Pomeron-Graviton Correspondence

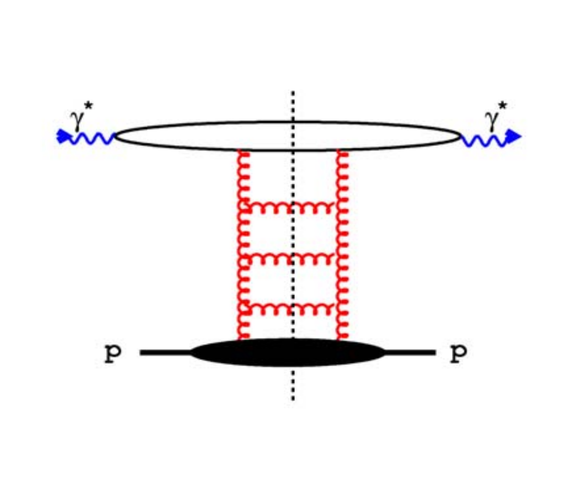
Evidence for the Discrete Asymptotically-Free BFKL Pomeron from HERA Data

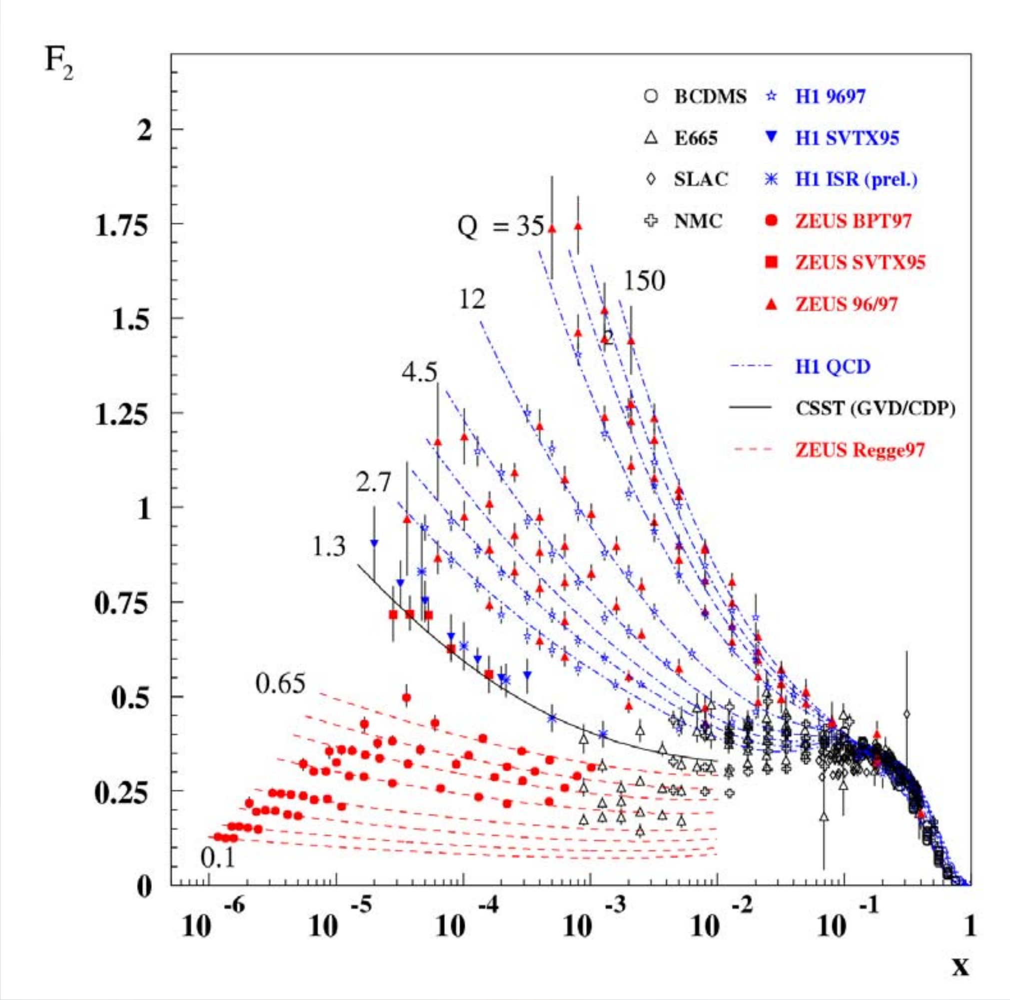
J. Ellis, H. Kowalski, D.A. Ross

Tok Meeting
Luty 2008 Warszawa 2008

Low-x Physics @ HERA

- At low x and high Q^2, steep rise in structure function = distribution of partons, integrated over k_T

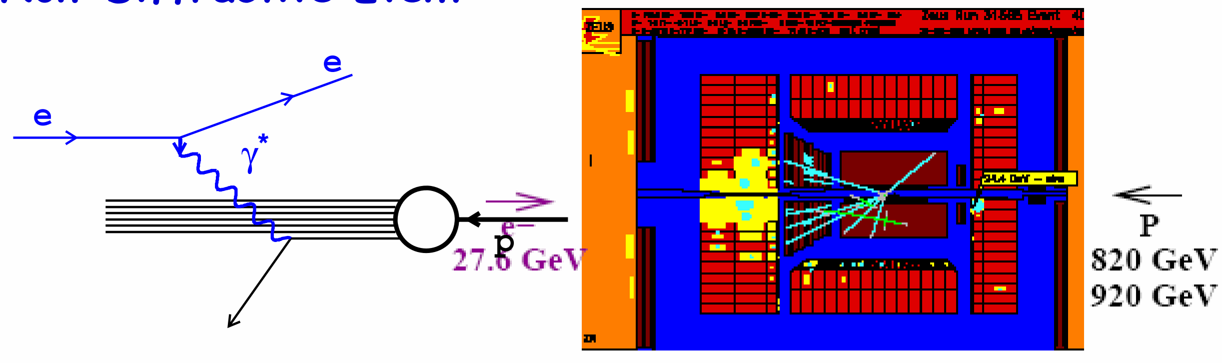




Behavior of F_2 is dominated by gluon density at small-x

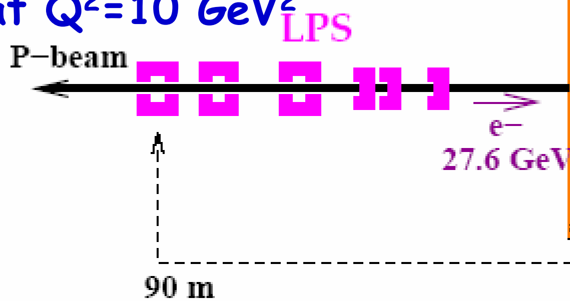
Hard Diffraction – the HERA surprise

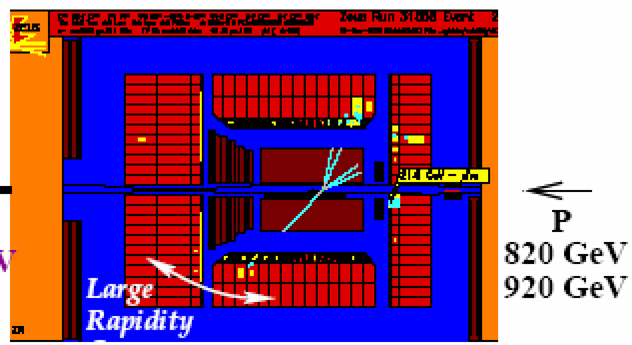
Non-Diffractive Event

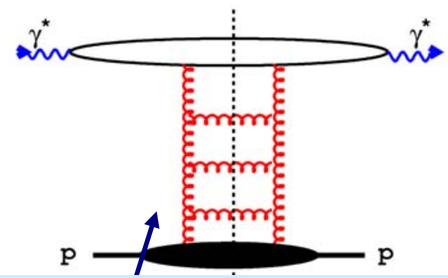


Diffractive Event

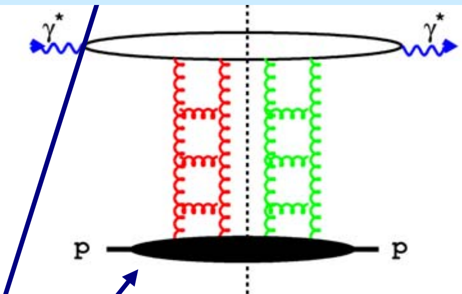
expected before HERA <0.01%, seen over 10% at Q^2=10 GeV^2







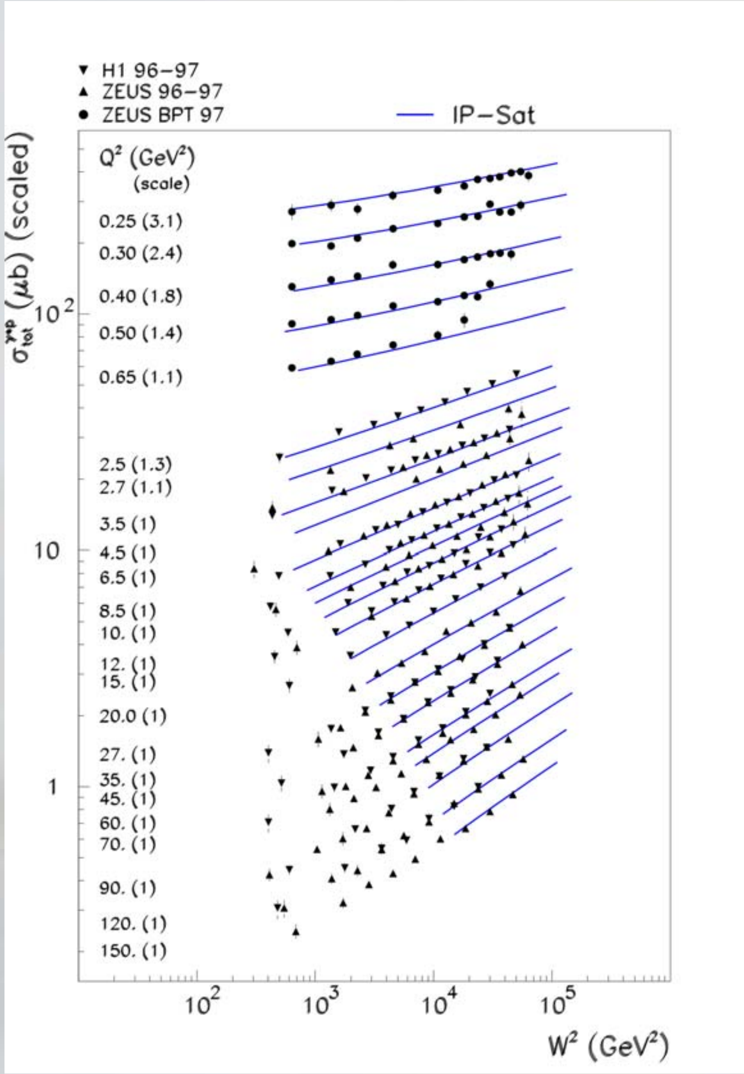
$$\tau_{qq} \approx \frac{1}{\Delta E} \approx \frac{1}{m_p x} \approx 10 - 1000 \quad \text{fm}$$



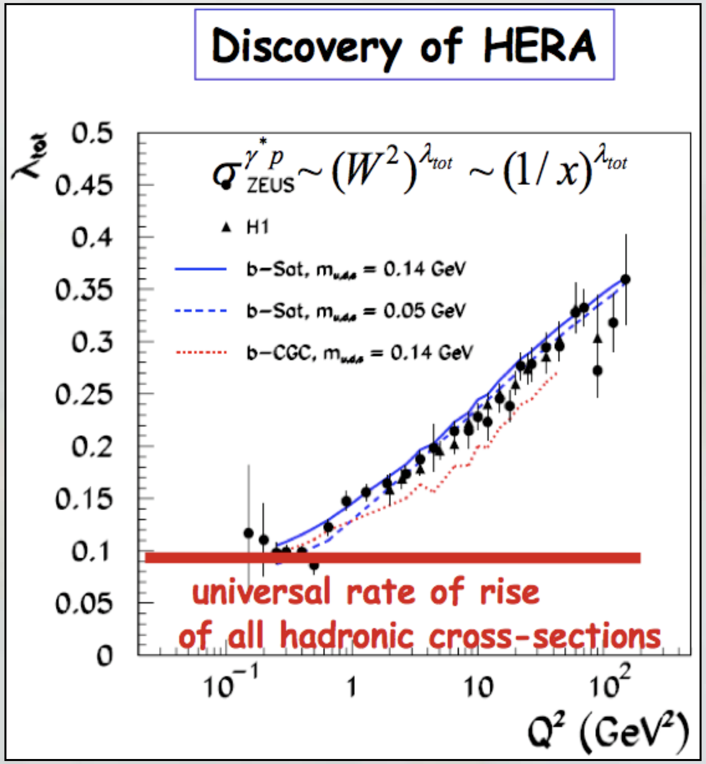
Diffraction at HERA is so large because it is a shadow of DIS (i.e. inelastic processes) ➔ dipole picture

$$\sigma_{tot}^{\gamma^* p} = \frac{1}{W^2} Im A_{el}(W^2, t = 0)$$

Low-x Physics @ HERA



- Increasing rate of growth with Q^2, well described by DGLAP evolution

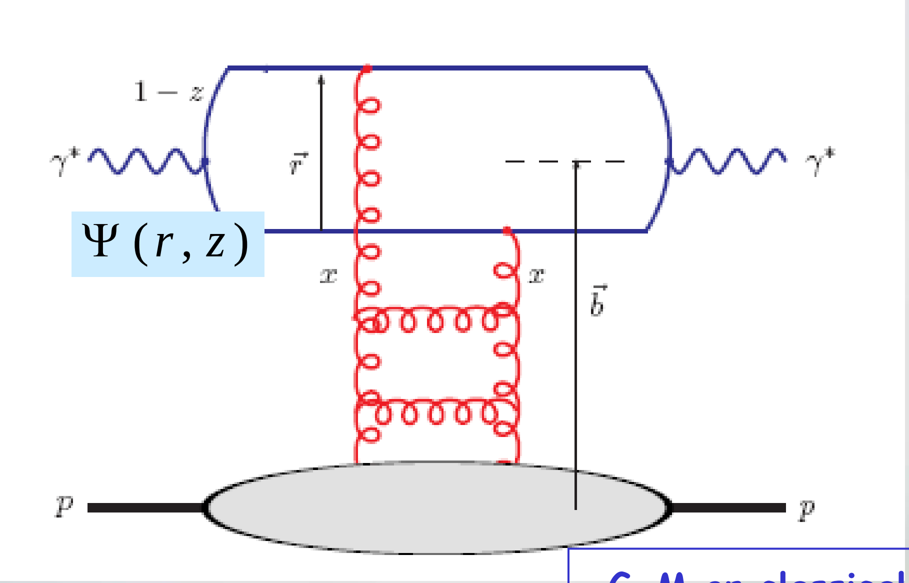


$$\sigma_{tot}^{\gamma p} = \frac{1}{W^2} Im A_{el}(W^2, t=0)$$

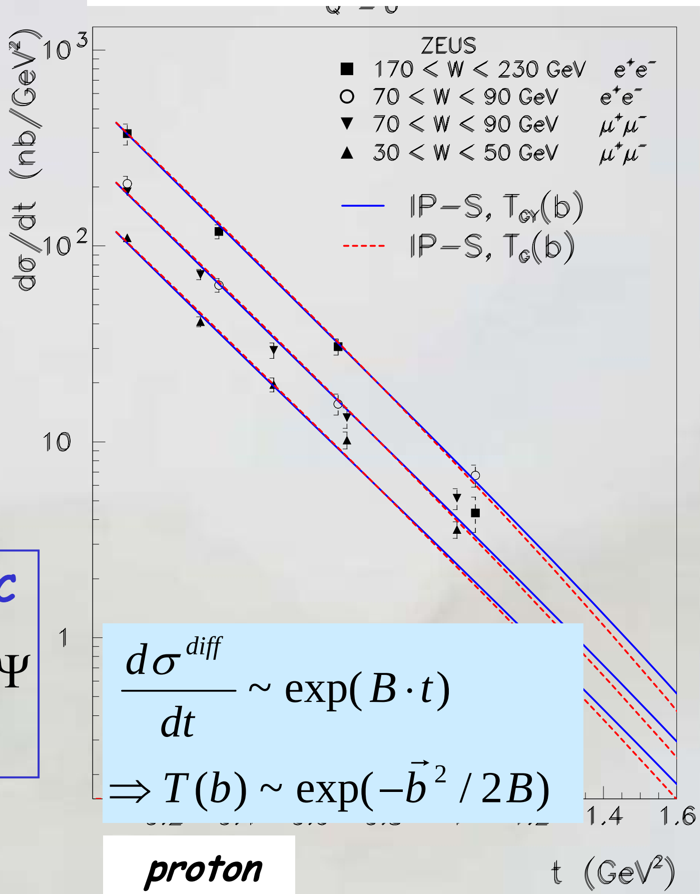
Dipole Models

equivalent to LO perturbative QCD for small dipoles

NNPZ, GLM, FKS, GBW, MMS
DGKP, BGBK, IIM, FSS……
KT - Kowalski, Teaney
KMW - Kowalski, Motyka, Watt



$\Psi(r, z)$



$$\frac{d\sigma^{diff}}{dt} \sim \exp(B \cdot t)$$

$$\Rightarrow T(b) \sim \exp(-\vec{b}^2 / 2B)$$

proton shape

G-M or classical CGC

$$\sigma_{tot}^{\gamma^* p} = \int d^2\vec{r} \int_0^1 dz \int d^2 b \Psi^* \cdot 2\left\{1 - \exp(-\frac{\Omega}{2})\right\}\Psi$$

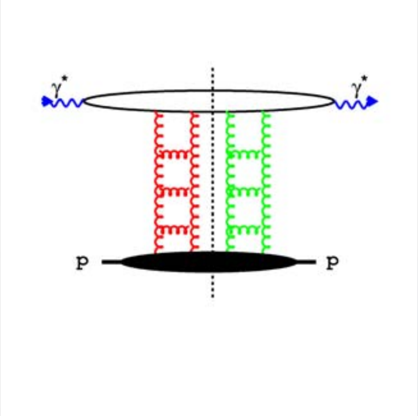
Optical Theorem

$$\Omega = \frac{\pi^2}{N_C} r^2 \alpha_s(\mu^2) x g(x, \mu^2) T(b)$$

$$\frac{d\sigma_{VM}^{\gamma^* p}}{dt} = \frac{1}{16\pi} \left| \int d^2\vec{r} \int d^2 b e^{-i\vec{b}\cdot\vec{\Delta}} \int_0^1 dz \Psi_{VM}^* \; 2\left\{1 - \exp(-\frac{\Omega}{2})\right\}\Psi \right|^2$$

KT

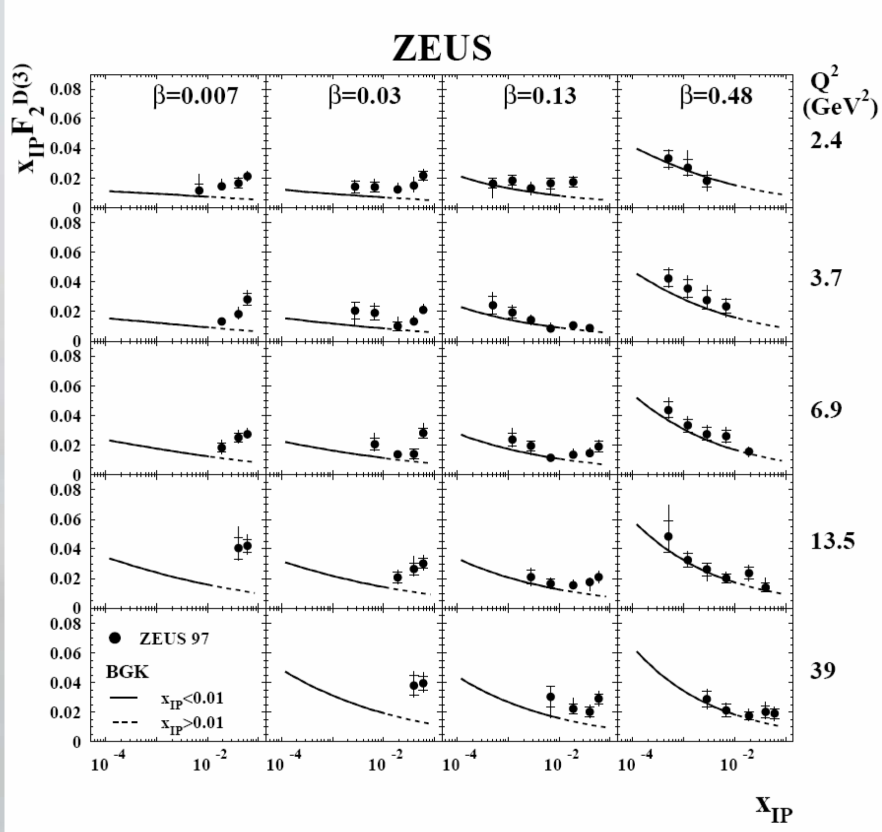
Dipole Model - gluon density convoluted with dipole wave functions
simultaneous prediction/description of many reactions

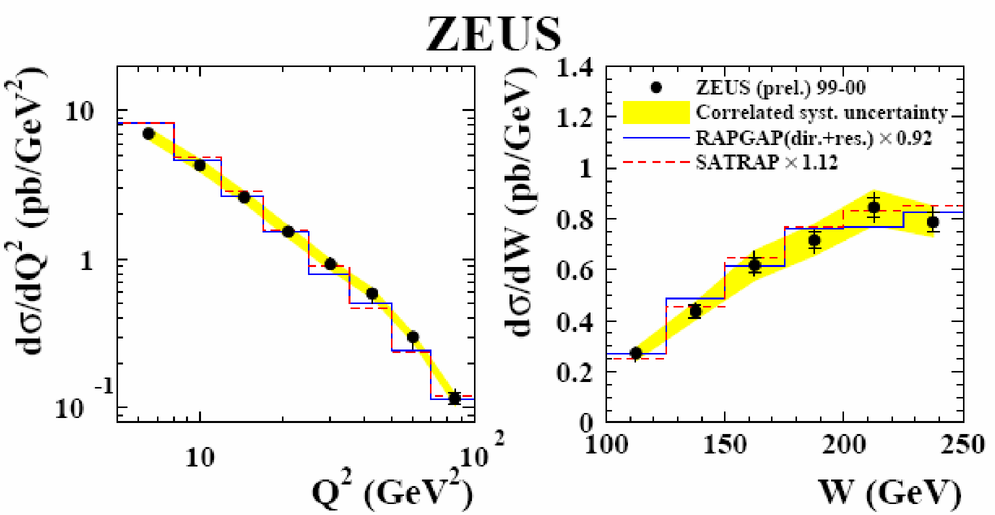


Diffractive Di-jets

$Q^2 > 5\ GeV^2$

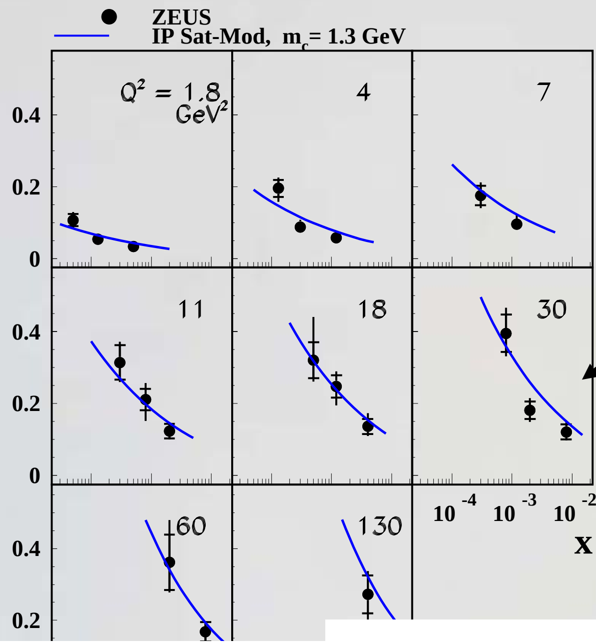
Inclusive Diffractive Cross Section



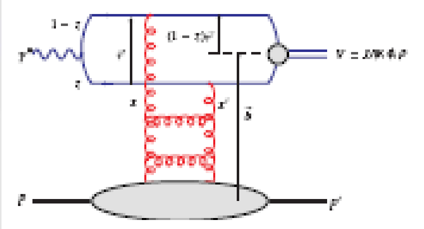


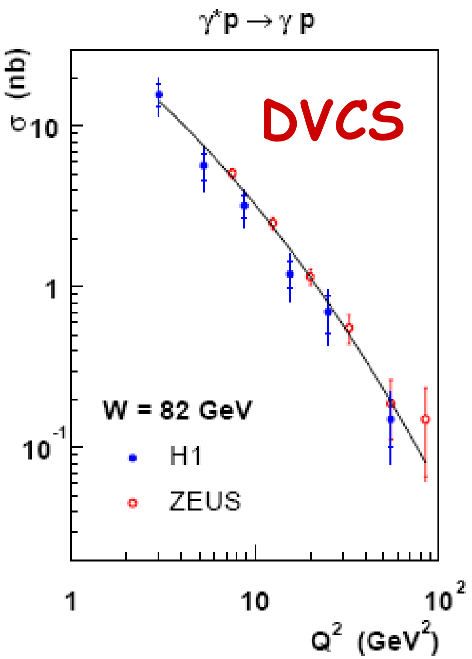
KT

$F_2{}^c$

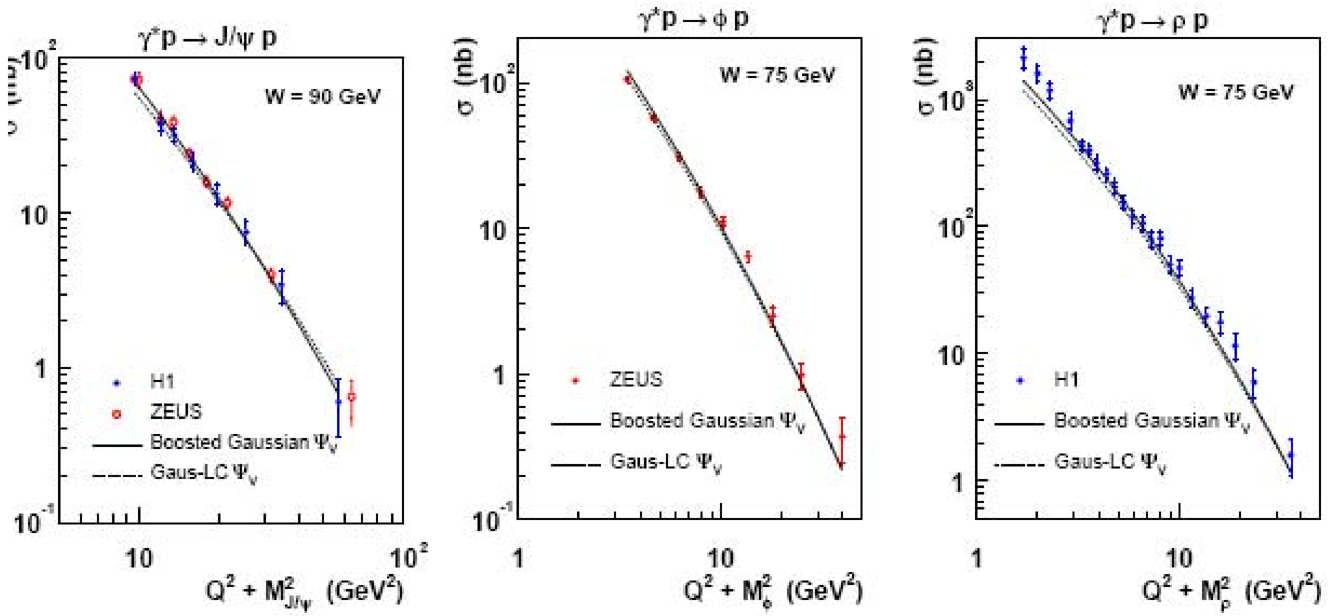


from gluon density convoluted with dipole wave functions we obtain simultaneous prediction/description of many reactions



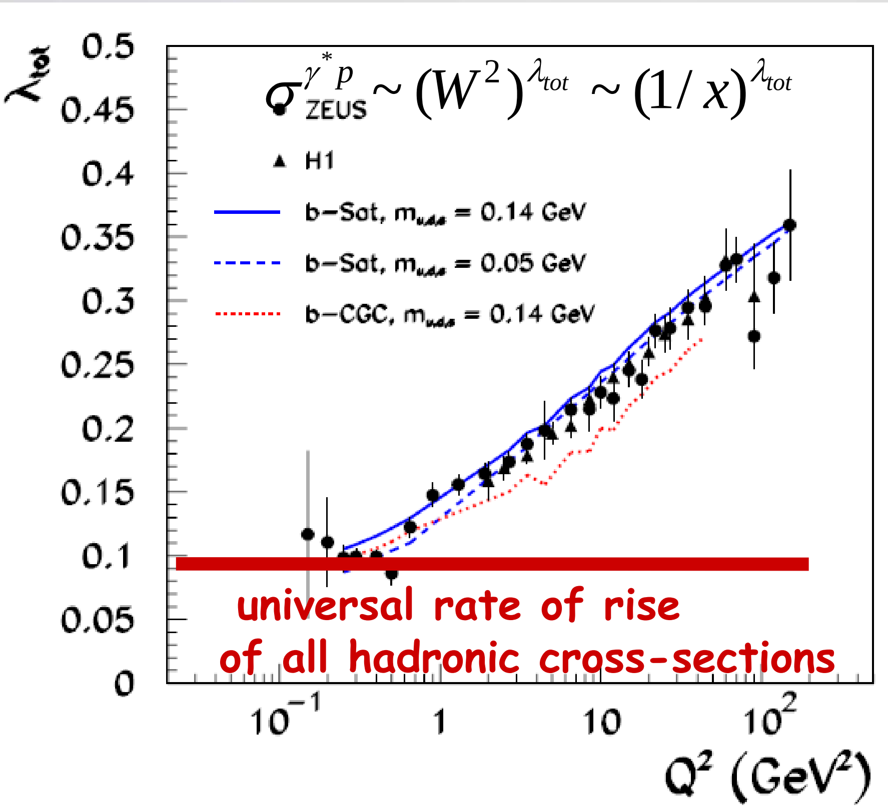


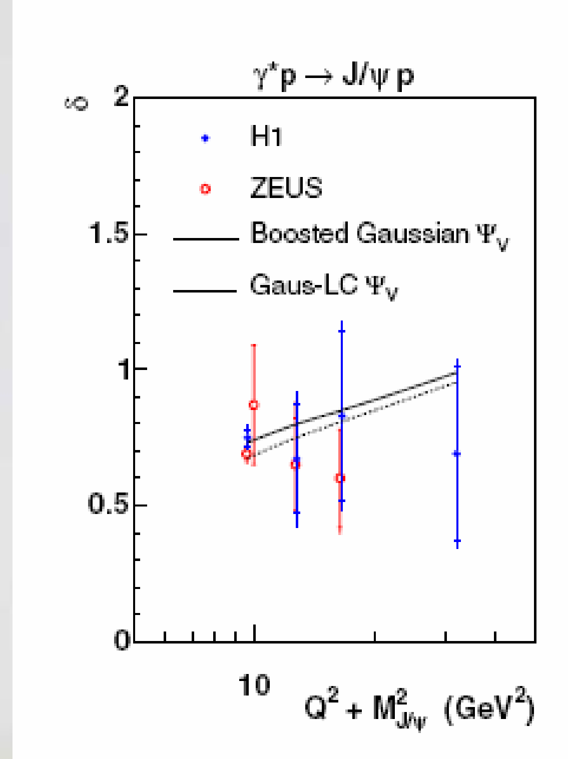
Vector Mesons

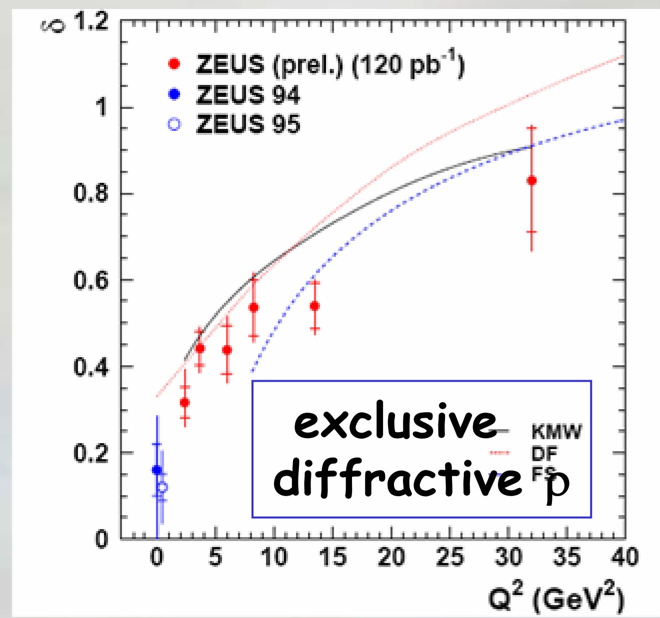


Note: educated guesses for VM wf work very wel

Discovery of HERA







Universality of the observed intercepts

➔ Universal, “Pomeron like” QCD object
soft and hard *IP* join together

Pomeron-Graviton Correspondence

String theory emerged out of
phenomenology of hadron-hadron scattering

$$\alpha(t) = \alpha_0 + \alpha' t$$

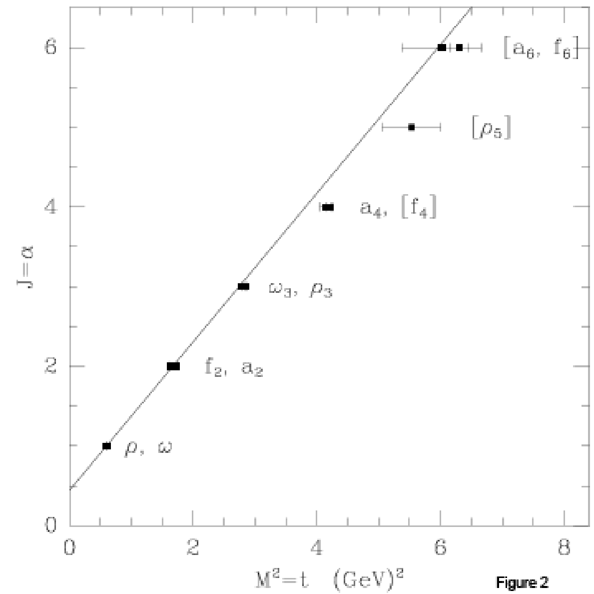


Figure 2

Dolan-Horn-Schmid duality between
s-channel and t-channel Regge-pole description
of hadronic X-sections

$$\sum_r \frac{g_r^2(t)}{s-(M_r-i\Gamma_r)^2} \simeq \beta(t)(-\alpha' s)^{\alpha(t)} \, .$$

➔ Veneziano amplitude

$$A_{\pi^+\pi^-\to\pi^+\pi^-}(s,t) = g_o^2 \frac{\Gamma[1-\alpha_\rho(t)]\Gamma[1-\alpha_\rho(s)]}{\Gamma[1-\alpha_\rho(s)-\alpha_\rho(t)]}$$

generalization by Virasoro ➔ dual models

$$A(s,t,u)=\beta\frac{\Gamma(\frac{1}{2}-\frac{1}{2}\alpha(s))\Gamma(\frac{1}{2}-\frac{1}{2}\alpha(t))\Gamma(\frac{1}{2}-\frac{1}{2}\alpha(u))}{\Gamma(1-\frac{1}{2}\alpha(t)-\frac{1}{2}\alpha(u))\Gamma(1-\frac{1}{2}\alpha(s)-\frac{1}{2}\alpha(u))\Gamma(1-\frac{1}{2}\alpha(s)-\frac{1}{2}\alpha(t))}$$

➔ mesons are open strings, closed strings necessary for unitarity

L - string length, $E = c\,L$, $J = \alpha' E^2$

Virasoro-amplitude for $\alpha(0)$=1 has a pole at $s = t = 0$
with J = 2, a graviton ➔ starting point for theory of quantum gravity

Superstring Theory, Green, Schwarz, Witten (1987)
R. Brower hep-th/0508036

Maldacena Conjecture

from the talk by J. Maldacena

Particle theory = gravity theory

Most supersymmetry QCD theory $=$ String theory on $AdS_5 \times S^5$ (J.M.)

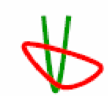
N colors

N = magnetic flux through S^5

Radius of curvature $R_{S^5} = R_{AdS_5} = \left(g_{YM}^2 \; N\right)^{1/4} l_s$

Duality:

$g^2 N$ is small → perturbation theory is easy – gravity is bad



$g^2 N$ is large → gravity is good – perturbation theory is hard

Strings made with gluons become fundamental strings.

From the talk by J. Maldacena

Most supersymmetric QCD

Supersymmetry

Ramond
Wess, Zumino

Bosons $\longleftrightarrow$ Fermions

Gluon $\longleftrightarrow$ Gluino

Many supersymmetries

$$\begin{array}{ccc} B1 & \longleftrightarrow & F1 \\ B2 & \longleftrightarrow & F2 \end{array}$$

Maximum 4 supersymmetries, $N = 4$ Super Yang Mills

A_μ	Vector boson	spin = 1
Ψ_α	4 fermions (gluinos)	spin = 1/2
Φ^I	6 scalars	spin = 0

All NxN matrices

SO(6) symmetry

Susy might be present in the real world but spontaneously broken at low energies.

We study this case because it is simpler.

$\boxed{\text{but } \beta = 0\text{, no asymptotic freedom}}$

Pomeron and Gauge/String Duality

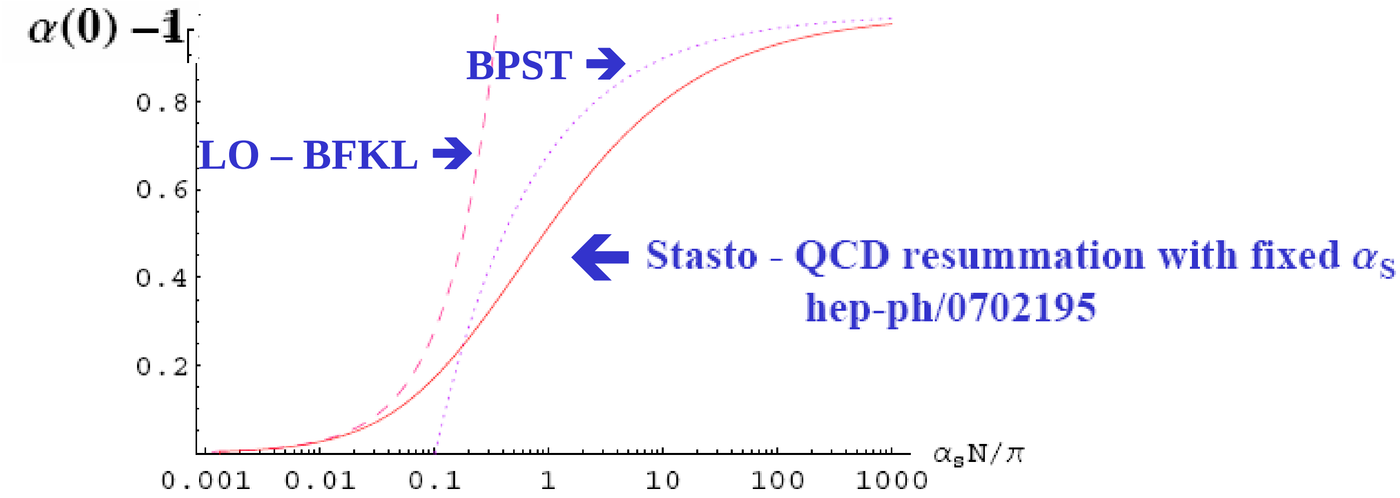
Brower, Polchinski, Strassler, and Tan, hep-th/0603115

Pomeron is a coherent color-singlet object, build from gluons, with universal properties; it is the object which is exchanged by any pair of hadrons that scatter at high energies.
In string theory, it is the object which is exchanged in tree level scattering in the Regge regime, it is not the graviton but the graviton's Regge traj.

$$\omega = \quad 2 - \frac{2}{\pi\sqrt{\overline{\alpha_s}}}, \quad \text{for string theory} \quad (\text{valid for } \overline{\alpha_s} \gg 1) \quad \textbf{in ADS/CFT}$$

in N=4 YM = Most Supersymmetric QCD

Kotikov, Lipatov, Onishchenko, Velizhanin, Physt. Lett. B 632, 754 (2006)



A Possible Pomeron-Graviton connection in the real world

How to combine Regge theory with DGLAP and BFKL ? Lipatov 1986

$$xg(x, Q^2) \approx \sum_{n=0} (1/x)^{\Delta_n} c_n(Q^2)$$

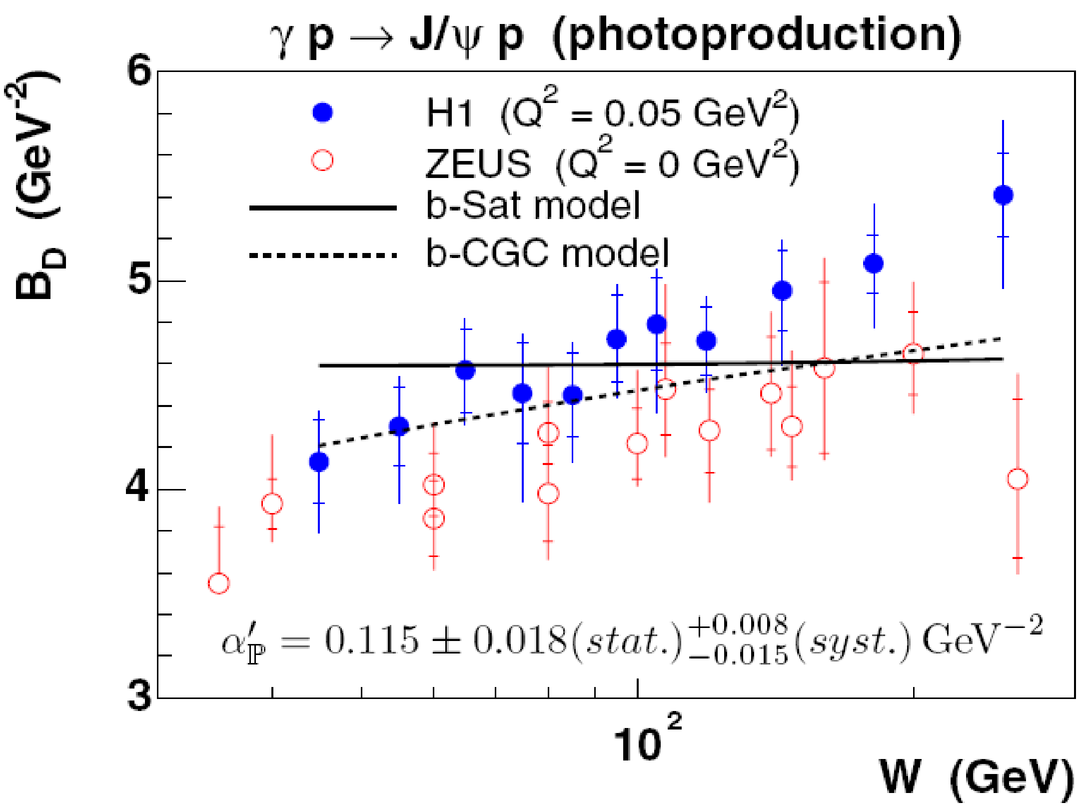
$$\Delta_n = J_n - 1 = \frac{c}{n+\delta}; \qquad 0 < \delta < 1$$

$$c_n \sim (\log Q^2)^{1/\Delta_n}$$

leading intercept $\Delta_0 \geq 0.4$
(no saturation effects)

Lipatov conjecture: quantum properties of the graviton determine the value of the leading Pomeron intercept
(leading intercept can be calculated in the gravitational string theory,
it cannot be calculated in the perturbative QCD)

measurement of ♋'

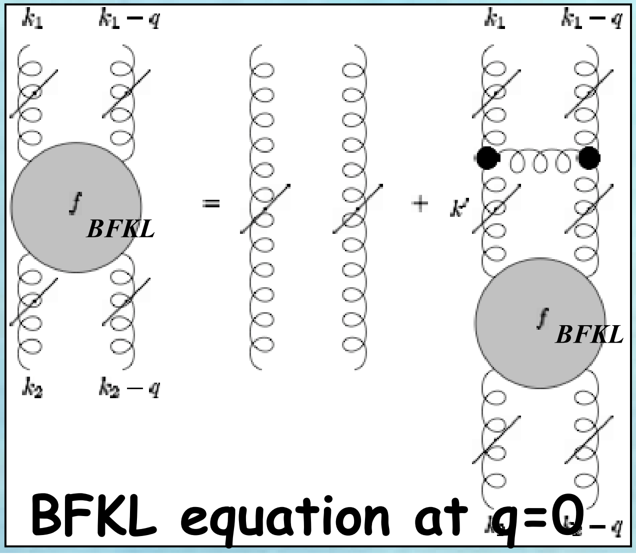


Significant slope is expected for a leading pomeron trajectory Lipatov (1986)

$$J_n(t) = 1 + \frac{c}{n + \delta(t)}$$

BPST ➔ $\alpha' = R^2 / g_{YM}\sqrt{N}$

Basics of BFKL



BFKL equation at q=0

Conformal invariance

solved by finding a complete set of eigenfunctions

$$\omega \tilde{f}(\omega, \mathbf{k_1}, \mathbf{k_2}) = \delta^2(\mathbf{k_1} - \mathbf{k2}) + \frac{\alpha C_A}{\pi^2} \int \frac{d^2\mathbf{k}'}{(\mathbf{k_1} - \mathbf{k}')^2} \left[\tilde{f}(\omega, \mathbf{k}', \mathbf{k_2}) - \frac{\mathbf{k_1}^2}{\mathbf{k}'^2 + (\mathbf{k}' - \mathbf{k_1})^2} \tilde{f}(\omega, \mathbf{k_1}, \mathbf{k_2}) \right]$$

Eigen-functions

$$f_\omega(k^2) = \frac{(k^2)^{i\nu}}{\sqrt{k^2}}$$

$$\omega = \bar{\alpha}_s \chi(\nu)$$

Characteristic function

$$\chi(\nu) = -2\gamma_E - \psi(1/2 + i\nu) - \psi(1/2 - i\nu)$$

ψ is the Digamma function

Green function

$$f_{BFKL}(\omega, k_1, k_2) = \int_{-\infty}^{\infty} d\nu \left(\frac{k_1^2}{k_2^2} \right)^{i\nu} \frac{1}{2\pi^2 k_1 k_2} \frac{1}{(\omega - \bar{\alpha}_s \chi(\nu))}$$

Green function

$$f_{BFKL}(\omega, k_1, k_2) = \int_{-\infty}^{\infty} d\nu \left(\frac{k_1^2}{k_2^2}\right)^{i\nu} \frac{1}{2\pi^2 k_1 k_2} \frac{1}{(\omega - \bar{\alpha_s}\chi(\nu))}$$

usually approximated by:

$$\chi(\nu) = 4\ln 2 - 14\zeta(3)\nu^2 + \ldots$$

$$f(\sqrt{s}, \mathbf{k_1}, \mathbf{k_2}) \sim \frac{1}{\mathbf{k_1}\mathbf{k_2}} s^{4\overline{\alpha_s}\ln(2)} \frac{1}{\sqrt{\ln(s)}} \exp\left\{\frac{-\ln^2(\mathbf{k_1}/\mathbf{k_2})}{14\zeta(3)\overline{\alpha_s}\ln(s)}\right\}$$

not used for DAFP

NLO BFKL with running α_s

NLO

$$\omega \equiv \chi(\alpha_s, \nu) = \bar{\alpha}_s (1 - A\bar{\alpha}_s) \chi_0 \left(\frac{1}{2} + \bar{\alpha}_s B + i\nu + \frac{\omega}{2} \right) + \bar{\alpha}_s^2 \chi_1(\nu).$$

Fadin, Lipatov
G. Salam
resummation

running coupling

$$\omega = \chi\left(\alpha_s(k), \nu_\omega(k)\right).$$

$$\omega = \chi(\alpha_s(k_{\rm crit}), 0).$$

property of χ:
largest ω at $\nu=0$

Airy functions are solving BFKL eq. around $k \sim k_{crit}$

$$\left[\frac{d^2}{d \ln(k^2/k_0^2)} + \frac{\beta_0}{2\pi} \frac{\dot{\chi}(\alpha_s(k_{\rm crit}), 0)}{\chi''(\alpha_s(k_{\rm crit}), 0)} \ln\left(\frac{k^2}{k_0^2}\right) \right] \overline{f_\omega}(k) = 0,$$

$$f_\omega(k^2) = \frac{\overline{f_\omega}(k)}{\sqrt{k^2}},$$

NLO BFKL with running α_s

solution away from k_{crit}

$$\overline{f_\omega}(k) = e^{\pm i\varphi_\omega(k)},$$

$$\varphi_\omega(k) = 2\int_k^{k_{\mathrm{crit}}} \frac{dk'}{k'} |\nu_\omega(k)| .$$

for all regions:

$$\begin{aligned}\overline{f_\omega}(k) &= \sqrt[3]{\varphi_\omega(k)}\left[J_{\frac{1}{3}}(\varphi_\omega(k)) + J_{-\frac{1}{3}}(\varphi_\omega(k))\right], \quad (k < k_{\mathrm{crit}}),\\ &= \sqrt{3}\sqrt[3]{\varphi_\omega(k)}K_{\frac{1}{3}}(\varphi_\omega(k)), \qquad (k > k_{\mathrm{crit}}),\end{aligned}$$

Matching the salutions at $k=k_{crit}$ determines the **phase of oscillations = $\pi/4$**

near $k \sim k_0$

$$\overline{f_\omega}(k) \sim \sin\left(\frac{\nu_\omega(k_0)}{k_0^2}\left(k^2 - k_0^2\right) - \eta\right).$$

Lipatov 86 ➔ encode the infrared behaviour of QCD by assuming a **fixed phase η at k_0**

Quantization condition

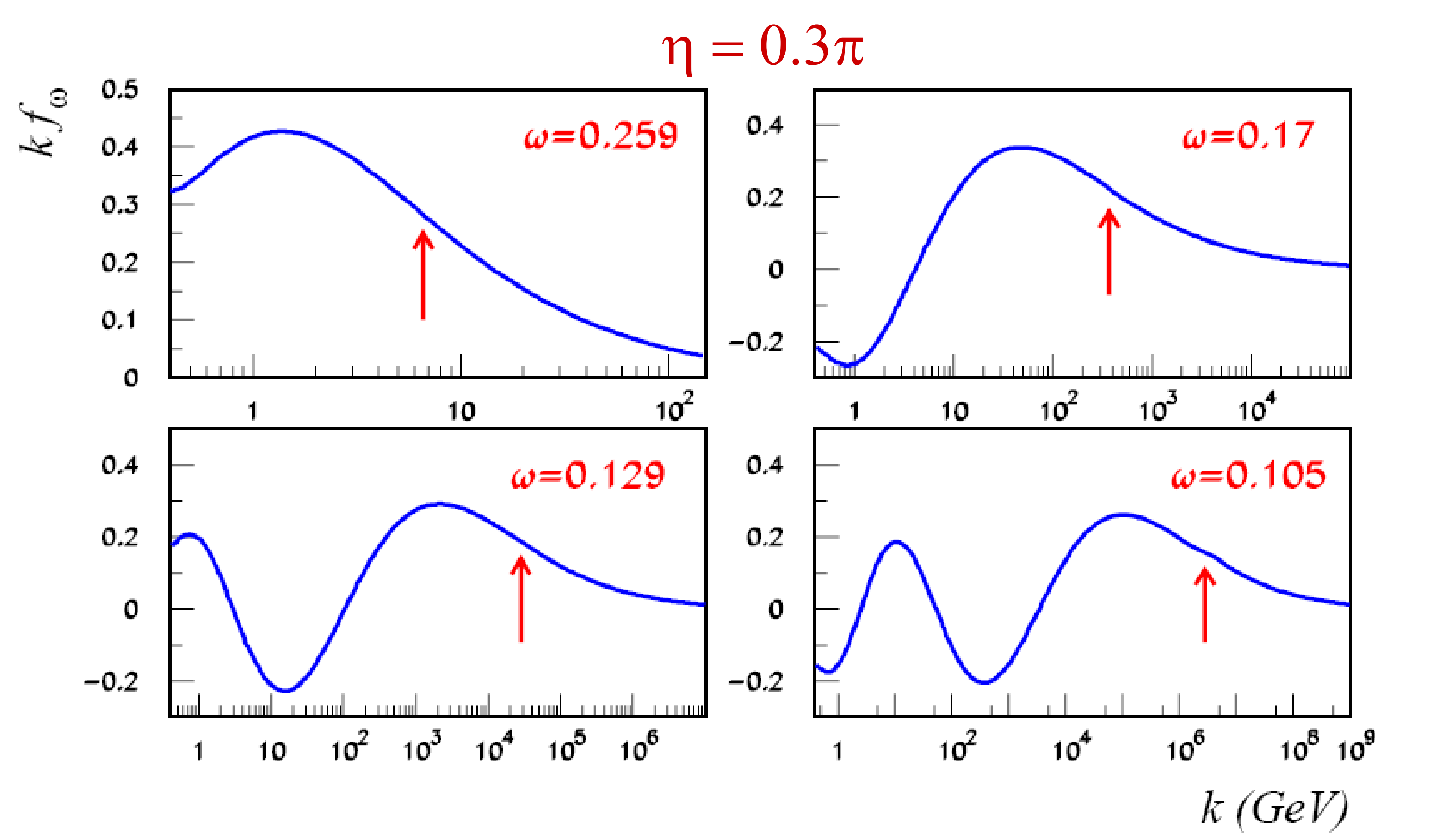
$$\varphi_\omega(k_0) \equiv 2\int_{k_0}^{k_{\mathrm{crit}}} \frac{dk'}{k'} |\nu_\omega(k)| = \left(n - \frac{1}{4}\right)\pi + \eta,$$

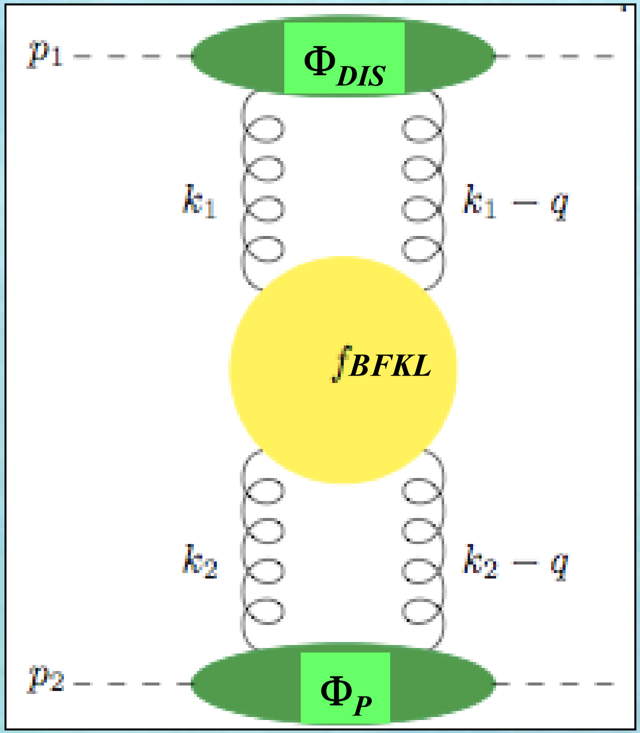
$$\omega = \chi\left(\alpha_s(k), \nu_\omega(k)\right).$$

$$\boxed{\varphi_\omega(k_0) \equiv 2\int_{k_0}^{k_{\rm crit}} \frac{dk'}{k'}\,|\nu_\omega(k)| = \left(n - \frac{1}{4}\right)\pi + \eta,}$$

solve for a fixed ω

just above k_0 ➔ $\overline{f_\omega}(k) \sim \sin\left(\frac{\nu_\omega(k_0)}{k_0^2}\left(k^2 - k_0^2\right) - \eta\right).$





Φ_{DIS} *known in QCD*

Φ_P *barely known*

structure function

$$F_2(x, Q^2) = \int_0^Q \frac{dk}{k} \Phi_{\rm DIS}(Q, k) x g(x, k),$$

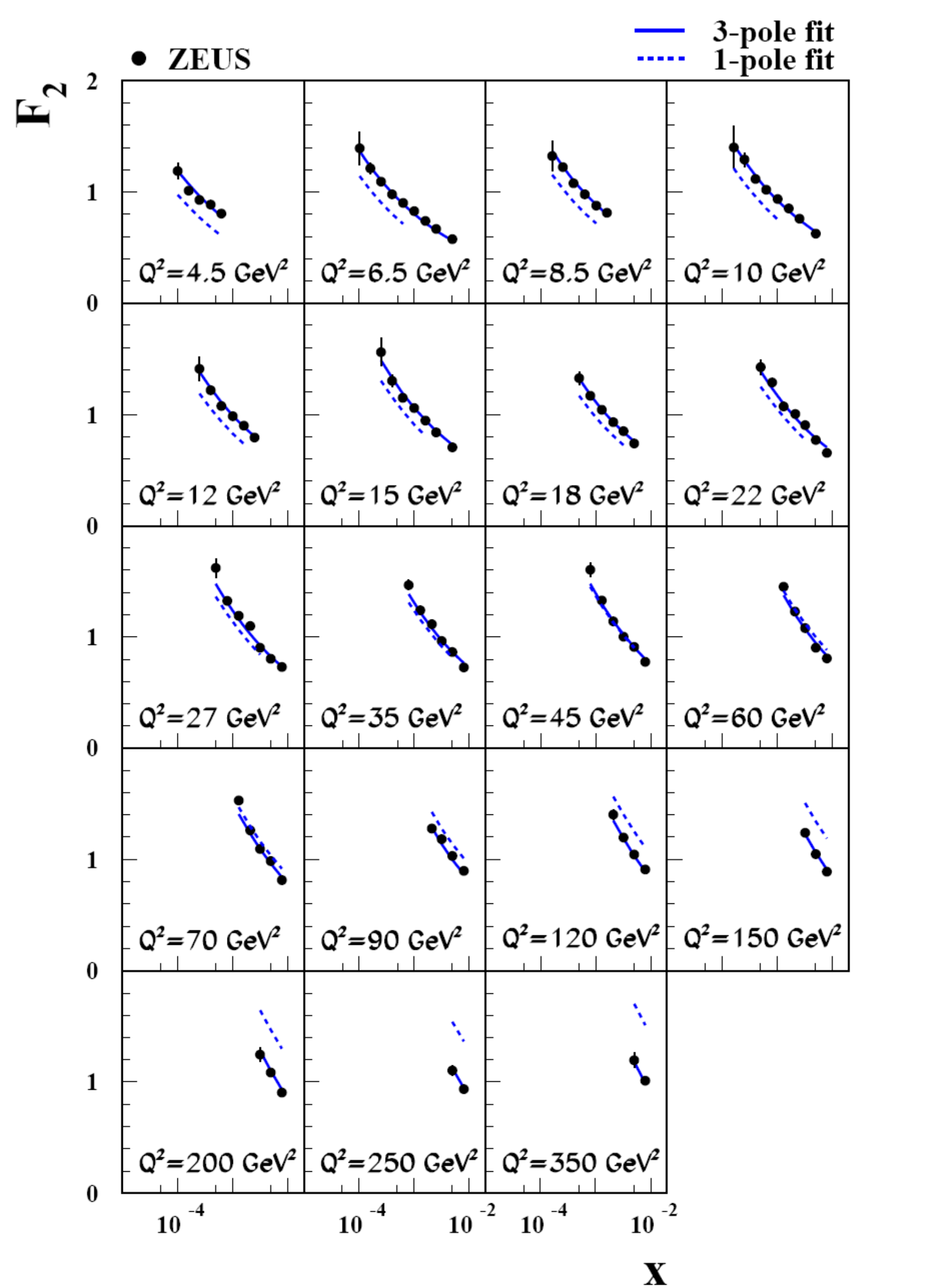
unintegrated gluon density

$$xg(x, k) = \sum_n \int \frac{dk'}{k'} \Phi_p(k') x^{-\omega_n} k^2 f^*_{\omega_n}(k') f_{\omega_n}(k),$$

$$xg(x, k) = \sum_n a_n x^{-\omega_n} k^2 f_{\omega_n}(k).$$

$$\Phi_p(k) = \sum_n a_n k^2 f_{\omega_n}(k),$$

$n-pole$	χ^2/N_{df}	a_1	a_2	a_3	a_4
1-pole	3900/101	0.033	-	-	-
2-pole	303/100	0.026	-0.030	-	-
3-pole	98.7/99	0.041	0.057	0.087	-
4-pole	98.4/98	0.043	0.095	0.16	0.049

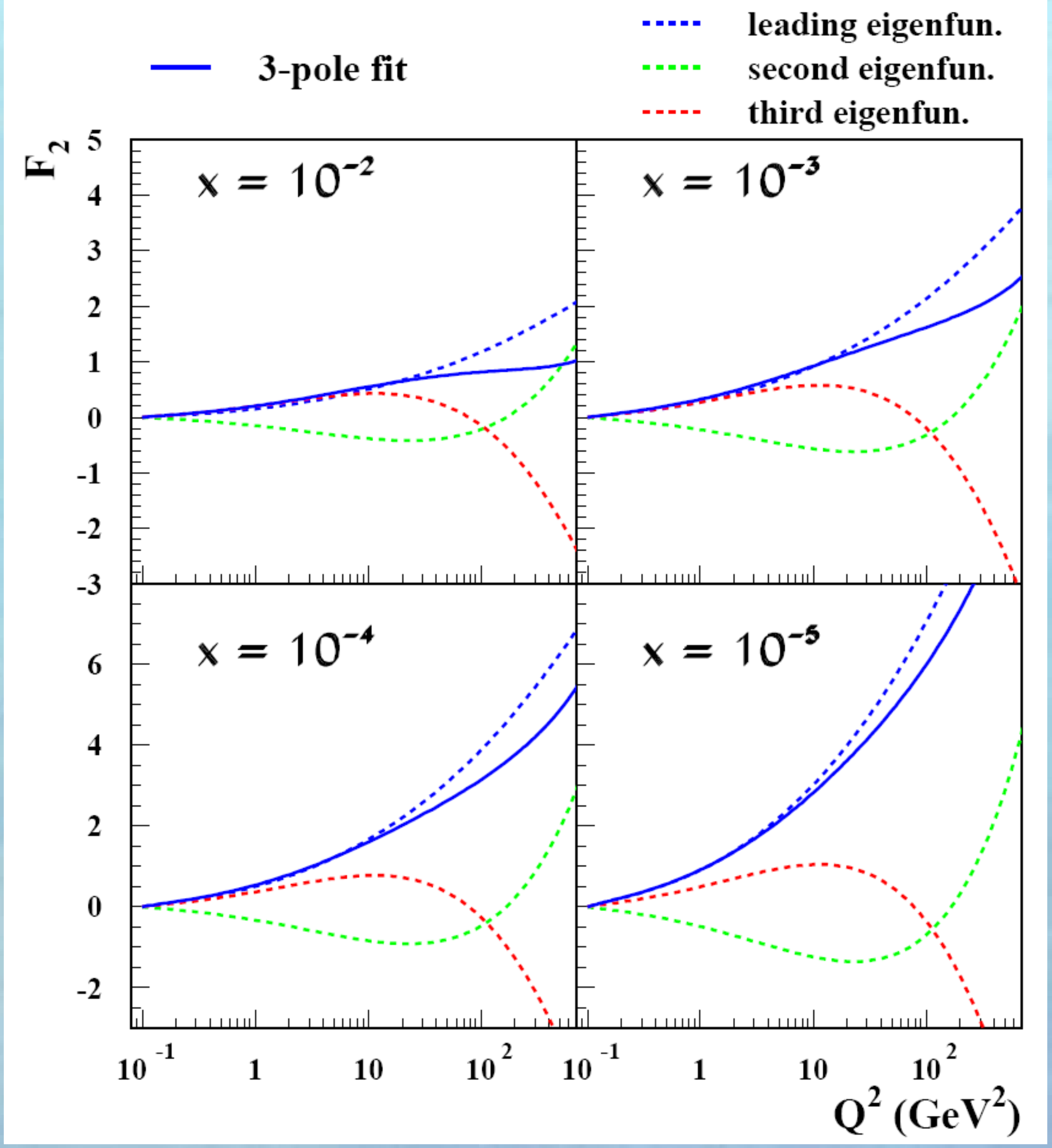


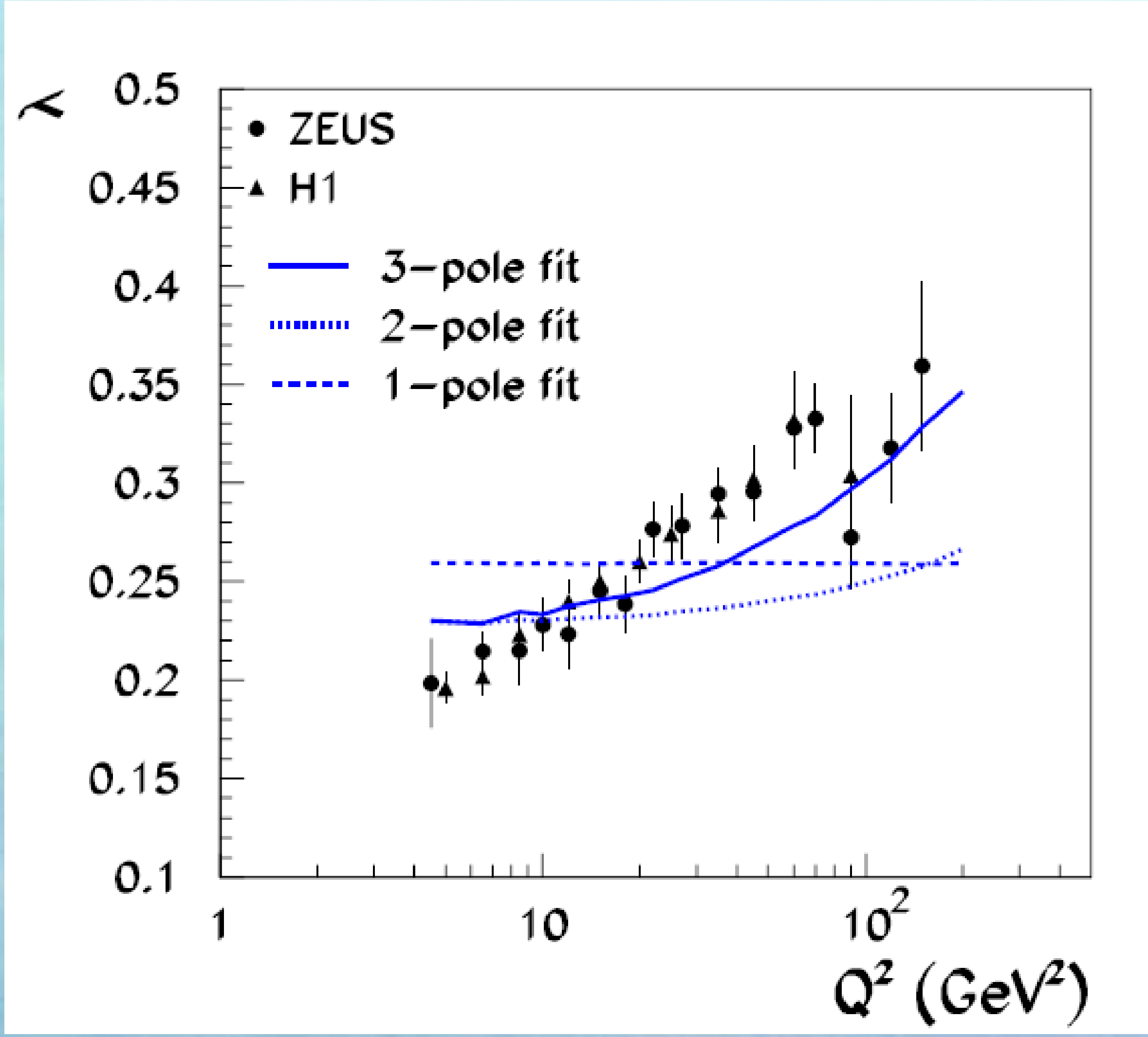
ZEUS
3-pole fit
1-pole fit
F2
Q²=4.5 GeV²
Q²=6.5 GeV²
Q²=8.5 GeV²
Q²=10 GeV²
Q²=12 GeV²
Q²=15 GeV²
Q²=18 GeV²
Q²=22 GeV²
Q²=27 GeV²
Q²=35 GeV²
Q²=45 GeV²
Q²=60 GeV²
Q²=70 GeV²
Q²=90 GeV²
Q²=120 GeV²
Q²=150 GeV²
Q²=200 GeV²
Q²=250 GeV²
Q²=350 GeV²
x

Contributions to F_2 of the individual eigenfunctions

good data description due to interferences

➔ phase η precisely determined





Sum of contributions with small eigenvalues can give a larger rate of rise than the leading eigenvalue !!!

Where Do BFKL and DGLAP Meet

Lipatov, private communication

Unintegrated BFKL gluon density (LO, no running α_s)

$$xg(x,k^2) = \int d\gamma \Phi_p(\gamma) \left(\frac{k^2}{\mu^2}\right)^\gamma x^{-\bar{\alpha_s}\chi(\gamma)} = \int d\gamma \Phi_p(\gamma) \exp(F(\gamma))$$

$\gamma = ½ + i\nu$

Saddle point

$$(F(\gamma))' = (\gamma \ln(k^2/\mu^2) + \bar{\alpha_s} \ln(1/x)\chi(\gamma))' = 0$$

$$\chi(\gamma) \;=\; \frac{1}{\gamma} - 2\zeta(3)\gamma^2 + \cdots \quad \Longrightarrow \quad \gamma^2 \;=\; \frac{\bar{\alpha}\ln(1/x)}{\ln(k^2/\mu^2)}$$

$$\omega \approx \bar{\alpha_s}/\gamma = \sqrt{\frac{\bar{\alpha_s}\ln(k^2/\mu^2)}{\ln(1/x)}}$$

valid if $\bar{\alpha}(k^2)\ln(1/x) \;\ll\; 1,$

equal to DLL limit of DGLAP (LO, no running α_s)

Conclusions

It is the first time that the Discrete Assymptotically Free (DAF)-Pomeron , with properties following directly from the first principles of QCD was successfully confronted with data

Gluon density described by the DAF-Pomeron has different (?) properties than the DGLAP one when extrapolated to lower-x and higher Q^2

Known problem of DGLAP: negative starting gluon density, lack of proportionality between the sea quarks and gluon distribution

Future: consequences for diffractive vector meson production, t-dependence
consequences for low-x, high Q^2 inclusive and exclusive processes at LHC
consequences for saturation physics

EKR

Pomeron – Graviton correspondence?