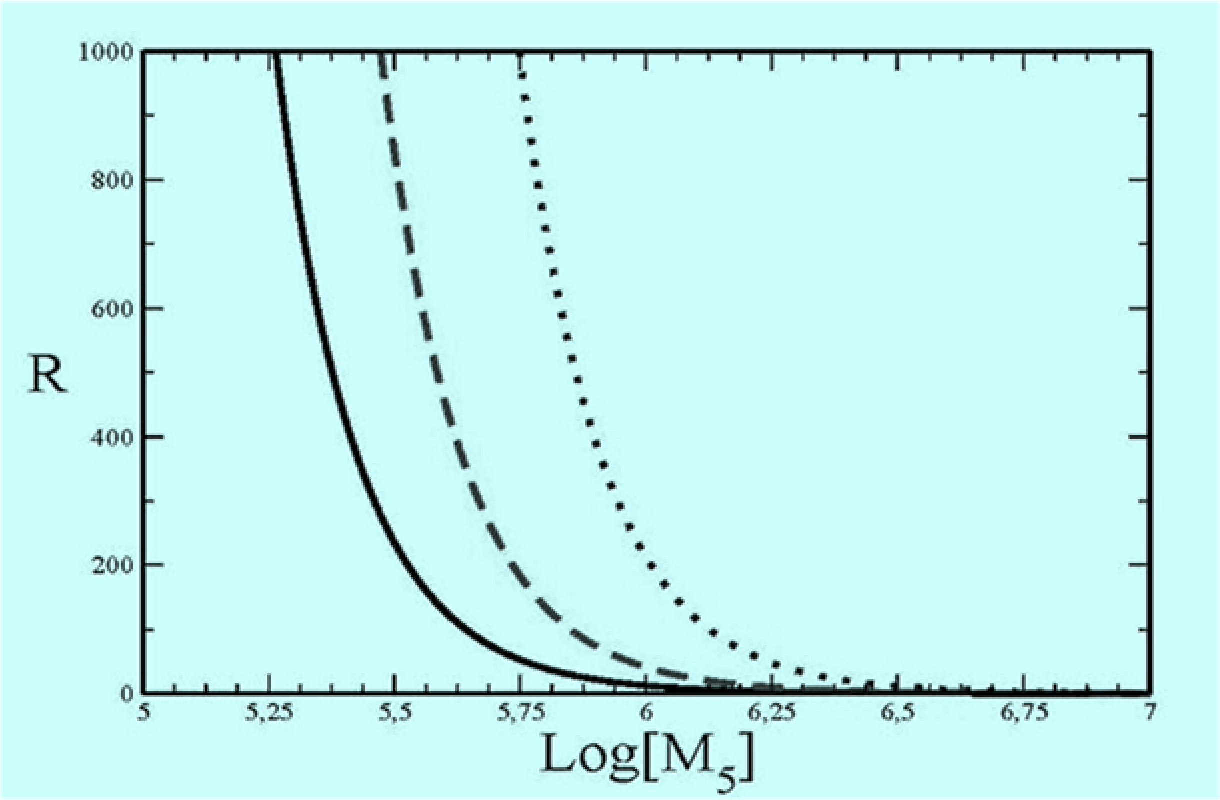
- R measures the enhancement/suppression in the relic abundance due to the brane cosmology.

$$R = \frac{\frac{a}{x_F} + \frac{3b}{x_F^2}}{3\sqrt{\frac{k_2}{k_3}} b \left(\sinh^{-1}\left(\sqrt{\frac{k_3}{k_2}} x_F^{-2}\right) - \sinh^{-1}\left(\sqrt{\frac{k_3}{k_2}} x_t^{-2}\right)\right) + a\left(\frac{1}{x}\ {}_2F_1\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{-k_3}{k_2 x^4}\right]\right)_{x_F}^{x_t} + \frac{a}{x_t} + \frac{3b}{x_t^2}}.$$

- R could be larger or smaller than one, depending on the values of the annihilation cross section parameters a, b, m_χ and M_5.

- Consider, as an example, the LSP as pure Bino: $b \simeq 8\pi\alpha_1^2 \frac{1}{m_\chi^2} \frac{1}{(1+x_{\tilde{l}_R})^2},$

$m_\chi \sim m_{\tilde{l}_R} \sim 100$ GeV, one finds $b \simeq \mathcal{O}(10^{-8})$ GeV^{-2}, which in the standard cosmology scenario leads to $\Omega_\chi h^2 \geq 0.1$.



- The enhancement/suppression factor R as a function of M_5 (GeV) for m_χ = 100 (solid curve), 200 (dashed curve) and 500 GeV (dotted curve).

- For $M_5 < 10^6$ the brane cosmology effect is quite large and R >>1

- The transition process from non-conventional cosmology to convention cosmology should be above the nucleosynthesis era (i.e., $T_t >$ 1 MeV). Thus $M_5 > 1.2 \times 10^4$.

- For $M_5 > 1.2 \times 10^6$, the ratio R becomes less than on and small suppression for $(\Omega_\chi h^2)_s$ can be obtained.

- This brane enhancement or suppression for the dark matter relic density could be favored or disfavored based on the value of the relic abundance in the standard scenario.

- If $(\Omega_\chi h^2)_s$ is already larger than the observational limit, as in the case of bino-like particle, then a suppression effect would be favored and hence M5 is constrained to be larger than 5×10^6 .

- For wino- or Higgsino-like particle where the standard computation leads to very small relic density, the enhancement effect will be favored and the constraint on M5 can be relaxed a bit .

- In general, it is remarkable that in this scenario the dark matter relic density imposes stringent constraint on the fundamental scale M_5 .

- DM relic density in brane cosmology with low reheating temperature has been also studied.

- In case of low reheating with non-equilibrium production and freeze out within brane cosmology:

$$Y_{\infty b} \simeq 0.02095 \times 10^{-6} \sqrt{\frac{\pi}{45}} g_\chi^2 g_*^{-3/2} M_{pl}\, m_\chi\, (9.3\, a + 3.7\, b)\,.$$

- In case the WIMP has large annihilation cross section and reaches the chemical equilibrium before reheating (m_χ=100, M_5=10^6):

$$\Omega_\chi h^2 \sim 1.1 \times 10^{-7}\, (95.2\, a - 4.12\, b)^{-1}\,.$$

- Now, with large annihilation cross section *O(10^{-6} -10^{-8})*, we can have

$\Omega_\chi h^2 \sim 0.1$

Conclusions

- we have analyzed the relic abundance of cold dark matter in brane cosmology.

- We investigated the brane comsology effect in two different scenarios, namely when the reheating temperature is higher or lower than the freeze-out temperature.

- We showed that with high reheating temperature, the relic density is enhanced with many order of magnitude for $M_5 < 10^6$.

- This imposes one of the strongest constraints on the scale of large extra dimensions.

- In case of low reheating temperature, we considered the possibility that WIMPs are in chemical equilibrium or non-equilibrium, which depends on the value of their annihilation cross section.

- We showed if WIMPs are in chemical non-equilibrium, then their relic density is very small and they can not account for the observational limits.

- While in case WIMPs reach chemical equilibrium before reheating, we showed that the relic density is enhanced by two order of magnitudes than the standard thermal scenario result.

- This enhancement can be considered as an interesting possibility for accommodating dark matter with large cross section, which is favored by the detection rate experiments.