Effective action of a five-dimensional domain wall

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work in collaboration
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arXiv:0812.2227 [hep-th]

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Outline

- Motivation - why this calculation?

  Branons and the fate of the zero mode of the 5D kink in the presence of gravity

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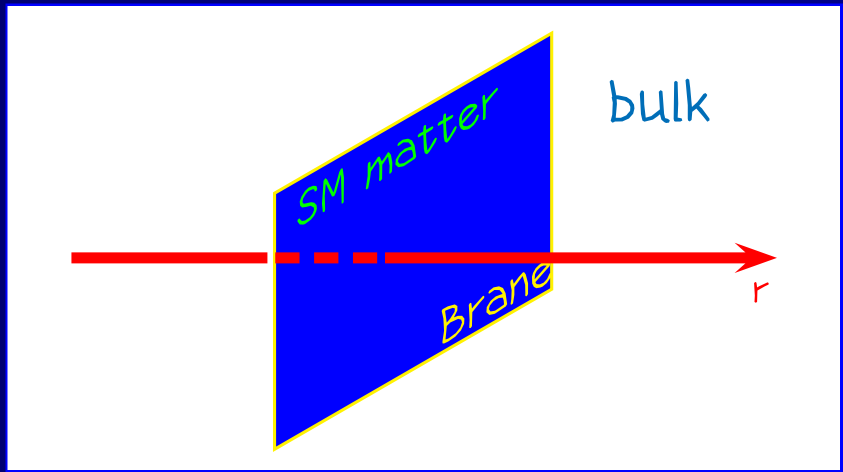
- Conclusions

FT brane-world models

Rubakov and Shaposhnikov, 1983
Akama, 1983

- Idea: Our universe is a ``brane'': a (3+1)-dimensional defect in a higher-dimensional field theory:



Thin branes – approximations of finite-width defects

SM particles: low-energy modes trapped on the defect ⇒ extra dimensions only visible in the very high energy experiments; 4D action – low energy effective action

Branons

Goldstone bosons of broken isometries of extra space

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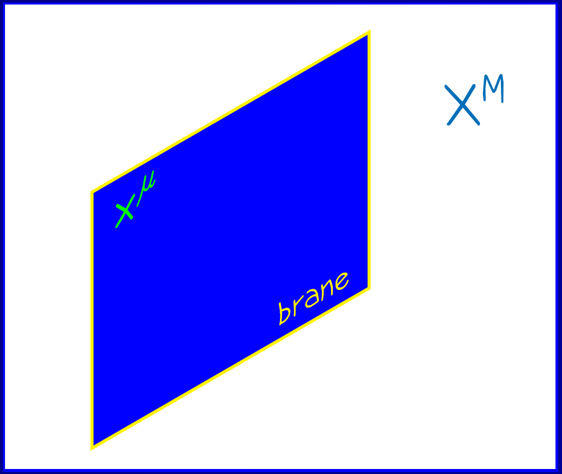

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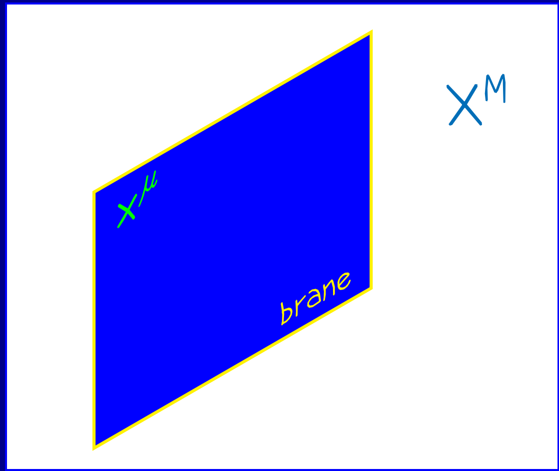
- brane (static gauge): $\begin{cases} X^\mu(x) = x^\mu \\ X^5(x) = Y(x) \end{cases}$
- induced metric: $g_{\mu\nu} = \eta_{\mu\nu} - \partial_\mu Y \partial_\nu Y$



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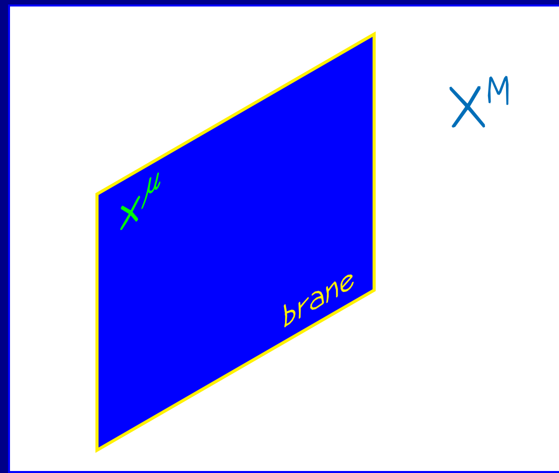

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- induced metric: $g_{\mu\nu} = \eta_{\mu\nu} - \partial_\mu Y \partial_\nu Y$
- Nambu-Goto action ($\mathcal{T}$ tension, branon $\tilde{Y}(x) = \sqrt{\mathcal{T}}\, Y(x)$)

$$S_{\mathrm{brane}} = - \int d^4x \sqrt{g}\, \mathcal{T}$$

$$= \int d^4x \left\{ -\mathcal{T} + \frac{1}{2} \partial_\mu \tilde{Y} \partial^\mu \tilde{Y} + \frac{1}{8\mathcal{T}} (\partial_\mu \tilde{Y} \partial^\mu \tilde{Y})^2 + \ldots \right\}$$

Branons

J. A. R. Cembranos, A. Dobado and A. L. Maroto,
``Brane-world dark matter'',
Phys. Rev. Lett. 90 (2003) 241301.

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Branon ~ zero mode of the 5D kink

Our study: gravitational backreaction of the brane important

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wide resonance $\Longleftrightarrow$ high probability to tunnel into the bulk

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$\Rightarrow$ Look into the effective action of the brane perturbations!

Domain wall – two scalars model

Action: $$S = \int d^4x dy \left[\frac{1}{2}\eta^{MN}\partial_M\Phi\partial_N\Phi + \frac{1}{2}\eta^{MN}\partial_M\Xi\partial_N\Xi - V(\Phi,\Xi) \right],$$

$$V(\Phi,\Xi) = \frac{\lambda}{4}\left(\Phi^2 - v^2\right)^2 + \frac{\tilde{\lambda}}{4}\Xi^4 + \frac{1}{2}M^2\Xi^2 + \frac{1}{2}\alpha(\Phi^2 - v^2)\Xi^2$$

Idea: Set up a brane as a domain wall. Compute the 4d low energy effective action.

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Idea: Set up a brane as a domain wall. Compute the 4d low energy effective action.

If $M^2 < \alpha v^2$ and $\lambda\tilde{\lambda}v^4 > (\alpha v^2 - M^2)^2$, the system has a degenerate GS ($\Phi_{GS} = \pm v, \Xi_{GS} = 0$)

$\Rightarrow$ domain wall interpolating between the two vacua;

kink configuration ($\Phi = v\tanh(ay), \Xi = 0$), $a^2 = \lambda v^2/2$, always solves EOM

Perturbations around ($\Phi = v\tanh(ar), \Xi = 0$)

Look at the perturbations around the background (using 4D Poincaré invariance):

$$\Phi(x,y) = \Phi_c(y) + \varphi(x^\mu, y) = \Phi_c(y) + \sum\!\!\!\!\!\!\int_n \; f_n(y)u_n(x)$$

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- $u_n(x)$, $v_n(x)$ – scalar fields from the 4D point of view
- $f_n(y)$, $h_n(y)$ – wave functions; determine the localization of the modes on the brane:

$$\begin{cases} -\partial_y^2 f + \left(4a^2 - \dfrac{6a^2}{\cosh^2(ar)}\right) f = m^2 f \\ -\partial_y^2 h + \left(M^2 - \dfrac{\alpha v^2}{\cosh^2(ar)}\right) h = \tilde{m}^2 h\ . \end{cases}$$

Spectrum of perturbations

Lowest lying states:

kink's zero mode: $\psi_0 = f_0(y)u_0(x) = N_0 \frac{va}{\cosh^2(ay)} u_0(x)$

massive mode of Ξ: $\psi_1 = h_1(y)v_1(x) = N_1 \frac{va}{\cosh^{\sigma}(ay)} v_1(x)$

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- mass of $v_1(x)$ is $\tilde{m}_1^2 = -\sigma^2 a^2 + M^2$

$\Rightarrow$ if we choose $M^2 = +\sigma^2 a^2 + \frac{\lambda v^2}{4}\varepsilon^2$, $|\varepsilon| \ll 1$

we get a light v_1: $\tilde{m}_1^2 = (\lambda v^2/4)\varepsilon^2$

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- rest of the spectrum:

for u_n's: heavy mode with mass $3a^2$, continuum from $4a^2$

for v_n's: continuum starts at $M^2 \approx \sigma a^2$, if other localized modes, then their masses $\mathcal{O}(a^2)$

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$a \gg \tilde{m}_1 \Rightarrow$ we can construct a sensible 4D low energy action

Collective coordinate

Presence of a zero mode ⇒ introduce a collective coordinate :

$$\Phi = \Phi_c(y - Y(x)) + \sum_{n\neq 0}\!\!\!\!\!\!\!\int f_n(y - Y(x))\, u_n(x)$$

$$\Xi = \xi(x, y) = h_1(y)v_1(x) + \sum_{n\neq 1}\!\!\!\!\!\!\!\int h_n(y - Y(x))\, v_n(x)$$

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Also, Y(x) is the transverse coordinate of the brane
implies direct connection with the geometric description
and easy comparaison with NG action.

Towards the effective action...

4D action:

$$S = \int d^4x \Big\{ -\mathcal{T} + \frac{1}{2}\partial^\mu \tilde{Y}\partial_\mu \tilde{Y} + \frac{1}{2}\partial^\mu v_1 \partial_\mu v_1 - \frac{1}{2}\tilde{m}_1^2 v_1^2 - \frac{\lambda_4^{(0)}}{4} v_1^4 + \lambda_{(2,2)}^{(0)} v_1^2\, \partial^\mu \tilde{Y}\partial_\mu \tilde{Y} + $$

$$+ \frac{1}{2}\sum\!\!\!\!\!\!\int_{n\neq 0} \left[\partial^\mu u_n \partial_\mu u_n - m_n^2 u_n^2\right] + \frac{1}{2}\sum\!\!\!\!\!\!\int_{n\neq 1} \left[\partial^\mu v_n \partial_\mu v_n - \tilde{m}_n^2 v_n^2\right] + \mathcal{L}_{\text{heavy}}^{\text{int}} \Big\}$$

where

$$\mathcal{T} = \int dy \Phi_c' = \frac{8a^3}{3\lambda}, \qquad \lambda_4^{(0)} = \tilde{\lambda}\int dy\, h_1^4 \sim \tilde{\lambda} a, \qquad \lambda_{(2,2)}^{(0)} = \frac{1}{\mathcal{T}}\int dy\, h_1'^2 \sim \frac{a^2}{\mathcal{T}} = \frac{\lambda}{a}$$

$$\mathcal{L}_{\text{heavy}}^{\text{int}} = \sum\!\!\!\!\!\!\int_{n\neq 0} J_n^{(1)} u_n + \frac{1}{2}\sum\!\!\!\!\!\!\int_{n,m\neq 0} J_{nm}^{(2)} u_n u_m + \sum\!\!\!\!\!\!\int_{n,m\neq 0} K_{nm}^\mu u_n \partial_\mu u_m +$$

$$+ \sum\!\!\!\!\!\!\int_{n\neq 1} \tilde{J}_n^{(1)} v_n + \frac{1}{2}\sum\!\!\!\!\!\!\int_{n,m\neq 1} \tilde{J}_{mn}^{(2)} v_n v_m + \sum\!\!\!\!\!\!\int_{n,m\neq 1} \tilde{K}_{nm}^\mu v_n \partial_\mu v_m + + \sum\!\!\!\!\!\!\int_{n\neq 0} \sum\!\!\!\!\!\!\int_{m\neq 1} \tilde{\tilde{J}}_{nm}^{(2)} u_n v_m + \ldots$$

Towards the effective action...

Effective action? Easy – neglect all the terms involving the heavy modes (only corrs to couplings, suppressed by the heavy scale):

$$S = \int d^4x \left\{ -\mathcal{T} + \frac{1}{2}\partial^\mu \tilde{Y} \partial_\mu \tilde{Y} + \frac{1}{2}\partial^\mu v_1 \partial_\mu v_1 - \frac{1}{2}\tilde{m}_1^2 v_1^2 - \frac{\lambda_4^{(0)}}{4} v_1^4 + \lambda_{(2,2)}^{(0)} v_1^2 \, \partial^\mu \tilde{Y} \partial_\mu \tilde{Y} \right\}$$

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Really?

Wrong! The heavy modes contribute significantly because of the trilinear interactions LLH even when $m_n \to \infty$ and cannot be simply thrown away!

S. Ranjbar-Daemi, A. Salvio, M. Shaposhnikov, 'On the decoupling of heavy modes in Kaluza-Klein theories' Nucl.Phys.B741:236-268,2006.

We have to be extra careful as we have an infinite tower of heavy modes and all of them will contribute...

Integrating out the heavy modes

We have to integrate out the heavy modes:

$$S_H = \int d^4x \left\{ \frac{1}{2}\partial^\mu H \partial_\mu H - \frac{1}{2} m_H^2 H^2 + JH \right\}$$

Effective action:

$$S_{eff} = -\frac{1}{2}\int d^4x d^4y\ J(x)\Delta_H(x-y)J(y) = \frac{1}{2M_H^2}\int d^4x\ J^2(x) + \ldots$$

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In our case, more involved...

Importance of the heavy modes

Integrating out the heavy modes:

$$
\begin{aligned}
S_{\text{eff}} = \int d^4x \Bigg\{ & -\mathcal{T} + \frac{1}{2}\partial^\mu \tilde{Y} \partial_\mu \tilde{Y} + \frac{1}{2\mathcal{T}^2} \sum\!\!\!\!\!\!\int_{n\neq 0} \frac{1}{m_n^2} \left(\int_{-\infty}^{\infty} dy\, \Phi_c' f_n' \right)^2 \left(\partial^\mu \tilde{Y} \partial_\mu \tilde{Y} \right)^2 \\
& + \frac{1}{2}\partial^\mu v_1 \partial_\mu v_1 - \frac{1}{2}\tilde{m}^2 v_1^2 \\
& - \left[\frac{\tilde{\lambda}}{4} \left(\int_{-\infty}^{\infty} dy\, h_1^4 \right) - \frac{1}{2\mathcal{T}^2} \sum\!\!\!\!\!\!\int_{n\neq 0} \frac{1}{m_n^2} \left(\int_{-\infty}^{\infty} dy\, \Phi_c' f_n' \right)^2 \right] v_1^4 \\
& + \frac{2}{\mathcal{T}} \sum\!\!\!\!\!\!\int_{n\neq 1} \frac{1}{\tilde{m}_n^2} \left(\int_{-\infty}^{\infty} dy\, h_1 h_n' \right)^2 \partial^\mu \tilde{Y} \partial^\nu \tilde{Y} \partial_\mu v_1 \partial_\nu v_1 \\
& + \left[\frac{1}{2\mathcal{T}} \int_{-\infty}^{\infty} dy\, h_1'^2 - \frac{\alpha}{\mathcal{T}} \sum\!\!\!\!\!\!\int_{n\neq 0} \frac{1}{m_n^2} \left(\int_{-\infty}^{\infty} dy\, \Phi_c' f_n' \right) \left(\int_{-\infty}^{\infty} dy\, \Phi_c' h_1^2 f_n \right) \right] v_1^2 \partial^\mu \tilde{Y} \partial_\mu \tilde{Y} \Bigg\}
\end{aligned}
$$

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& + \frac{1}{2}\partial^\mu v_1 \partial_\mu v_1 - \frac{1}{2}\tilde{m}^2 v_1^2 \\
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& + \underbrace{\left[\frac{1}{2\mathcal{T}}\int_{-\infty}^{\infty} dy\, h_1'^2 - \frac{\alpha}{\mathcal{T}} \sum_{n\neq 0}\!\!\!\!\!\!\!\int \; \frac{1}{m_n^2}\left(\int_{-\infty}^{\infty} dy\, \Phi_c' f_n'\right)\left(\int_{-\infty}^{\infty} dy\, \Phi_c' h_1^2 f_n\right)\right]}_{=\,0!} v_1^2 \partial^\mu\tilde{Y}\partial_\mu\tilde{Y} \Bigg\}
\end{aligned}
$$

Connection with NG action

We expect our effective action:

$$S_{eff} = \int d^4x \Big\{ -\mathcal{T} + \frac{1}{2}\partial^\mu \tilde{Y}\partial_\mu \tilde{Y} + \frac{1}{8\mathcal{T}}\left(\partial^\mu \tilde{Y}\partial_\mu \tilde{Y}\right)^2 + \frac{1}{2}\partial^\mu v_1 \partial_\mu v_1 - \frac{1}{2}\tilde{m}^2 v_1^2$$
$$- \left(\frac{\tilde{\lambda}}{4} - \frac{\lambda\sigma^2}{16}\right)\left(\int_{-\infty}^{\infty} dy\, h_1^4\right) v_1^4 + \frac{1}{2\mathcal{T}}\partial^\mu \tilde{Y}\partial^\nu \tilde{Y}\partial_\mu v_1 \partial_\nu v_1 \Big\}$$

to coincide with the Nambu-Goto action:

$$S_{NG} = \int d^4x \sqrt{-g}\left\{ -\mathcal{T} + \frac{1}{2}\partial^\mu v_1 \partial_\mu v_1 - \frac{1}{2}\tilde{m}^2 v_1^2 - \frac{\lambda_4}{4} v_1^4 \right\}$$
$$= \int d^4x \Big\{ -\mathcal{T} + \frac{1}{2}\partial^\mu \tilde{Y}\partial_\mu \tilde{Y} + \frac{1}{8\mathcal{T}}\left(\partial^\mu \tilde{Y}\partial_\mu \tilde{Y}\right)^2 + \frac{1}{2}\partial^\mu v_1 \partial_\mu v_1 - \frac{1}{2}\tilde{m}^2 v_1^2 - \frac{\lambda_4}{4} v_1^4$$
$$+ \frac{1}{2\mathcal{T}}\partial^\mu \tilde{Y}\partial^\nu \tilde{Y}\partial_\mu v_1 \partial_\nu v_1 - \frac{1}{4\mathcal{T}}\partial^\mu \tilde{Y}\partial_\mu \tilde{Y}\partial^\nu v_1 \partial_\nu v_1 + \frac{\tilde{m}_1^2}{4\mathcal{T}}\partial^\mu \tilde{Y}\partial_\mu \tilde{Y} v_1^2 + \ldots \Big\}$$

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$$- \left(\frac{\tilde{\lambda}}{4} - \frac{\lambda \sigma^2}{16}\right) \left(\int_{-\infty}^{\infty} dy\, h_1^4\right) v_1^4 + \frac{1}{2\mathcal{T}}\partial^\mu \tilde{Y} \partial^\nu \tilde{Y} \partial_\mu v_1 \partial_\nu v_1 \Big\}$$

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The two actions are almost identical – but not exactly...

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Are branon interactions modified by curvature effects?

Geometric description

Yes! As for NG, the effects of branons can be rewritten in purely geometric terms:

$$S_{eff} = \int d^4x\sqrt{-g}\left\{-\mathcal{T} - \frac{(\pi^2-6)\mathcal{T}}{24a^2}R + \frac{1}{2}\partial^\mu v_1\partial_\mu v_1 - \frac{1}{2}\tilde{m}^2 v_1^2 - \frac{\lambda_4}{4}v_1^4 \right.$$
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where $R = -\frac{1}{\mathcal{T}}\partial^\alpha\partial^\nu\tilde{Y}\partial_\alpha\partial_\nu\tilde{Y}$ is the Ricci scalar

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- Comment 2: Curvature, but no graviton! Only one d.o.f

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- These corrections are likely to be important to describe branon interactions correctly!
- Including bulk metric perturbations? To be looked into soon. Expect scalar-tensor gravity.