DE SITTER VACUA IN 4D SIMPLE EXTENDED SUPERGRAVITY

or

GRAVITINO DRESSED DE SITTER VACUA

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Conventions: We use late greek letters for curve space indices, i.e., $\mu, \nu .. = 0, 1, 2, 3$, early greek letters for flat space indices i.e., $\alpha, \beta ... = 0, 1, 2, 3$. In addition we use $i, j, m, ... = 1, 2, 3$ for curve indices and $a, b, ... = 1, 2, 3$ for flat indices except otherwise explicitly declared. Curvature tensors and γ algebra are defined as

$$R_{\mu\nu}{}^{\alpha\beta} = \partial_\mu \omega_\nu{}^{\alpha\beta} + \omega_\mu{}^{\alpha\gamma}\omega_{\nu\gamma}{}^{\beta} - (\mu \leftrightarrow \nu)\,, \tag{1}$$

$$R_\mu{}^\alpha = e^\nu_\beta R_{\mu\nu}{}^{\alpha\beta}, \ \ , R = e^\mu_\alpha R_\mu{}^\alpha \tag{2}$$

$$\{\gamma_\alpha, \gamma_\beta\} = 2\eta_{\mu\nu}\,, \quad \eta_{\mu\nu} = (-,+,+,+)\gamma_5^2 = +1\,, \quad \epsilon^{0123} = +1, \tag{3}$$

Gravitini are Majorana spinors and the latter satisfy

$$\bar{\lambda}\gamma_{\alpha_1}...\gamma_{\alpha_n}\chi = (-)^n \bar{\chi}\gamma_{\alpha_n}...\gamma_{\alpha_1}\lambda \tag{4}$$

I. INTRODUCTION

Although Minkowski and anti-de Sitter backgrounds are quite common in supergravity, the existence of de Sitter vacua has been questioned. For example there exists a $\mathcal{N}=1$ supergravity with a cosmological term Townsend (1977) with lagrangian

$$\mathcal{L} = \frac{1}{2}eR - \frac{1}{2}\bar{\psi}_\mu \Gamma^{\mu\nu\rho} D_\nu \psi_\rho \ + \ 6eg^2 \ + \ 2eg\bar{\psi}_\mu \Gamma^{\mu\nu}\psi_\nu \tag{5}$$

It is invariant under

$$\delta e^a_\mu = \bar{\epsilon}\gamma^a \psi_\mu \,, \tag{6}$$

$$\delta\psi_\mu = 2D_\mu \epsilon \ + \ g\gamma_\mu \epsilon \tag{7}$$

From the sign of the cosmological term we see that this theory has an anti-de Sitter vacuum. An opossite cosmological term (for a de Sitter background to exists) will lead to inconsistencies (imaginary action, etc.)

Similarly, one may add a cosmological term in the $\mathcal{N} = 2$ theory. In this case, insisting on the existence of a de Sitter vacuum, the bosonic part of the action turns out to be (in the mostly-plus convention for the metric)

$$\mathcal{L} = \frac{1}{2}R + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \dots \tag{8}$$

so that de Sitter local supersymmetry has vector-ghosts. On the group-theoretic side, one may prove that there are no non-trivial representations of the de Sitter O(3,2) algebra on a positive Hilbert space.

This conclusion was further supported by the fact that:

higher-dimensional supergravities do not admit stable ground states with a positive cosmological constant Gibbons, deWit et al., Maldacena-Nunez

Why looking for de Sitter vacua in first place?

For various reasons:

- The simplest model accounting for the observed accelerated expansion involves backgrounds with a tiny positive cosmological constant.
- de Sitter, anti-de Sitter and Minkowski spaces are spaces of maximal symmetry.

- QFT in de Sitter space has particular features not shared by QFT on Minkowski spacetime.
- theoretical reasons
- ...

For these reasons, we see a lot of activity in searching for de Sitter (dS) vacua in the effective four-dimensional supergravity description of string theory compactifications in the $\mathcal{N} = 1$ context. Effects like gaugino condensation or background fluxes may stabilize the moduli fields. The generic vacuum is, however, anti-de Sitter (AdS) or Minkowski space, and it appears as an exception the non-supersymmetric dS vacua. There are various proposals for the uplifting of an $\mathcal{N} = 1$ 4D Minskowski or AdS vacuum to a dS one, like for example in the KKLT model where non-perturbative effects as well as $\bar{D}3$ supersymmetry breaking terms leads to a stable volume modulus and a fine-tuned cosmological constant. However all these considerations have assumed vanishing fermionic fields.

Here we would like to explore the possibility of de Sitter vacua in supergravity when fermionic fields are turned on.

II. $\mathcal{N}$=2 SUPERGRAVITY

The simplest extended supergravity is the $\mathcal{N} = 2$ extended model. It was constructed by Ferrara and van Nieuwenhuizen (1979) by coupling the (2,3/2) graviton multiplet to the (3/2,1) matter multiplet of the $\mathcal{N} = 1$ theory. Thus, it contains:

a graviton,	a U(1) gauge field	two gravitini
$g_{\mu\nu}$	A_μ	$\psi^I_\mu, \quad (I = 1, 2)$

(9)

It is described by the action

$$\mathcal{L} = \frac{e}{2\kappa^2} R(e,\omega) - \frac{i}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}^I_\mu \gamma_5 \gamma_\nu D_\rho(\omega)\psi^I_\sigma - \frac{e}{4}F^2_{\mu\nu} \tag{10}$$

$$+\frac{\kappa}{4\sqrt{2}}\bar{\psi}^I_\mu \left[e(F^{\mu\nu} + \hat{F}^{\mu\nu}) + \frac{i}{2}\gamma_5(\tilde{F}^{\mu\nu} + \hat{\tilde{F}}^{\mu\nu}) \right] \psi^I_\nu \tag{11}$$

where, $\tilde{F}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$,

$$D_\mu(\omega) = \partial_\mu + \frac{1}{8}\omega_{\mu\alpha\beta}[\gamma^\alpha, \gamma^\beta] \,, \tag{12}$$

and $\hat{F}_{\mu\nu}$ is the supercovariant photon curl

$$\hat{F}_{\mu\nu} = F_{\mu\nu} - \frac{\kappa}{\sqrt{2}}\epsilon^{IJ}\bar{\psi}^I_{[\mu}\psi^J_{\nu]} \tag{13}$$

In the 1.5 formalism, the spin connection ω is not independent variable. It contains ψ-torsion and it is given by

$$\omega_{\mu\alpha\beta} = \overset{\circ}{\omega}_{\mu\alpha\beta} + K_{\mu\alpha\beta} \tag{14}$$

where

$$\overset{\circ}{\omega}_{\mu\alpha\beta} = \frac{1}{2}e^\nu_\alpha\left(\partial_\mu e_{\beta\nu} - \partial_\nu e_{\beta\mu}\right) - \frac{1}{2}e^\nu_\beta\left(\partial_\mu e_{\alpha\nu} - \partial_\nu e_{\alpha\mu}\right) \tag{15}$$

is the torsion-free part of the connection and

$$K_{\mu\alpha\beta} = \frac{\kappa^2}{4}\left(\bar{\psi}^I_\mu\gamma_\alpha\psi^I_\beta - \bar{\psi}^I_\mu\gamma_\beta\psi^I_\alpha + \bar{\psi}^I_\alpha\gamma_\mu\psi^I_\beta\right) \tag{16}$$

is the contortion tensor. Torsion itself is easily found to be

$$T^\alpha{}_{\mu\nu} = D_\nu e^\alpha_\mu - D_\mu e^\alpha_\nu = \frac{\kappa^2}{2}\bar{\psi}^I_\mu\gamma^\alpha\psi^I_\nu\,. \tag{17}$$

This action is invariant then under the supersymmetry transformations (with $\delta\omega_{\mu\alpha\beta} = 0$)

$$\delta e^\alpha_\mu = \kappa\bar{\epsilon}^I\gamma^\alpha\psi^I_\mu\,, \quad \delta A_\mu = \frac{\kappa}{\sqrt{2}}\bar{\epsilon}^I\psi^J_\mu\epsilon^{IJ} \tag{18}$$

$$\delta\psi^I_\mu = \frac{2}{\kappa}D_\mu(\omega)\epsilon^I + \frac{\kappa}{2\sqrt{2}}\epsilon^{IJ}\Big(\hat{F}_{\mu\lambda}\gamma^\lambda + \frac{i}{2}e\hat{\tilde{F}}^{\mu\nu}\gamma^\lambda\gamma^5\Big)\,\epsilon^J$$

III. SUPERGRAVITY IS A TELEPARALLEL THEORY

Supergravity can be formulated in such a way that it has zero curvature but non-zero torsion. (The same as standard GR). Ferrara and van Nieuwenhuizen (1979) For example the telleparallel action of the $\mathcal{N}=1$ supergravity is

$$S = \int d^4x \left(-\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_5\gamma_\nu\partial_\rho\psi_\sigma - \frac{1}{8}\Omega^2_{\mu\nu a} - \Omega_{\mu\nu a}\Omega^{\mu\nu a} + \frac{1}{2}{\Omega^\lambda{}_{\nu\lambda}}^2 \right) \tag{19}$$

where

$$\Omega_{\mu\nu a} = -\partial_\mu e_{a\nu} + \partial_\nu e_{a\mu} + \frac{\kappa^2}{2}\bar{\psi}_\mu\gamma_a\psi_\nu \tag{20}$$

The spin connection

$$\omega_{ab\mu} = 0 \quad \text{so that} \quad {R_{\mu\nu}}^{\alpha\beta} = \partial_\mu {\omega_\nu}^{\alpha\beta} + {\omega_\mu}^{\alpha\gamma}{\omega_{\nu\gamma}}^{\beta} - (\mu \leftrightarrow \nu) = 0$$

(the Riemann tesnor vanish). However, torsion is not zero

$$T_{a\mu\nu} = \frac{\kappa^2}{2}\bar{\psi}_\mu \gamma_a \psi_\nu .$$

The fact hat supergravity as well as GR can be formulated in “flat” space with zero curvature and non-zero torsion is simply a rewritting of the same theory and it is considered as a curiosity rather a result of fundamental importance.

But why ψ-torsion may allow for dS solution?

It is a possible as may be seen by considering Raychaudhuri equation For example, in the case of a timelike geodesic congruence, Raychaudhuri equation reads

$$\dot{\theta} = -\frac{1}{3}\theta^2 - \sigma^2 + \omega^2 - R_{ab}u^a u^b$$

where

$$\sigma^2 = \sigma_{ab}\sigma^{ab} \geq 0 \quad (\sigma_{ab} \;\; \text{shear})$$

$$\omega^2 = \omega_{ab}\omega^{ab} \geq 0 \quad (\omega_{ab} \;\; \text{vorticity})$$

$$\theta = \nabla_a u^a \quad (\text{expansion})$$

In the presence of torsion Raychaudhuri equation turns out to be

$$\dot{\theta} = -\frac{1}{3}\theta^2 - \sigma^2 + \omega^2 - R_{ab}u^a u^b + S(\sigma, \omega, T)$$

IV. SOLUTIONS OF THE $\mathcal{N}$=2 SUPERGRAVITY

Question: Are there solutions where not only gravity but also gravitini are turned on?

Answer: Yes by ensuring that the graviphoton A_μ can consistently set to zero.

This is possible for example if the gravitini satisfy

$$\epsilon^{IJ}\bar{\psi}^I_{[\mu}\psi^J_{\nu]} = 0\,, \tag{21}$$

$$\epsilon^{\mu\nu\rho\sigma}\partial_\rho\left(\bar{\psi}^I_\mu\gamma^5\psi^J_\nu\right)\epsilon^{IJ} = 0\,. \tag{22}$$

In this case we can consistently set the graviphoton field

$$A_\mu = 0$$

as this choice satisfies its equation of motion. Then the rest of the field equations for gravity and gravitini fields are

$$G_\alpha{}^\nu = -\frac{i}{2}\kappa^2\epsilon^{\mu\nu\rho\sigma}\bar{\psi}^I_\mu\gamma_5\gamma_\alpha D_\rho\psi^I_\sigma \tag{23}$$

$$\epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu D_\rho\psi^I_\sigma = 0 \tag{24}$$

where

$$G_\alpha{}^\nu = R_\alpha{}^\nu(e,\omega) - \frac{1}{2}e^\alpha_\nu R(e,\omega) \tag{25}$$

is the Einstein tensor. It should be stressed that the Einstein tensor in eqs.(23,25) is calculated with the spin-connection ω in eq.(14) which contains torsion terms as well. Thus, it is not symmetric as the right hand side of eq.(23) is not symmetric either. In standard GR for example we have the Bianchi identities

$$\epsilon^{\nu\rho\sigma\kappa} R^{\mu}{}_{\nu\rho\sigma} = 0 \tag{26}$$

$$R^{\mu}{}_{\nu[\kappa\lambda;\sigma]} = 0 \tag{27}$$

However, in supergravity, due to ψ-torsion, the Riemann tensor $R^{\mu}{}_{\nu\rho\sigma}$ is not totally antisymmetric in its three lower indices but rather satisfies the Bianchi identity

$$\epsilon^{\nu\rho\sigma\kappa} R^{\mu}{}_{\nu\rho\sigma} = -\frac{\kappa^2}{2} \epsilon^{\nu\rho\sigma\kappa} \bar{\psi}^I_\nu \gamma^\mu D_\nu \psi^I_\sigma \,. \tag{28}$$

from where we find that

$$G_{[\lambda\kappa]} = R_{[\lambda\kappa]} = \frac{1}{2} \left(D_\kappa T^\mu_{\lambda\mu} + D_\lambda T^\mu_{\mu\kappa} + D_\mu T^\mu_{\kappa\lambda} \right) \tag{29}$$

GRAVITINO DRESSED BACKGROUNDS

Our aim here is to solve the supergravity equations

$$G_\alpha{}^\nu = -\frac{i}{2}\kappa^2 \epsilon^{\mu\nu\rho\sigma}\bar{\psi}^I_\mu \gamma_5 \gamma_\alpha D_\rho \psi^I_\sigma \tag{30}$$

$$\epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu D_\rho \psi^I_\sigma = 0 \tag{31}$$

We will assume a background metric of the form

$$ds^2 = -dt^2 + a(t)^2 \left(dx_1^2 + dx_2^2 + dx_3^2\right) \tag{32}$$

and time depended gravitino fields

$$\psi^I_\mu = \psi^I_\mu(t) \tag{33}$$

By making the ansatz

$$\psi_0^1 = \psi\,, \quad \psi_i^1 = \gamma_i\,\psi\,,$$

$$\psi_0^2 = \theta\,, \quad \psi_i^2 = -\gamma_i\,\theta\,, \tag{34}$$

where

$$\psi = (u_1, u_2, u_1, u_2)\,, \qquad \theta = (-u_1, u_2, -u_1, u_2) \tag{35}$$

one may easily check that eq.(22) is satisfied for anticommuting (Grassmann) u_1, u_2. We will satisfy eq.(22) in a moment consistently with a vanishing graviphoton field. Then, the only non-zero components of the torsion (17) are

$$T^a{}_{it} = 4\kappa^2 a(t) u_1 u_2\,\delta_i^a \tag{36}$$

On the other hand, in the orthonormal frame

$$e^0 = dt, \quad e^a = a(t)dx_a \tag{37}$$

we find that the spin connection ω is given by

$$\omega_{0a} = -\left(\frac{\dot{a}}{a} - H\right)\delta_{ab}e^b\,, \quad \omega_{a0} = \left(\frac{\dot{a}}{a} - H\right)\delta_{ab}e^b \tag{38}$$

where

$$H = 4\kappa^2 a(t)^2 u_2 u_1 \tag{39}$$

The gravitini equations are written as

$$\partial_t(\gamma_i\psi) = \frac{1}{2}\omega_{iab}\sigma^{ab}\psi\,, \qquad \partial_t(\gamma_i\theta) = \frac{1}{2}\omega_{iab}\sigma^{ab}\theta$$

which turns out to be

$$\partial_t(a\psi) + \frac{a}{2}\left(\frac{\dot{a}}{a} - H\right)\gamma_0\psi = 0\,,$$
$$\partial_t(a\theta) + \frac{a}{2}\left(\frac{\dot{a}}{a} - H\right)\gamma_0\theta = 0 \tag{40}$$

It follows from the equations above that

$$u_I = \frac{u_I^{(0)}}{a}\,, \quad I = 1, 2\,, \tag{41}$$
$$\frac{\dot{a}}{a} = H\,, \tag{42}$$

where $u_I^{(0)} = \text{const.}$ and

$$H = 4\kappa^2 a(t)^2 u_2 u_1 = 4\kappa^2 u_2^{(0)} u_1^{(0)} \tag{43}$$

is also constant. It is easy to verify then that (41) satifies eq.(22) justifying that we have consistently set the graviphoton field to zero. The Einstein equations turned then out to be

$$G_{\mu\nu}(\omega) = 0\,. \tag{44}$$

Since for the solution (42), the spin connection $\omega = 0$ and clearly eq.(44) is satisfied. The solution to eq.(42) is

$$a(t) = e^{Ht}\,, \tag{45}$$

so that the background metric turns out to be the de Sitter metric

$$ds^2 = -dt^2 + e^{2Ht}\left(dx_1^2 + dx_2^2 + dx_3^2\right)\,. \tag{46}$$

Isometries

It should be noted that the introduction of a non-zero gravitino fields as in eq.(33) are not consistent with the symmetries of the background. The background (46) has an $SO(1,4)$ symmetry generated by the 10 Killing vectors $\xi_{\mathbf{A}}$ with

$$\xi^0_{(0)} = 1, \;\; \xi^k_{(0)} = -Hx^k \,, \qquad \xi^0_{(\alpha)} = 0, \;\; \xi^k_{\alpha} = \delta^k_{\alpha} \,, \qquad \xi^0_{(\lambda)} = 0, \;\; \xi^k_{\lambda} = \epsilon^k{}_{\lambda l} x^l \,,$$
$$\xi^0_{(s)} = x^s, \;\; \xi^k_{(s)} = H\left[\delta^{ks}\left(r^2 - \frac{e^{-2Ht}}{H^2}\right) - x^k x^s\right] , \tag{47}$$

where $(k = 1,2,3)$, $(\alpha = 1,2,3)$, $(\lambda = 4,5,6)$ and $(s = 7,8,9)$. Then, for the Killing vectors (47) and the gravitini in eq.(33,34), we find that,

$$\delta_{\xi_A}\psi^I_\mu = \mathcal{L}_{\xi_A}\psi^I_\mu = D_\mu \epsilon^I_A \tag{48}$$

where ϵ is the field depended supersymmetry parameter

$$\epsilon_A^I = \xi_A^\mu \psi_\mu^I \tag{49}$$

and $\mathcal{L}$ denotes the Lie derivative. As a result, the gravitino is invariant under the $SO(1,4)$ de Sitter group up to a sypersymmetry transformation.

CONCLUSIONS

- There are unexploited issues in supergravity theory
- Fermionic dressed backgrounds might be interesting in cosmology
- Supersymmetry breaking
- Torsion: non-vanising gravitino leads in general to non-zero torsion. Can we measure the the latter?