Warsaw, 4-7 February '09

Models of Neutrino Masses and Mixings

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Message from the CERN DG Heuer

6 February 2009

As a result of the LHC Performance Workshop, Chamonix :

Under a proposal submitted to CERN management, we will have physics data in late 2009, and there is a strong recommendation to run the LHC through the winter and on to autumn 2010 until we have substantial quantities of data for the experiments. With this change to the schedule, our goal for the LHC's first running period is an integrated luminosity of more than 200 pb-1 operating at 5 TeV per beam, sufficient for the first new physics measurements to be made.

Back to neutrino masses and mixings

I now review some ideas on model building

Old models are more generic and qualitative than present models

Anarchy
Semianarchy
Lopsided models
$U(1)_{FN}$
••••••

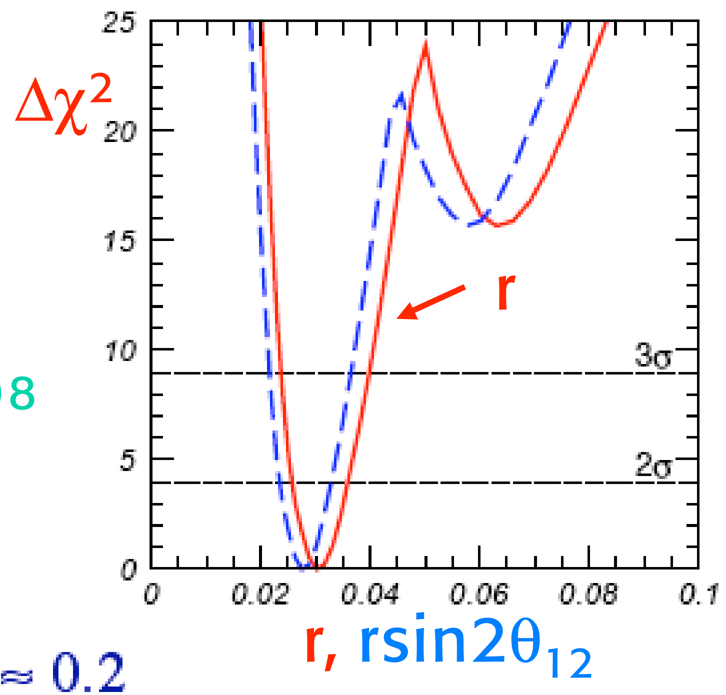
With better data the range for each mixing angle has narrowed and models have become more quantitative

e.g Tribimaximal mixing, A4, S4

General remarks

• After KamLAND, SNO and WMAP.... not too much hierarchy is found in ν masses:

$$r \sim \Delta m^2_{sol}/\Delta m^2_{atm} \sim 1/30$$



Only a few years ago could be as small as 10^{-8}!

Precisely at 3σ: $0.025 < r < 0.039$

Schwetz et al '08

or

$m_{heaviest} < 0.2 - 0.7$ eV
$m_{next} > \sim 8\ 10^{-3}$ eV

For a hierarchical spectrum: $\frac{m_2}{m_3} \approx \sqrt{r} \approx 0.2$

Comparable to $\lambda_C = \sin\theta_C$: $\lambda_C \approx 0.22$ or $\sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$

Suggests the same "hierarchy" parameters for q, l, ν
(small powers of λ_C) $\longrightarrow$ e.g. θ_{13} not too small!

- Still large space for non maximal 23 mixing

  2-σ interval $0.37 < \sin^2\theta_{23} < 0.60$ Fogli et al '08

  Maximal θ_{23} theoretically hard

- θ_{13} not necessarily too small
  probably accessible to exp.

  Very small θ_{13} theoretically hard

For some time people considered limiting models with θ_{13}= 0 and θ_{23} maximal and θ_{12} generic

The most general mass matrix for θ_{13}= 0 and θ_{23} maximal is given by (after ch. lepton diagonalization!!!):

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix}$$

Neglecting Majorana phases it depends on 4 real parameters (3 mass eigenvalues and 1 mixing angle: θ_{12})

Inspired models based on μ–τ symmetry

Grimus, Lavoura..., Ma,.... Mohapatra, Nasri, Hai-Bo Yu ....

Actually, at present, since KamLAND, the most accurately known angle is θ_{12}

G.L.Fogli et al'08

At $\sim 1\sigma$: $\boxed{\sin^2\theta_{12} = 0.294\text{-}0.331}$

By adding $\sin^2\theta_{12} \sim 1/3$ to $\theta_{13} \sim 0$, $\theta_{23} \sim \pi/4$:

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Harrison, Perkins, Scott '02

Some additional ingredient other than μ–τ symmetry needed!

Comparison with experiment:

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

At 1σ: G.L.Fogli et al'08

$\sin^2\theta_{12} = 1/3$: 0.29-0.33

$\sin^2\theta_{23} = 1/2$: 0.41-0.54

$\sin^2\theta_{13} = 0$: $< \sim 0.02$

The HPS mixing is clearly a very good approx. to the data!

Also called:
Tri-Bimaximal mixing

$$\nu_3 = \frac{1}{\sqrt{2}}(-\nu_\mu + \nu_\tau)$$

$$\nu_2 = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)$$

By adding $\sin^2\theta_{12} \sim 1/3$ to $\theta_{13} \sim 0$, $\theta_{23} \sim \pi/4$:

Tribimaximal Mixing

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix} \longrightarrow m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

$$\sin^2 2\theta_{12} = \frac{8y^2}{(x-w-z)^2 + 8y^2}$$

$m_1 = x - y$
$m_2 = x + 2y$
$m_3 = x - y + 2v$

The 3 remaining parameters are the mass eigenvalues

Tribimaximal Mixing

A simple mixing matrix compatible with all present data

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

In the basis of diagonal ch. leptons:

$m_\nu = U \mathrm{diag}(m_1, m_2, m_3) U^T$

$$m_\nu = \frac{m_3}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} + \frac{m_2}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \frac{m_1}{6}\begin{bmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{bmatrix}$$

Eigenvectors: $m_3 \rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ $\quad m_2 \rightarrow \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad m_1 \rightarrow \frac{1}{\sqrt{6}}\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

Note: mixing angles independent of mass eigenvalues

Compare with quark mixings $\lambda_C \sim (m_d/m_s)^{1/2}$

- For the HPS mixing matrix all mixing angles are fixed to particularly symmetric values

Sparked interest in constructing models that can naturally produce this highly ordered structure (very far from anarchy!)

Models based on the A4 discrete symmetry (even permutations of 1234) offer a minimal solution

Ma...;
GA, Feruglio, hep-ph/0504165, hep-ph/0512103
GA, Feruglio, Lin hep-ph/0610165
GA, Feruglio, Hagedorn, 0802.0090 [hep-ph]
Y. Lin, 0804.2867 [hep-ph]........

Larger finite groups: T', $\Delta(27)$, S4

Feruglio et al;
Chen, Mahanthappa;
Frampton, Kephart; Lam;
Bazzocchi et al ....

Alternative models based on $SU(3)_F$ or $SO(3)_F$ or their finite subgroups

Verzielas, G. Ross King ....
....

Lindner-Manchester '07

List of models with flavor symmetries

(incomplete, by symmetry):

S_3: Pakvasa et al. (1978) Derman (1979), Ma (2000), Kubo et al. (2003), Chen et al. (2004), Grimus et al. (2005), Dermisek et al. (2005), Mohapatra et al. (2006), ...

S4: Pakvasa et al. (1979), Derman et al. (1979), Lee et al. (1994), Mohapatra et al. (2004), Ma (2006), Hagedorn, ML and Mohapatra (2006), Caravaglios et al. (2006), ...

A_4: Wyler (1979), Ma et al. (2001), Babu et al. (2003), Altarelli et al. (2005,2006), He et al. (2006) ...

D_4: Seidl (2003), Grimus et al. (2003,2004), Kobayashi et al. (2005), ...

D_5: Ma (2004), Hagedorn et al. (2006).

D_n: Chen et al. (2005), Kajiyama et al. (2007), Frampton et al. (1995,1996,2000), Frigerio et al. (2005), Babu et al. (2005), Kubo (2005), ...

T': Frampton et al. (1994,2007), Aranda et al. (1999,2000), Feruglio et al. (2007), Chen and Mahanthappa (2007)

Δ_n: Kaplan et al. (1994), Chou et al. (1997), de Medeiros Varzielas et al. (2005), ...

T_7: Luhn et al.

Model building

Quality factors for models:
(higher standards by now!)

- Based on the most general lagrangian compatible with some simple symmetry or dynamical principle
- Should be complete: address at least charged leptons and neutrinos ($U_{P\text{-}NMS} = U^{+}_{e} U_{\nu}$, and the gauge symmetry connects ch. leptons and LH neutrinos)
- As many as possible small parameters (masses and mixings) should be naturally explained as a consequence.
- The necessary VEV configuration should be a minimum of the most general potential for a region of parameter space
- The stability under radiative corrections and higher dim operators must be checked
- Simplicity, economy of fields and parameters, predictivity...

A4

A4 is the discrete group of even perm's of 4 objects.
(the inv. group of a tetrahedron). It has $4!/2 = 12$ elements.

A4 transformations can be written in terms of S and T as:

$1, T, S, ST, TS, T^2, TST, STS, ST^2, T^2S, T^2ST, TST^2$

with: $S^2 = T^3 = (ST)^3 = 1$ [$(TS)^3 = 1$ also follows]

An element is abcd which means 1234 --> abcd

C_1: $1 = 1234$
C_2: $T = 2314$ $ST = 4132$ $TS = 3241$ $STS = 1423$
C_3: $T^2 = 3124$ $ST^2 = 4213$ $T^2S = 2431$ $TST = 1342$
C_4: $S = 4321$ $T^2ST = 3412$ $TST^2 = 2143$

C_1, C_2, C_3, C_4 are equivalence classes

x, x′ in same class if $[x' \sim gxg^{-1}]$

g: group element

A4 has 4 inequivalent irreducible representations:
a triplet and 3 different singlets

$$3,\ 1,\ 1',\ 1''$$

(promising for 3 generations!)

Note:

as many representations as equivalence classes

$\sum d_i^2 = 12$ $\qquad$ $9+1+1+1=12$

Note: many models tried S3
S3 has no triplets but only 2 , 1, 1′
A4 is better in the lepton sector

Mohapatra, Nasri, Yu
Koide
Kubo et al
Kaneko et al
Caravaglios et al
Morisi
Picariello......

Three singlet inequivalent represent'ns:

Recall:
$S^2 = T^3 = (ST)^3 = 1$

$$\begin{cases} 1: & S=1,\ T=1 \\ 1': & S=1,\ T=\omega \\ 1'': & S=1,\ T=\omega^2 \end{cases}$$

$$\omega = \exp i\frac{2\pi}{3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$\omega^3 = 1$$

$$1 + \omega + \omega^2 = 0$$

$$\omega^2 = \omega^*$$

The only irreducible 3-dim represent'n is obtained by:

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

(S-diag basis)

An equivalent form:

$$VV^\dagger = V^\dagger V = 1$$

$$S' = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} = VSV^\dagger \qquad T' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{bmatrix} = VTV^\dagger \qquad V = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$$

(T-diag basis)

Cabibbo '78

A4 has only 4 irreducible inequivalent represt'ns: 1,1',1",3

Table of Multiplication:
1'x1'=1"; 1"x1"=1';1'x1"=1
3x3=1+1'+1"+3+3

A4 is well fit for 3 families!

Ch. leptons l ~ 3

e^c, μ^c, τ^c ~ 1, 1", 1'

In the S-diag basis consider 3: (a_1,a_2,a_3) $\xrightarrow{S}$ $(a_1,-a_2,-a_3)$

(a_1,a_2,a_3) $\xrightarrow{T}$ (a_2,a_3,a_1)

For $3_1=(a_1,a_2,a_3)$, $3_2=(b_1,b_2,b_3)$ we have in $3_1 x 3_2$:

$$1 = a_1b_1 + a_2b_2 + a_3b_3$$

$$1' = a_1b_1 + \omega^2 a_2b_2 + \omega a_3b_3$$

$$1" = a_1b_1 + \omega a_2b_2 + \omega^2 a_3b_3$$

$$3 \sim (a_2b_3, a_3b_1, a_1b_2)$$

$$3 \sim (a_3b_2, a_1b_3, a_2b_1)$$

e.g. $1" = a_1b_1+\omega a_2b_2+\omega^2 a_3b_3 \xrightarrow{T} a_2b_2+\omega a_3b_3+\omega^2 a_1b_1 =$
$= \omega^2 [a_1b_1+\omega a_2b_2+\omega^2 a_3b_3]$

(under S, 1" is invariant)

In the T-diagonal basis we have:

$$VV^\dagger = V^\dagger V = 1$$

$$S' = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} = VSV^\dagger \qquad T' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{bmatrix} = VTV^\dagger \qquad V = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$$

Cabibbo '78

For $3_1=(a_1,a_2,a_3)$, $3_2=(b_1,b_2,b_3)$ we have in $3_1 x 3_2$:

$$1 = a_1b_1 + a_2b_3 + a_3b_2$$

$$1' = a_3b_3 + a_1b_2 + a_2b_1$$

$$1" = a_2b_2 + a_1b_3 + a_3b_3$$

We will see that in this basis the charged leptons are diagonal

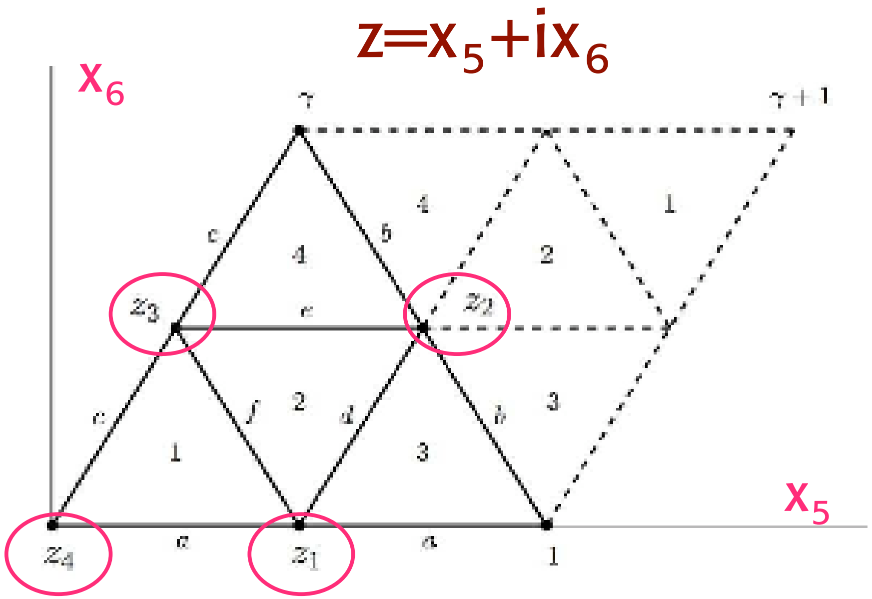
$$3_{symm} \sim \frac{1}{3}(2a_1b_1 - a_2b_3 - a_3b_2, 2a_3b_3 - a_1b_2 - a_2b_1, 2a_2b_2 - a_1b_3 - a_3b_1)$$

$$3_{antisymm} \sim \frac{1}{2}(a_2b_3 - a_3b_2, a_1b_2 - a_2b_1, a_1b_3 - a_3b_1)$$

What can be the origin of A4?

A4 (or some other discrete group) could arise from extra dimensions (by orbifolding with fixed points) as a remnant of 6-dim spacetime symmetry:

G.A.,F. Feruglio&Y. Lin, NP B775(2007)31



A torus with identified points:

$z \to z + 1$

$z \to z + \gamma \quad \gamma=\exp(i\pi/3)$

and a parity $z \to -z$

leads to 4 fixed points (equivalent to a tethraedron).

There are 4D branes at the fixed points where the SM fields live (additional gauge singlets are in the bulk)

⊕ A4 interchanges the fixed points

Under A4 the most common classification is:

lepton doublets $l \sim 3$

$e^c, \mu^c, \tau^c \sim 1, 1'', 1'$ respectively

A4 breaking gauge singlet flavons $\phi_S, \phi_T, \xi, (\xi') \sim 3, 3, 1, (1)$

For SUSY version: driving fields $\phi'_S, \phi'_T, \xi_0 \sim 3, 3, 1$

with the alignment:

!!!

$$\boxed{\begin{aligned} \langle \varphi_T \rangle &= (v_T, 0, 0) \\ \langle \varphi_S \rangle &= (v_S, v_S, v_S) \\ \langle \xi \rangle = u \quad &, \quad \langle \tilde{\xi} \rangle = 0 \end{aligned}}$$

In all versions there are additional symmetries:

- e.g. a broken $U(1)_F$ symmetry to ensure hierarchy of charged lepton masses
- some discrete symmetries Z_n to restrict allowed couplings

Structure of the model (a 4-dim SUSY version)

GA, Feruglio, hep-ph/0512103

$$w_l = y_e e^c(\varphi_T l) + y_\mu \mu^c(\varphi_T l)' + y_\tau \tau^c(\varphi_T l)'' + (x_a \xi + \tilde{x}_a \tilde{\xi})(ll) + x_b(\varphi_S ll) + h.c. + \ldots$$

shorthand: Higgs and cut-off scale Λ omitted, e.g.:

$$y_e e^c(\varphi l) \sim y_e e^c(\varphi l) h_d/\Lambda, \qquad x_a \xi(ll) \sim x_a \xi(l h_u l h_u)/\Lambda^2$$

In T-diag basis:

with this alignment:

!!!

$$\boxed{\begin{aligned} \langle \varphi_T \rangle &= (v_T, 0, 0) \\ \langle \varphi_S \rangle &= (v_S, v_S, v_S) \\ \langle \xi \rangle = u \quad &, \quad \langle \tilde{\xi} \rangle = 0 \end{aligned}}$$

Ch. leptons are diagonal

$$m_l = v_T \frac{v_d}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

ν's are tri-bimaximal

$$m_\nu = \frac{v_u^2}{\Lambda} \begin{pmatrix} a + 2b/3 & -b/3 & -b/3 \\ -b/3 & 2b/3 & a - b/3 \\ -b/3 & a - b/3 & 2b/3 \end{pmatrix}$$

$$a \equiv x_a \frac{u}{\Lambda} \qquad b \equiv x_b \frac{v_T}{\Lambda}$$

recall:

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

For the 4-dim SUSY version (written in the T-diag basis)

In this basis the ch. leptons are diagonal!

$$w_l = y_e e^c(\varphi_T l) + y_\mu \mu^c(\varphi_T l)' + y_\tau \tau^c(\varphi_T l)'' + (x_a \xi + \tilde{x}_a \tilde{\xi})(ll) + x_b(\varphi_S ll) + h.c. + \ldots$$

One more singlet is needed for vacuum alignment

The superpotential (at leading order):

$$\begin{aligned} w_d &= M(\varphi_0^T \varphi_T) + g(\varphi_0^T \varphi_T \varphi_T) \\ &+ g_1(\varphi_0^S \varphi_S \varphi_S) + g_2 \tilde{\xi}(\varphi_0^S \varphi_S) + g_3 \xi_0(\varphi_S \varphi_S) + g_4 \xi_0 \xi^2 + g_5 \xi_0 \xi \tilde{\xi} + g_6 \xi_0 \tilde{\xi}^2 \end{aligned}$$

and the potential $V = \sum_i \left| \frac{\partial w}{\partial \phi_i} \right|^2 + m_i^2 |\phi_i|^2 + \ldots$

The assumed simmetries are summarised here

Field	l	e^c	μ^c	τ^c	$h_{u,d}$	φ_T	φ_S	ξ	$\tilde{\xi}$	φ_0^T	φ_0^S	ξ_0
A_4	3	1	$1'$	$1''$	1	3	3	1	1	3	3	1
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	ω	1	ω	ω
$U(1)_R$	1	1	1	1	0	0	0	0	0	2	2	2
$U(1)_F$		2q	q	1								

The driving field have zero VEV. So the minimization is:

$$\begin{aligned}
\frac{\partial w}{\partial \varphi_{01}^T} &= M\varphi_{T1} + \frac{2g}{3}(\varphi_{T1}^2 - \varphi_{T2}\varphi_{T3}) = 0 & \frac{\partial w}{\partial \varphi_{01}^S} &= g_2\tilde{\xi}\varphi_{S1} + \frac{2g_1}{3}(\varphi_{S1}^2 - \varphi_{S2}\varphi_{S3}) = 0\\
\frac{\partial w}{\partial \varphi_{02}^T} &= M\varphi_{T3} + \frac{2g}{3}(\varphi_{T2}^2 - \varphi_{T1}\varphi_{T3}) = 0 & \frac{\partial w}{\partial \varphi_{02}^S} &= g_2\tilde{\xi}\varphi_{S3} + \frac{2g_1}{3}(\varphi_{S2}^2 - \varphi_{S1}\varphi_{S3}) = 0\\
\frac{\partial w}{\partial \varphi_{03}^T} &= M\varphi_{T2} + \frac{2g}{3}(\varphi_{T3}^2 - \varphi_{T1}\varphi_{T2}) = 0 & \frac{\partial w}{\partial \varphi_{03}^S} &= g_2\tilde{\xi}\varphi_{S2} + \frac{2g_1}{3}(\varphi_{S3}^2 - \varphi_{S1}\varphi_{S2}) = 0
\end{aligned}$$

$$\frac{\partial w}{\partial \xi_0} = g_4\xi^2 + g_5\xi\tilde{\xi} + g_6\tilde{\xi}^2 + g_3(\varphi_{S1}^2 + 2\varphi_{S2}\varphi_{S3}) = 0$$

Solution:

$$\varphi_T = (v_T, 0, 0) \quad , \qquad v_T = -\frac{3M}{2g}$$

$$\begin{aligned}
\tilde{\xi} &= 0\\
\xi &= u\\
\varphi_S &= (v_S, v_S, v_S) \quad , \qquad v_S^2 = -\frac{g_4}{3g_3}u^2
\end{aligned}$$

So, at LO TB mixing is exact

When NLO corrections are included from operators of higher dimension in the superpotential each mixing angle receives corrections of the same order $\delta\theta_{ij} \sim o(VEV/\Lambda)$

As the maximum allowed corrections to θ_{12} (and pehaps also to θ_{23}) are $o(\lambda_C^2)$, we need $VEV/\Lambda \sim o(\lambda_C^2)$ and we expect:

$\theta_{13} \sim o(\lambda_C^2)$ measurable in next run of exp's

(T2K starts at the end of '09)

Note that for TB mixing in A4 it is important that no flavons transforming as 1′ and 1″ exist

Recently Lam claimed that for “a natural” TB model the smallest group is S4 (instead A4 is a subgroup of S4)

This is because he calls “natural” a model only if all possible flavons are introduced

We do not accept this criterium:

In physics we call natural a model if the lagrangian is the most general given the symmetry and the representations of the fields
(for example the SM is natural even if only Higgs doublets are present)

TB mixing corresponds to m
in the basis where
charged leptons are diagonal

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

m is the most general matrix invariant under
SmS = m and $A_{23}mA_{23}$= m with:

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

$$A_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

2-3
symmetry

Invariance under S can be made automatic in A4 while
invariance under A_{23} happens if 1′ and 1″ flavons are absent.

Charged lepton masses are a generic diagonal matrix, invariant under T
(or ηT with η a phase):

$$m_l = v_T \frac{v_d}{\Lambda}\begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

$$T^\dagger m_l T = m_l$$

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

$$\boxed{\begin{aligned} \langle \varphi_T \rangle &= (v_T, 0, 0) \\ \langle \varphi_S \rangle &= (v_S, v_S, v_S) \\ \langle \xi \rangle = u \quad &, \quad \langle \tilde{\xi} \rangle = 0 \end{aligned}}$$

The aligment works because based on A4 group theory:

ϕ_T breaks A4 down to G_T
ϕ_S breaks A4 down to G_S
(G_T, G_S: subgroups generated by T, S)

Recent directions of research:

- Different (larger) finite groups

Ma;
Kobayashi et al;
Luhn, Nasri, Ramond [$\Delta(3n^2)$];
Bazzocchi et al; .....

- Trying to improve the quark mixings

Carr, Frampton
Feruglio et al
Frampton, Kephart.....

- Construct GUT models with approximate tribimaximal mixing

Ma, Sawanaka, Tanimoto; Ma;
Morisi, Picarello, Torrente Lujan; Bazzocchi et al;
de Madeiros Verzielas, King, Ross [$\Delta(27)$];
King, Malinsky [$SU(4)_C xSU(2)_L xSU(2)_R$]; Antusch et al;
Chen, Mahanthappa
GA, Feruglio, Hagedorn.........

Extension to quarks

If we take all fermion doublets as 3 and all singlets as 1, 1′, 1″ (as for leptons): $Q_i \sim 3$, $u^c, d^c \sim 1$, $c^c, s^c \sim 1'$, $t^c, b^c \sim 1''$

Then u and d quark mass matrices, like for charged leptons, are BOTH diagonal in the T-diagonal basis

As a result V_{CKM} is unity: $V_{CKM} = U_u^{+} U_d \sim 1$

So, in first approx. (broken by loops and higher dim operators), ν mixings are HPS and quark mixings ~identity

Corrections are far too small to reproduce quark mixings eg λ_C (for leptons, corrections cannot exceed $o(\lambda_C^2)$). But even those are essentially the same for u and d quarks)

A4 is simple and economic for leptons

One problem is how to extend the model to quarks

Aranda, Carone, Lebed
Carr, Frampton
Feruglio et al
Chen, Mahanthappa

Also one would like a GUT model with all fermion masses and mixings reproduced, which includes TB mixing for ν's from A4

NOT straightforward to embed these models in a GUT: for A4 to commute with SU(5) one needs

If $l \sim 3$ then all $F_i \sim 5_i^* \sim 3$, so that $d^c_i \sim 3$
if $e^c, \mu^c, \tau^c \sim 1, 1'', 1'$ then all $T_i \sim 10_i \sim 1, 1'', 1'$

Widespread feeling that A4 cannot be unified in a satisfactory way.
Here we show a counterexample

Here is our A4 GUT model (0802.0090[hep-ph])

A SUSY SU(5) Grand Unified Model of Tri-Bimaximal Mixing from A_4

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Abstract

We discuss a grand unified model based on SUSY SU(5) in extra dimensions and on the flavour group $A_4 \times U(1)$ which, besides reproducing tri-bimaximal mixing for neutrinos with the accuracy required by the data, also leads to a natural description of the observed pattern of quark masses and mixings.

SUSY-SU(5) GUT with A4

Key ingredients:

- SUSY

  In general SUSY is crucial for hierarchy, coupling unification and p decay
  Specifically it makes simpler to implement the required alignment

- GUT's in 5 dimensions

  In general GUT's in ED are most natural and effective
  Here also contribute to produce fermion hierarchies

- Extended flavour symmetry: $A4 x U(1) x Z_3 x U(1)_R$

  $U(1)_R$ is a standard ingredient of SUSY GUT's in ED

  Hall-Nomura'01

GUT's in extra dimensions

- Minimal SUSY-SU(5), -SO(10) models are in trouble
- More realistic models are possible but they tend to be baroque (e.g. large Higgs representations)

Recently a new idea has been developed and looks promising: unification in extra dimensions

Kawamura
GA, Feruglio
Hall, Nomura;
Hebecker, March-Russell;
Hall, March-Russell, Okui, Smith
Asaka, Buchmuller, Covi
....

Factorised metric

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + h_{ij}(y) dy^i dy^j$$

The compactification radius $R \sim 1/M_{GUT}$ (not so large!)

Virtues:

- No baroque large Higgs representations
- SUSY and SU(5) breaking by orbifolding
- Doublet-triplet splitting problem solved
- New handles for p decay, flavour hierarchies

SUSY-SU(5) GUT with A4

Key ingredients:

Reduces to R-parity when SUSY is broken at m_{soft}

Froggatt-Nielsen

- GUT's in 5 dimensions
- Extended flavour symmetry: $A4 \times U(1) \times Z_3 \times U(1)_R$

Keeps ϕ_S and ϕ_T separate

Field	N	F	T_1	T_2	T_3	H_5	$H_{\bar{5}}$	φ_T	φ_S	$\xi,\ \tilde{\xi}$	θ	θ''	φ_0^T	φ_0^S	ξ_0
SU(5)	1	$\bar{5}$	10	10	10	5	$\bar{5}$	1	1	1	1	1	1	1	1
A_4	3	3	$1''$	$1'$	1	1	$1'$	3	3	1	1	$1''$	3	3	1
U(1)	0	0	3	1	0	0	0	0	0	0	-1	-1	0	0	0
Z_3	ω	ω	ω	ω	ω	ω	ω	1	ω	ω	1	1	1	ω	ω
$U(1)_R$	1	1	1	1	1	0	0	0	0	0	0	0	2	2	2

U(1) breaking flavons

driving fields for alignment

: in bulk

ED effects contribute to the fermion mass hierarchies

A bulk field is related to its zero mode by: $B = \frac{1}{\sqrt{\pi R}} B^0 + \ldots$

This produces a suppression parameter for couplings with bulk fields $s \equiv \frac{1}{\sqrt{\pi R \Lambda}} < 1$

Λ: UV cutoff

- In bulk: N=2 SUSY Yang-Mills fields + H_5, H_5^{bar} + T_1, T_2, T_1', T_2'

(doubling of bulk fermions to obtain chiral massless states at y=0)

also crucial to avoid too strict mass relations for 1,2 families:

(b-τ unification only for 3rd family)

- All other fields on brane at y=0 (in particular N, F, T_3)

Superpotential terms on the brane
($T_{1,2}$ represent either $T_{1,2}$ or $T'_{1,2}$)

Up masses

$$\begin{aligned} w_{up} \;=\;& \frac{1}{\Lambda^{1/2}} H_5 T_3 T_3 + \frac{\theta''}{\Lambda^2} H_5 T_2 T_3 + \frac{\theta''^2}{\Lambda^{7/2}} H_5 T_2 T_2 + \frac{\theta\theta''^2}{\Lambda^4} H_5 T_1 T_3 \\ +\;& \frac{\theta^4}{\Lambda^{11/2}} H_5 T_1 T_2 + \frac{\theta\theta''^3}{\Lambda^{11/2}} H_5 T_1 T_2 + \frac{\theta^5\theta''}{\Lambda^{15/2}} H_5 T_1 T_1 + \frac{\theta^2\theta''^4}{\Lambda^{15/2}} H_5 T_1 T_1 \end{aligned}$$

Down and charged lepton masses

$$\begin{aligned} w_{down} \;=\;& \frac{1}{\Lambda^{3/2}} H_{\bar{5}} (F\varphi_T)'' T_3 + \frac{\theta}{\Lambda^3} H_{\bar{5}} (F\varphi_T)' T_2 + \frac{\theta^3}{\Lambda^5} H_{\bar{5}} (F\varphi_T) T_1 + \frac{\theta''^3}{\Lambda^5} H_{\bar{5}} (F\varphi_T) T_1 \\ +\;& \frac{\theta''}{\Lambda^3} H_{\bar{5}} (F\varphi_T)'' T_2 + \frac{\theta^2\theta''}{\Lambda^5} H_{\bar{5}} (F\varphi_T)' T_1 + \frac{\theta\theta''^2}{\Lambda^5} H_{\bar{5}} (F\varphi_T)'' T_1 + \ldots \quad , \end{aligned}$$

Neutrino masses from see-saw
(correct relation bewteen m_ν and M_{GUT})

$$w_\nu = \frac{y^D}{\Lambda^{1/2}} H_5 (NF) + (x_a \xi + \tilde{x}_a \tilde{\xi})(NN) + x_b(\varphi_S NN)$$

$$m_u = \begin{pmatrix} s^2t^5t'' + s^2t^2t''^4 & s^2t^4 + s^2tt''^3 & stt''^2 \\ s^2t^4 + s^2tt''^3 & s^2t''^2 & st'' \\ stt''^2 & st'' & 1 \end{pmatrix} sv_u^0 \sim \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \lambda v_u^0$$

dots=0 in 1st approx

fixed by higher dim operators & corrections to alignment (see later)

$$m_d = \begin{pmatrix} st^3 + st''^3 & \dots & \dots \\ st^2t'' & st & \dots \\ stt''^2 & st'' & 1 \end{pmatrix} v_T s v_d^0 \sim \begin{pmatrix} \lambda^4 & \dots & \dots \\ \lambda^4 & \lambda^2 & \dots \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} v_T \lambda v_d^0$$

$$m_e = \begin{pmatrix} st^3 + st''^3 & st^2t'' & stt''^2 \\ \dots & st & st'' \\ \dots & \dots & 1 \end{pmatrix} v_T s v_d^0 \sim \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^4 \\ \dots & \lambda^2 & \lambda^2 \\ \dots & \dots & 1 \end{pmatrix} v_T \lambda v_d^0$$

with

$$\frac{\langle \varphi_T \rangle}{\Lambda} = (v_T, 0, 0) \quad , \quad \frac{\langle \varphi_S \rangle}{\Lambda} = (v_S, v_S, v_S) \quad , \quad \frac{\langle \xi \rangle}{\Lambda} = u \quad \frac{\langle \theta \rangle}{\Lambda} = t \quad , \quad \frac{\langle \theta'' \rangle}{\Lambda} = t''$$

$s \sim t \sim t'' \sim \lambda \sim 0.22$ $\qquad$ $v_T \sim \lambda^2 \sim m_b/m_t$ $\qquad$ $v_S,\ u \sim \lambda^2$

For ν's after see-saw

$$m_\nu = \frac{1}{3a(a+b)} \begin{pmatrix} 3a+b & b & b \\ b & \frac{2ab+b^2}{b-a} & \frac{b^2-ab-3a^2}{b-a} \\ b & \frac{b^2-ab-3a^2}{b-a} & \frac{2ab+b^2}{b-a} \end{pmatrix} \frac{s^2(v_u^0)^2}{\Lambda}$$

with

$$a \equiv \frac{2x_a u}{(y^D)^2} \quad , \qquad b \equiv \frac{2x_b v_S}{(y^D)^2}$$

m_ν is of the form

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix} \longrightarrow U = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & +1/\sqrt{2} \end{pmatrix}$$

charged lepton diagonalization for dots=0 contributes λ^4, λ^8, λ^4 terms to 12, 13, 23

with

$$m_1 = \frac{1}{(a+b)} \quad , \qquad m_2 = \frac{1}{a} \quad , \qquad m_3 = \frac{1}{(b-a)}$$

or

$$\frac{2}{m_2} = \frac{1}{m_1} - \frac{1}{m_3}$$

$$\boxed{\begin{aligned} r &\equiv \Delta m^2_{sol}/\Delta m^2_{atm} \\ \Delta m^2_{sol} &\equiv |m_2|^2 - |m_1|^2 \\ \Delta m^2_{atm} &\equiv \left||m_3|^2 - |m_1|^2\right| \end{aligned}}$$

$$r = \frac{|1 - z|^2 |z + \bar{z} + |z|^2|}{2|z + \bar{z}|}$$

$$z \equiv \frac{b}{a}$$

For z~+1 a viable normal hierarchy spectrum
while z~-2 would give an inverse hierarchy solution

z~+1, normal hierarchy
is the most natural:

$$\sqrt{\Delta m^2_{atm}} \approx \frac{s^2 (v_u^0)^2}{|a| \Lambda \sqrt{r}}$$

$$\sum_i |m_i| \approx (0.06 - 0.07) \text{ eV}$$

$$|m_{ee}| \approx 0.007 \text{ eV}$$

$$\begin{aligned} |m_1|^2 &= \frac{1}{3} \Delta m^2_{atm}\, r + ... \\ |m_2|^2 &= \frac{4}{3} \Delta m^2_{atm}\, r + ... \\ |m_3|^2 &= \left(1 + \frac{r}{3}\right) \Delta m^2_{atm} + ... \\ |m_{ee}|^2 &= \frac{16}{27} \Delta m^2_{atm}\, r + ... \quad , \end{aligned}$$

Finally:

By taking $s \sim t \sim t'' \sim \lambda \sim 0.22$ $v_T \sim \lambda^2 \sim m_b/m_t$ $v_S, u \sim \lambda^2$

a good description of all quark and lepton masses is obtained.
As for all U(1) models only $o(\lambda^p)$ predictions can be given (modulo o(1) coeff.s)

TB mixing for neutrinos is reproduced in first approximation

Quark hierarchies force corrections to TB mixing to be $o(\lambda^2)$
(in particular we predict $\theta_{13} \sim o(\lambda^2)$, accessible at T2K).

A moderate fine tuning is needed to fix λ_C and r (nominally of $o(\lambda^2)$ and 1 respectively)

Normal hierarchy is favoured, degenerate ν's are excluded

Thus:

The A4 approach to TB neutrino mixing is shown to be compatible with quark masses and mixings in a GUT model

The unification with quarks fixes the size of the expected deviations from TB mixing: all mixing angles should deviate by o(λ^2) from the TB values

A normal hierarchy spectrum is indicated with

$$\frac{2}{m_2} = \frac{1}{m_1} - \frac{1}{m_3}$$

$$\sum_i |m_i| \approx (0.06 - 0.07)\ \mathrm{eV}$$

$$|m_{ee}| \approx 0.007\ \mathrm{eV}$$

If θ_{13} is found near its present bound (e.g $o(\lambda_C)$) this would hint that TB is accidental and bimaximal mixing (BM) could be a better first approximation

There is an intriguing empirical relation:

$$\theta_{12} + \theta_C = (47.0 \pm 1.7)^o \sim \pi/4$$

Raidal

Suggests bimaximal mixing in 1st approximation, corrected by charged lepton diagonalization.

Recall that

$$\lambda_C \approx 0.22 \text{ or } \sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$$

While $\theta_{12} + o(\theta_C) \sim \pi/4$ is easy to realize, exactly $\theta_{12} + \theta_C \sim \pi/4$ is more difficult: no compelling model

Minakata, Smirnov

Here we construct a model where BM mixing holds in 1st approximation and is then corrected by terms $o(\lambda_C)$

Revisiting Bimaximal Neutrino Mixing in a Model with S_4 Discrete Symmetry

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soon to appear on the web

BM mixing

$\theta_{12} = \theta_{23} = \pi/4,\ \theta_{13} = 0$

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

By adding $\sin^2\theta_{12} \sim 1/2$ to $\theta_{13} \sim 0$, $\theta_{23} \sim \pi/4$:

Bimaximal Mixing

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix} \longrightarrow m_{\nu BM} = \begin{pmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{pmatrix}$$

$$\sin^2 2\theta_{12} = \frac{8y^2}{(x-w-z)^2 + 8y^2}$$

$$m_1 = x + \sqrt{2}y$$

$$m_2 = x - \sqrt{2}y$$

$$m_3 = 2z - x$$

BM corresponds to $\tan^2\theta_{12} = 1$
while exp.: $\tan^2\theta_{12} = 0.45 \pm 0.04$
so a large correction is needed

The 3 remaining parameters are the mass eigenvalues

Bimaximal Mixing

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

In the basis of diagonal ch. leptons:

$m_\nu = U \mathrm{diag}(m_1, m_2, m_3) U^T$

$$m_{\nu BM} = \left[\frac{m_3}{2} M_3 + \frac{m_2}{4} M_2 + \frac{m_1}{4} M_1\right]$$

$$M_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \quad M_2 = \begin{pmatrix} 2 & -\sqrt{2} & -\sqrt{2} \\ -\sqrt{2} & 1 & 1 \\ -\sqrt{2} & 1 & 1 \end{pmatrix}, \quad M_1 = \begin{pmatrix} 2 & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & 1 & 1 \\ \sqrt{2} & 1 & 1 \end{pmatrix}$$

Eigenvectors: $(\sqrt{2}, 1, 1)/2, \ (-\sqrt{2}, 1, 1)/2 \ , \ (0, 1, -1)/\sqrt{2}$

S4: Group of permutations of 4 objects (24 transformations)

Irreducible representations: 1, 1', 2, 3, 3'

$$T^4=S^2=(ST)^3=(TS)^3=1$$

1 $\quad T = 1 \qquad S = 1$

2 $\quad T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad S = \frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$

3 $\quad T = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & i \end{pmatrix} \qquad S = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$

1 <-> 1' and 3<-> 3' by changing S, T <-> -S, -T

BM mixing corresponds to m in the basis where charged leptons are diagonal

$$m_{\nu BM} = \begin{pmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{pmatrix}$$

m is the most general matrix invariant under SmS = m and $A_{23}mA_{23}$= m with:

$$S = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \qquad A_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

2-3 symmetry

Invariance under S can be made automatic in S4 while invariance under A23 happens if the flavon content is suitable

	l	e^c	μ^c	τ^c	ν^c	$h_{u,d}$	θ	φ_l	χ_l	ψ_l^0	χ_l^0	ξ_ν	φ_ν	ξ_ν^0	φ_ν^0
S_4	3	1	$1'$	1	3	1	1	3	$3'$	2	$3'$	1	3	1	3
Z_4	1	-1	-i	-i	1	1	1	i	i	-1	-1	1	1	1	1
$U(1)_{FN}$	0	2	1	0	0	0	-1	0	0	0	0	0	0	0	0
$U(1)_R$	1	1	1	1	1	1	0	0	0	2	2	0	0	2	2

$$w_l = \frac{y_e^{(1)}}{\Lambda^2}\frac{\theta^2}{\Lambda^2}e^c(l\varphi_l\varphi_l) + \frac{y_e^{(2)}}{\Lambda^2}\frac{\theta^2}{\Lambda^2}e^c(l\chi_l\chi_l) + \frac{y_e^{(3)}}{\Lambda^2}\frac{\theta^2}{\Lambda^2}e^c(l\varphi_l\chi_l) + + \frac{y_\mu}{\Lambda}\frac{\theta}{\Lambda}\mu^c(l\chi_l)' + \frac{y_\tau}{\Lambda}\tau^c(l\varphi_l) + \dots$$

$$w_\nu = y(\nu^c l) + M\Lambda(\nu^c\nu^c) + a(\nu^c\nu^c\xi_\nu) + b(\nu^c\nu^c\varphi_\nu) + \dots$$ see-saw

$$\frac{\langle\xi_\nu\rangle}{\Lambda} = E \qquad \frac{\langle\varphi_\nu\rangle}{\Lambda} = \begin{pmatrix}0\\1\\-1\end{pmatrix}C \qquad \frac{\langle\varphi_l\rangle}{\Lambda} = \begin{pmatrix}0\\1\\0\end{pmatrix}A \qquad \frac{\langle\chi_l\rangle}{\Lambda} = \begin{pmatrix}0\\0\\1\end{pmatrix}B$$

In this model BM mixing is exact at LO

For the special flavon content chosen only θ_{12} and θ_{13} are corrected from the charged lepton sector by terms of $o(\lambda_C)$ (large correction!) while θ_{23} gets smaller corrections (great!) [for a richer flavon content also $\delta\theta_{23} \sim o(\lambda_C)$]

The only fine-tuning needed is to account for r ~ 1/30
[In most A4 models r ~ 1 would be expected as l, ν^c ~ 3]

An experimental indication for this model would be that θ_{13} is found near its present bound at T2K