Multi-field inflation and adiabatic-entropy correlations at the Hubble crossing

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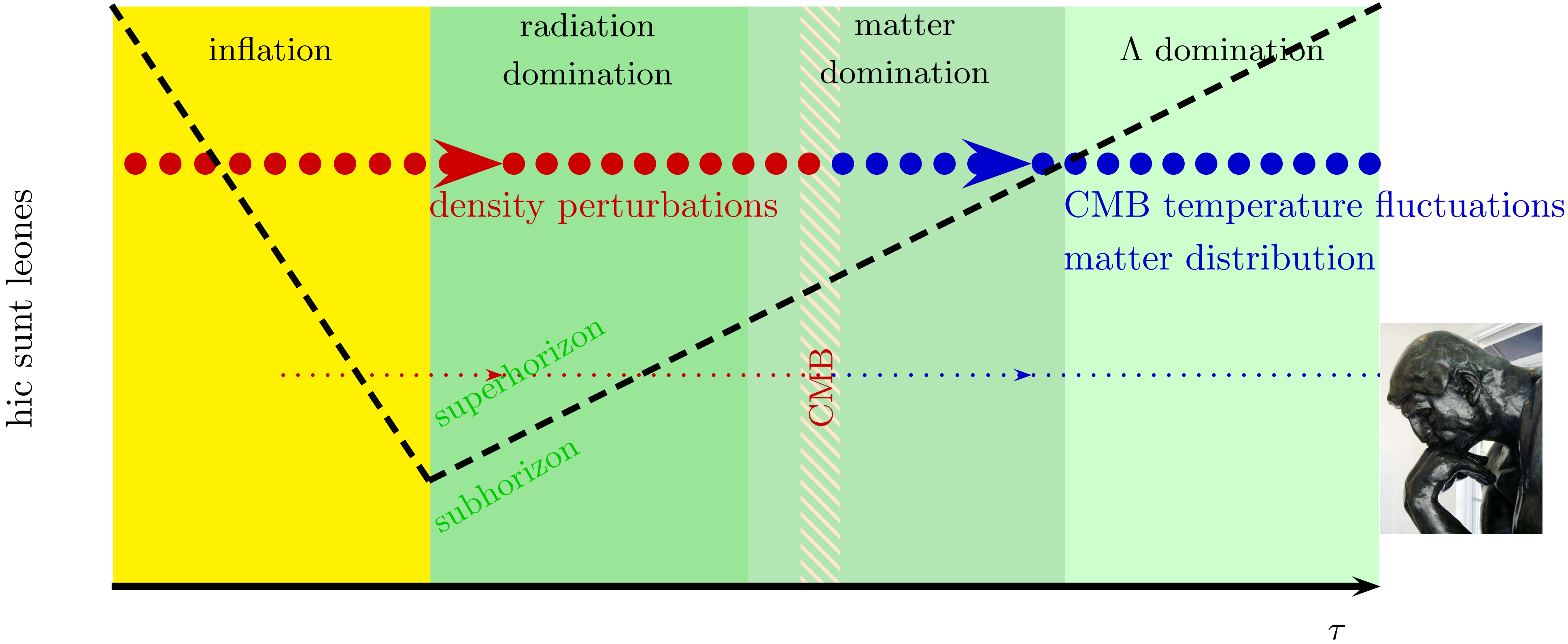
with Z. Lalak, D. Langlois and S. Pokorski

Why is cosmological inflation so lovable?

- solves the horizon problem
- solves the flatness problem
- predicts the generation of primordial density perturbations ($\longrightarrow$ the rest of this talk)

Road map


hic sunt leones
inflation
radiation domination
matter domination
Λ domination
density perturbations
CMB temperature fluctuations
matter distribution
superhorizon
subhorizon
CMB
τ

Multi-field inflation – non-canonical kinetic terms

The action (gravity and a scalar field):

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2 R}{2} + \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) + \frac{e^{2b(\phi)}}{2}(\partial_\mu\chi)(\partial^\mu\chi) - V(\phi,\chi)\right].$$

Start from the longitudinal gauge:

$$\mathrm{d}s^2 = (1+2\Phi)\mathrm{d}t^2 - a^2(t)(1-2\Phi)\mathrm{d}\mathbf{x}\cdot\mathrm{d}\mathbf{x}$$

Decompose:

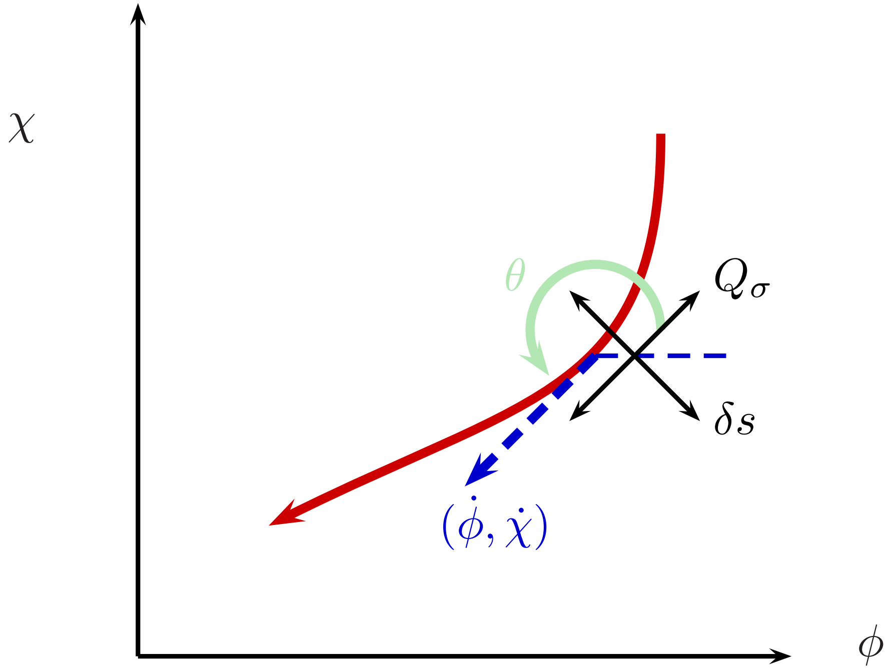
$$\phi = \phi(t) + \delta\phi(t,\mathbf{x}) \qquad \chi = \chi(t) + \delta\chi(t,\mathbf{x})$$

and introduce gauge-invariant Mukhanov-Sasaki variables:

$$Q_\phi = \delta\phi + \frac{\dot\phi}{H}\Phi \qquad Q_\chi = \delta\chi + \frac{\dot\chi}{H}\Phi$$

Multi-field inflation

Gordon-Wands-Bassett-Maartens adiabatic-entropy decomposition (as adapted by Di Marco and Finelli):



$$Q_\sigma \equiv \cos\theta\, Q_\phi + e^b \sin\theta\; Q_\chi$$

$$\delta s \equiv -\sin\theta\, Q_\phi + e^b \cos\theta\, e^b\, Q_\chi$$

$$\cos\theta \equiv \dot{\phi}/\dot{\sigma},\ \sin\theta \equiv e^b \dot{\chi}/\dot{\sigma}$$

$$\dot{\sigma}^2 \equiv \dot{\phi}^2 + e^{2b}\dot{\chi}^2$$

adiabatic perturbations: $\mathcal{R} \equiv \frac{H}{\dot{\sigma}} Q_\sigma$

entropy perturbations: $\mathcal{S} \equiv \frac{H}{\dot{\sigma}} \delta s$.

Multi-field inflation

Define $u_\sigma = a\, Q_\sigma$, $u_s = a\, \delta s$. Write e.o.m.'s as:

$$\left[\left(\frac{\mathrm{d}^2}{\mathrm{d}\tau^2} + k^2 - \frac{2+3\epsilon}{\tau^2}\right) + 2\tilde{E}\frac{1}{\tau}\frac{\mathrm{d}}{\mathrm{d}\tau} + \tilde{M}\frac{1}{\tau^2}\right]\begin{pmatrix} u_\sigma \\ u_s \end{pmatrix} = 0$$

with

$$\tilde{E} = \begin{pmatrix} 0 & -\eta_{\sigma s} \\ \eta_{\sigma s} & 0 \end{pmatrix} + \begin{pmatrix} 0 & \xi s_\theta^3 \\ -\xi s_\theta^3 & 0 \end{pmatrix}$$

$$\tilde{M} = \begin{pmatrix} -6\epsilon + 3\eta_{\sigma\sigma} & 4\eta_{\sigma s} \\ 2\eta\sigma s & 3\eta_{ss} \end{pmatrix} + \begin{pmatrix} 3\xi s_\theta^2 c_\theta & -4\xi s_\theta^3 \\ -2\xi s_\theta^3 & -3\xi c_\theta(1+s_\theta^2) \end{pmatrix}$$

and $\xi = \sqrt{2} b_\phi M_P \sqrt{\epsilon}$. Rewrite:

$$\left[\left(\frac{\mathrm{d}^2}{\mathrm{d}\tau^2} + k^2 - \frac{2+3\epsilon}{\tau^2}\right) + \tilde{E}\left(\frac{1}{\tau}\frac{\mathrm{d}}{\mathrm{d}\tau} - \frac{1}{\tau^2}\right) + (\tilde{M} + \tilde{E})\frac{1}{\tau^2}\right]\begin{pmatrix} u_\sigma \\ u_s \end{pmatrix} = 0$$

Multi-field inflation

Results:

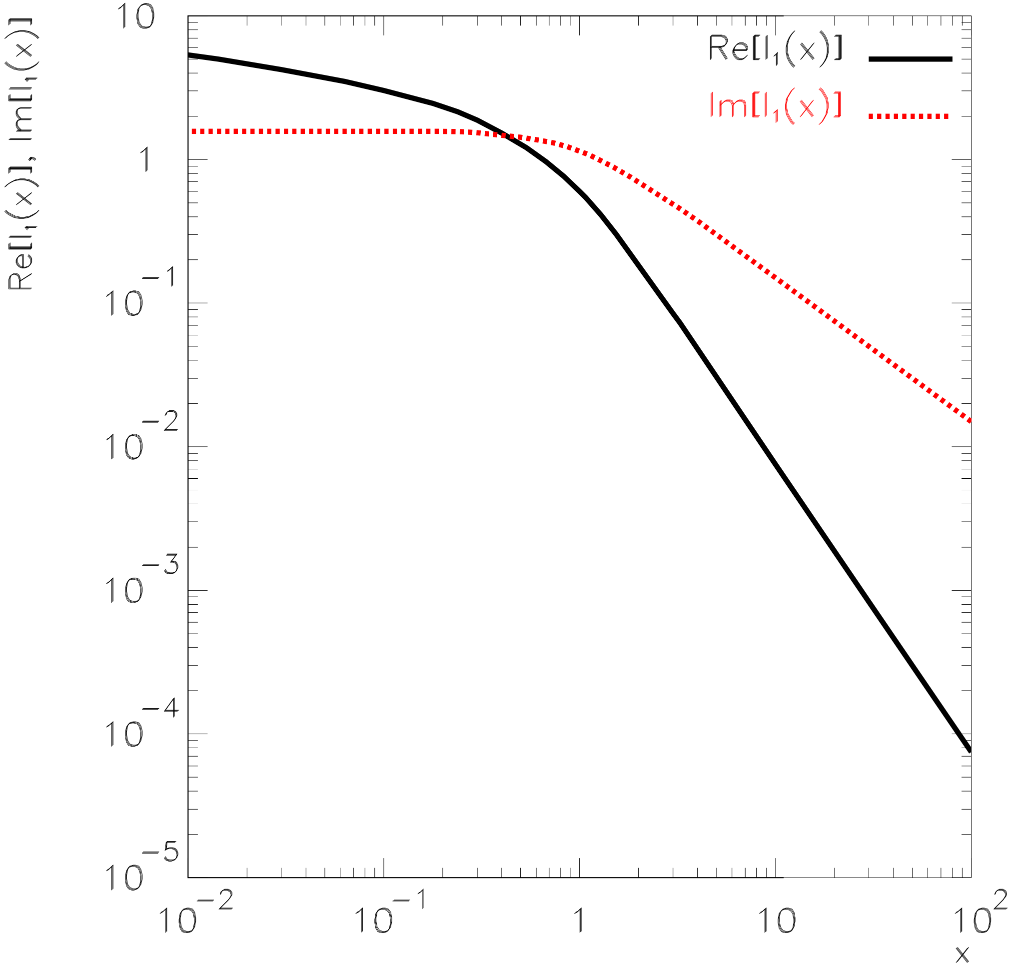
$$\begin{aligned}\mathcal{P}_{\mathcal{R}} &= \mathcal{P}^{(0)}\left(1+\frac{k^2}{a^2H^2}\right)(1+a_1)\\ \mathcal{C}_{\mathcal{RS}} &= \mathcal{P}^{(0)}\left(1+\frac{k^2}{a^2H^2}\right)a_2\\ \mathcal{P}_{\mathcal{S}} &= \mathcal{P}^{(0)}\left(1+\frac{k^2}{a^2H^2}\right)(1+a_3)\end{aligned}$$

where $\mathcal{P}^{(0)} = (H^2/2\pi\dot{\sigma})^2$ evaluated at the given scale k/aH and

$$\begin{aligned}a_1 &= -2\frac{k}{aH}\epsilon + \mathrm{Re}[I_1]\left(-2\xi s_\theta^2 c_\theta + 6\epsilon - 2\eta_{\sigma\sigma}\right)\\ a_2 &= 2\mathrm{Re}[I_1](\xi s_\theta^3 - \eta_{\sigma s})\\ a_3 &= -2\frac{k}{aH}\epsilon + \mathrm{Re}[I_1]\left(2\xi c_\theta(1+s_\theta^2) + 2\epsilon - 2\eta_{ss}\right)\end{aligned}$$

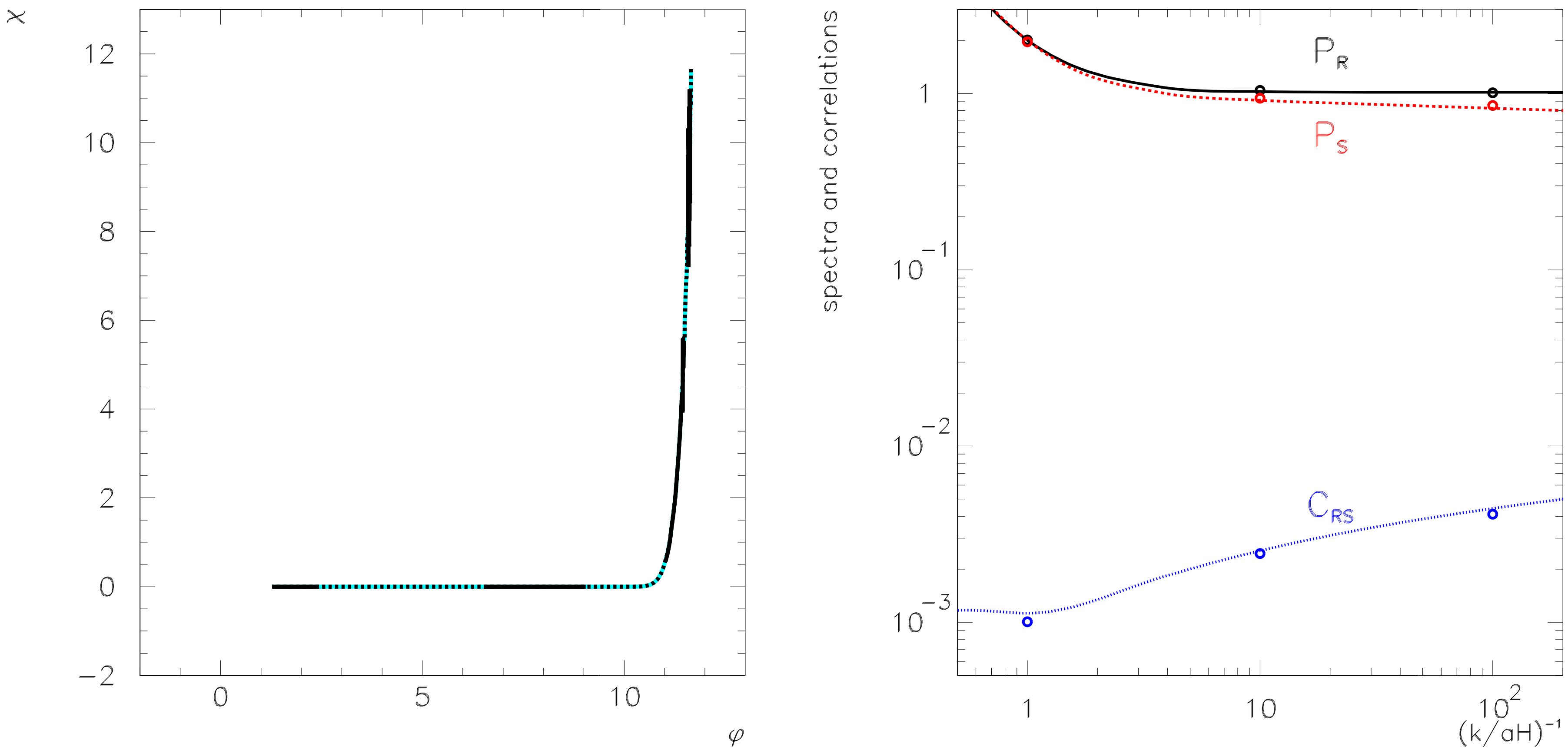
with $\xi = \sqrt{2} b_\phi M_P \sqrt{\epsilon}$ and

$$I_1(x) = \frac{2\imath}{x+\imath} - i\pi e^{-2\imath x} + \frac{x-\imath}{x+\imath}\left(\mathrm{Ci}(2x) + \imath\,\mathrm{si}(2x)\right)e^{-2\imath x}$$


Re[I$_1$(x)]
Im[I$_1$(x)]
Re[I$_1$(x)], Im[I$_1$(x)]
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-2}
10^{-1}
1
10
10^{2}
x

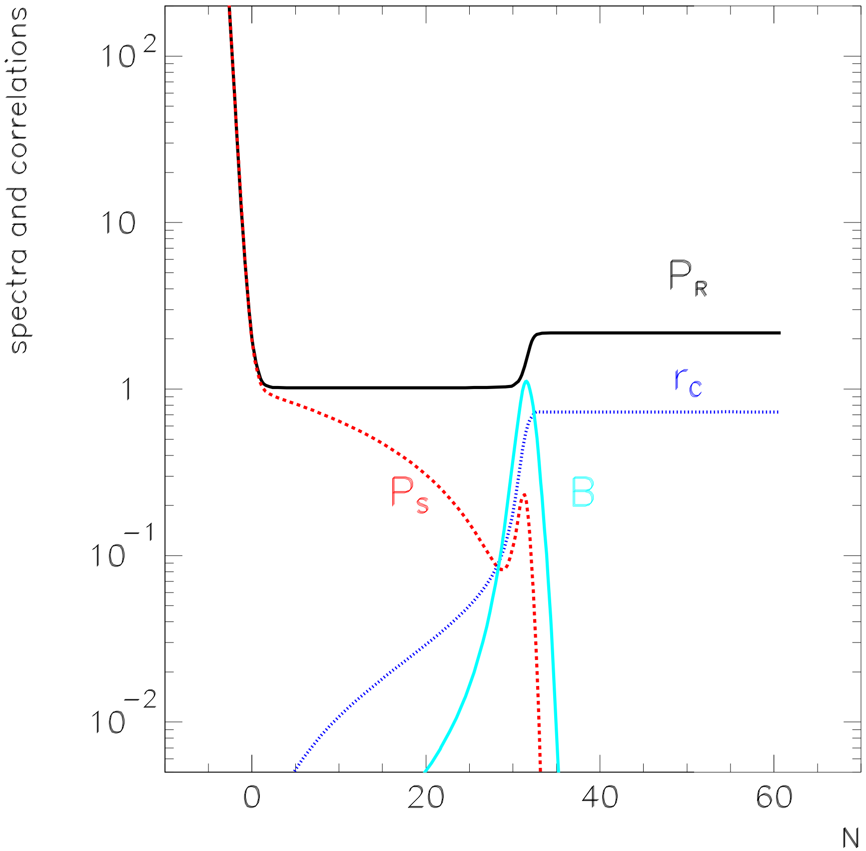
Multi-field inflation

$$V(\phi,\chi) = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2 \text{ with } m_\chi = 7m_\phi$$



Multi-field inflation

On super-Hubble scales:



$$\dot{Q}_\sigma = AHQ_\sigma + BH\delta s \qquad \dot{\delta s} = DH\delta s$$

$$A = -\eta_{\sigma\sigma} + 2\epsilon$$

$$B = -2\eta_{\sigma s} \approx 2\frac{\mathrm{d}\theta}{\mathrm{d}N}$$

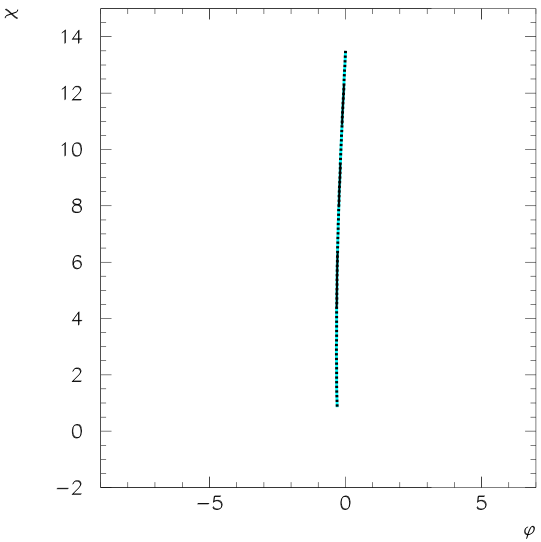
$$D = -\eta_{ss}$$

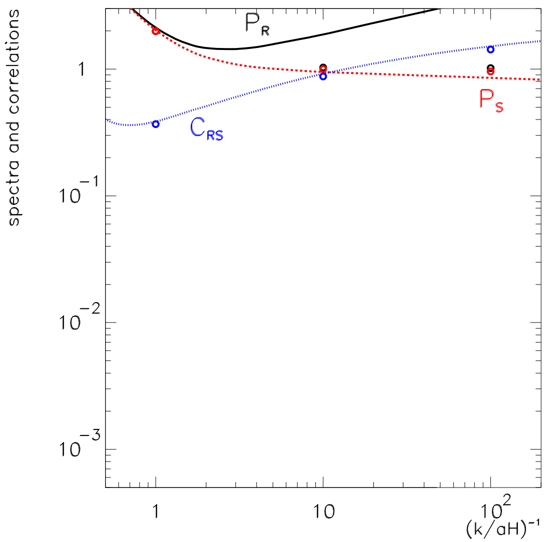
$$r_C = \frac{|C_{\mathcal{RS}}|}{\sqrt{\mathcal{P_R P_S}}}$$

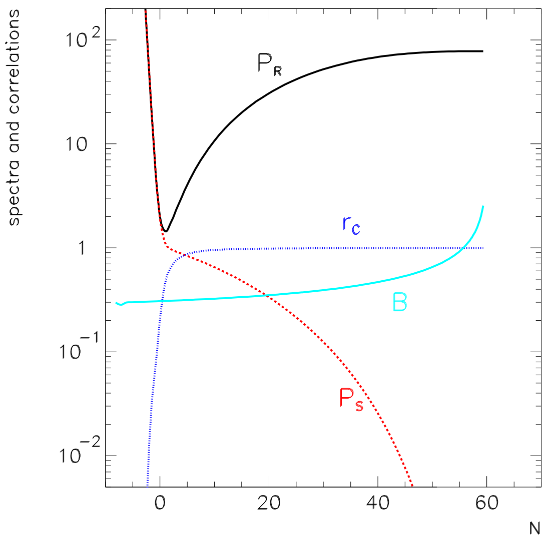
Multi-field inflation

$$V(\phi,\chi) = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2 \text{ with } m_\chi = m_\phi$$

$$b(\phi) = -\phi/M_P$$







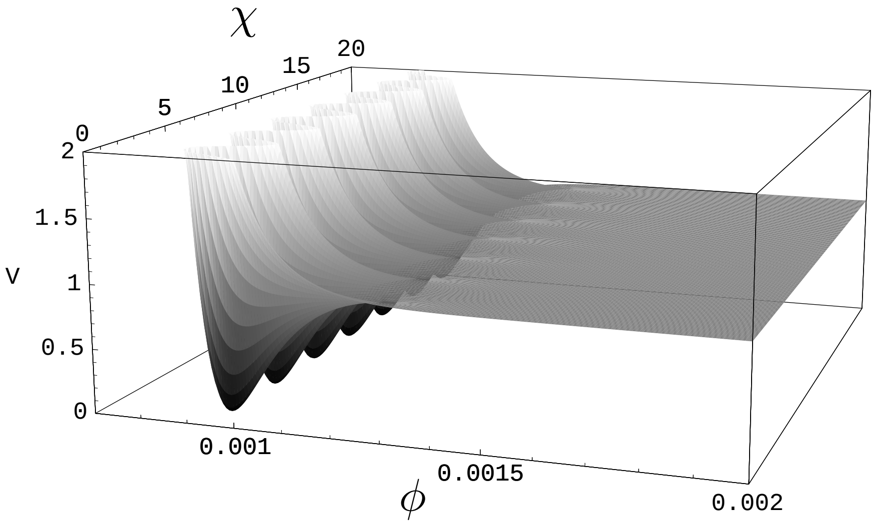
Multi-field inflation

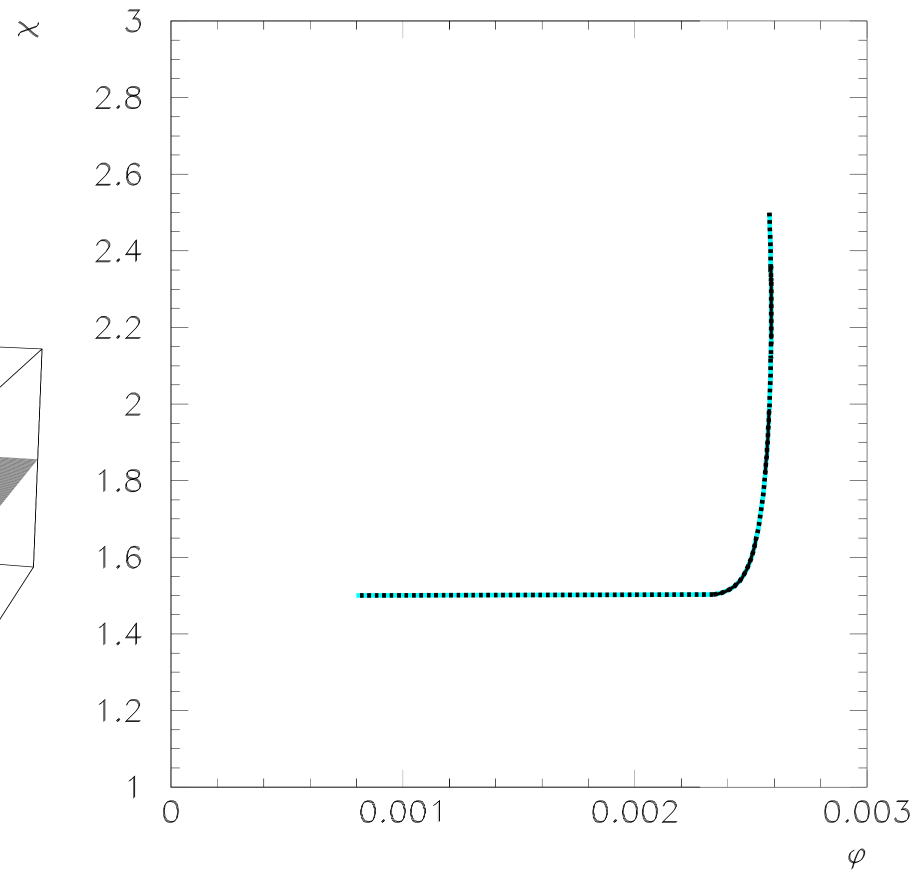
$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2 R}{2} + \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) + \frac{e^{2b(\phi)}}{2}(\partial_\mu\chi)(\partial^\mu\chi) - V(\phi,\chi)\right]$$

$$V(\phi,\chi) = V_0 + V_1\sqrt{\psi(\phi)}e^{-2\beta_1\psi(\phi)} + V_2\psi(\phi)e^{-\beta_1\psi(\phi)}\cos(\beta_2\chi)$$

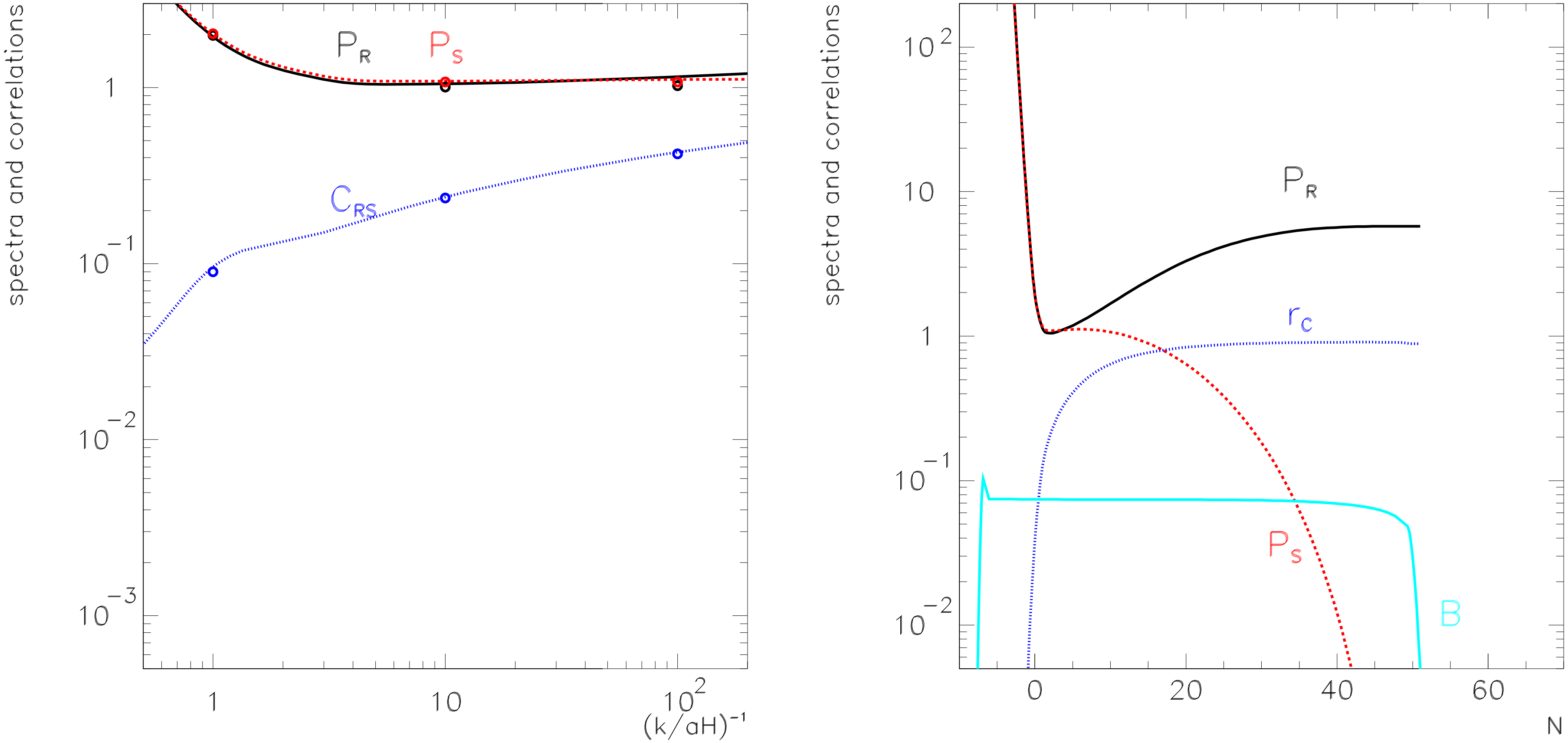
$$b(\phi) = b_0 - \frac{1}{3}\ln\left(\frac{\phi}{M_P}\right)$$

(Bond *et al.*, Dec 2006)





Multi-field inflation



Conclusions

In multi-field inflation:

- interesting and nontrivial dynamics
  - at the Hubble crossing,
  - on super-Hubble scales
- generalization to non-canonical kinetic terms