

*On the fate of the Standard Model  
in view of the Higgs discovery*

Gino Isidori

[ *INFN, Frascati & CERN* ]

- ▶ Introduction
- ▶ The Higgs potential at high energies
- ▶ Stability and metastability bounds
- ▶ Vacuum stability at NNLO
- ▶ Speculations on Planck-scale dynamics
- ▶ Conclusions

## ► Introduction

All known phenomena in particle physics (*leaving aside a few cosmological observations*) can be described with good accuracy by a remarkably simple (*effective*) theory:

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}}(A_a, \psi_i) + \mathcal{L}_{\text{Symm. Break.}}(\phi, A_a, \psi_i)$$

- *Natural*
- Experimentally tested with high accuracy
- Stable with respect to quantum corrections (UV insensitive)
- Highly symmetric

$$\mathcal{L}_{\text{gauge}} = \sum_a -\frac{1}{4g_a^2} (F_{\mu\nu}^a)^2 + \sum_{\psi} \sum_i \bar{\psi}_i i\not{D} \psi_i$$

- $\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$  *local symmetry*
- *Global flavor symmetry*

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- *Natural*
  - Experimentally tested with high accuracy
  - Stable with respect to quantum corrections (UV insensitive)
  - Highly symmetric [*gauge + favor symmetries*]
  - *Ad hoc*
  - Necessary to describe data [*the electroweak symmetry forbid masses for all the elementary particles observed so far...*]
  - Not stable with respect to quantum corrections (UV sensitive)
  - Origin of the flavor structure of the model [*and of all the problems of the model...*]

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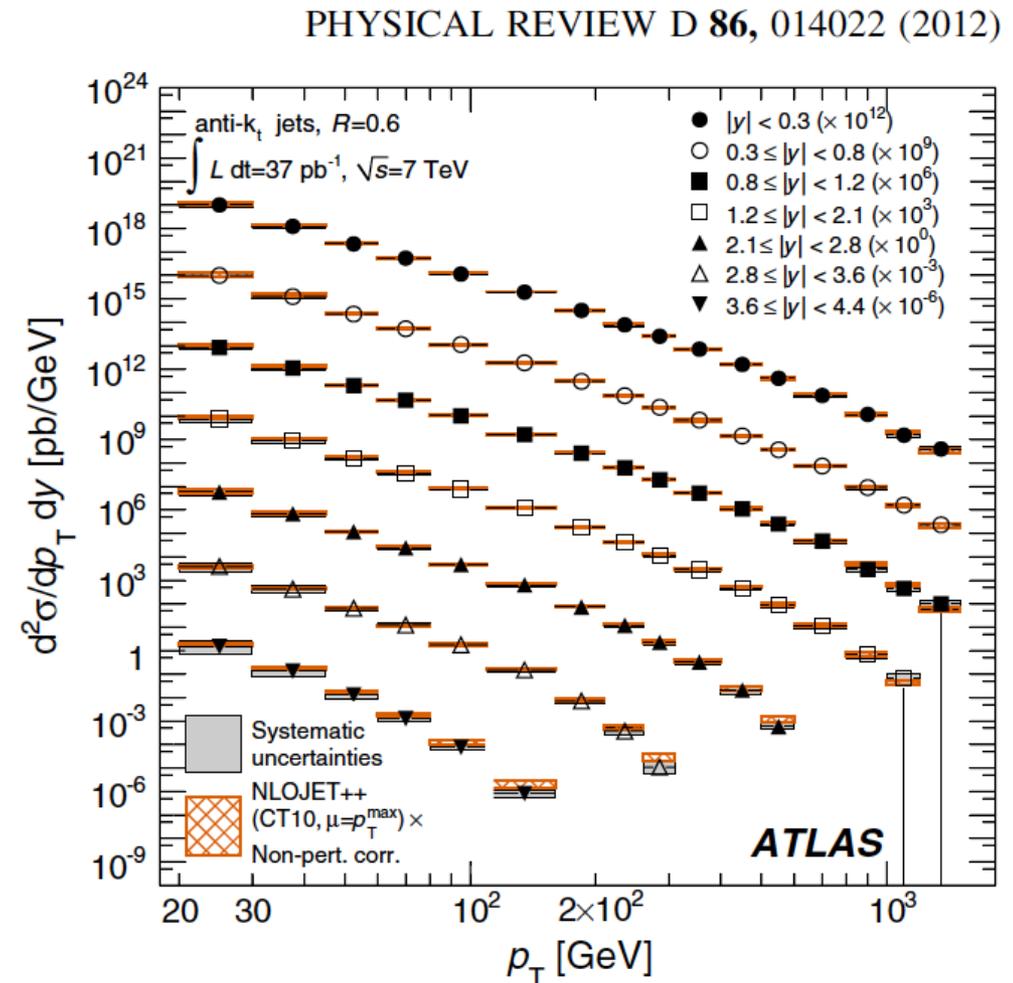
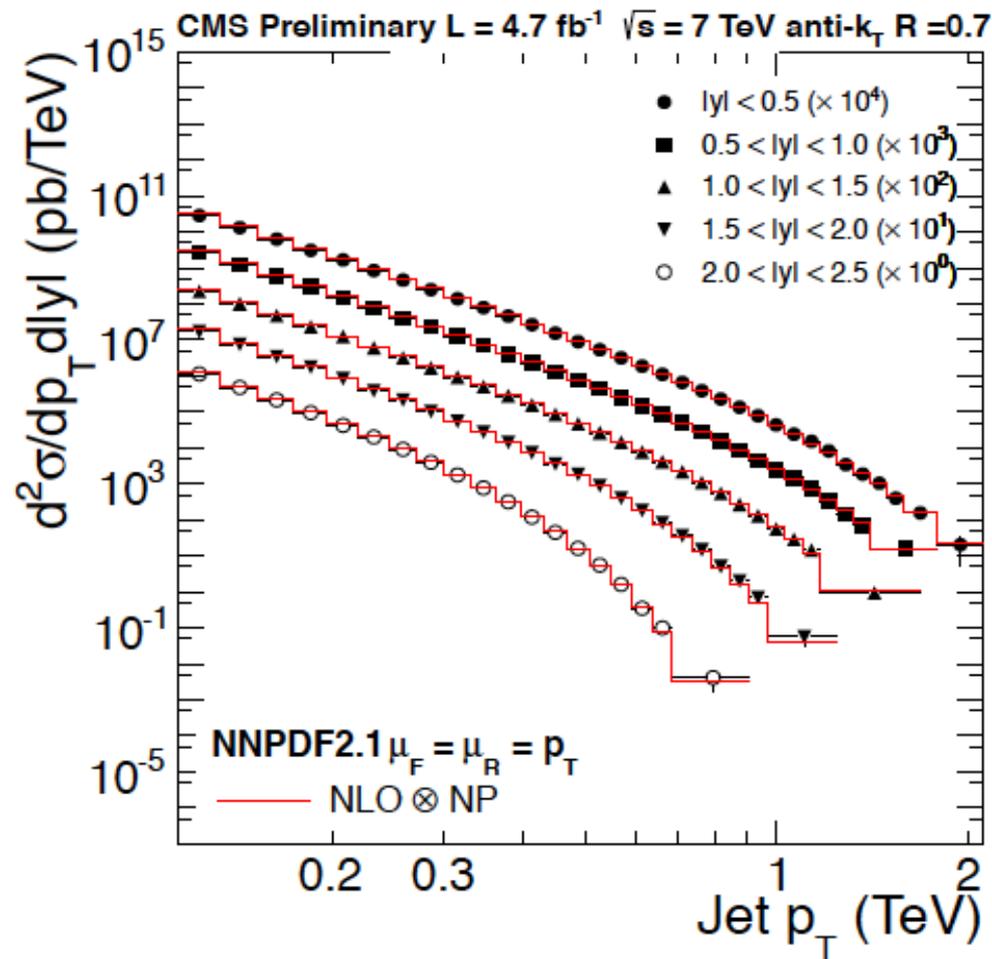
*Elegant & stable,  
but also a bit boring...*

- *Ad hoc*
- Necessary to describe data  
[*we couldn't live in a fully symmetric world...*]

*Ugly & unstable, but is what  
makes nature interesting...!*

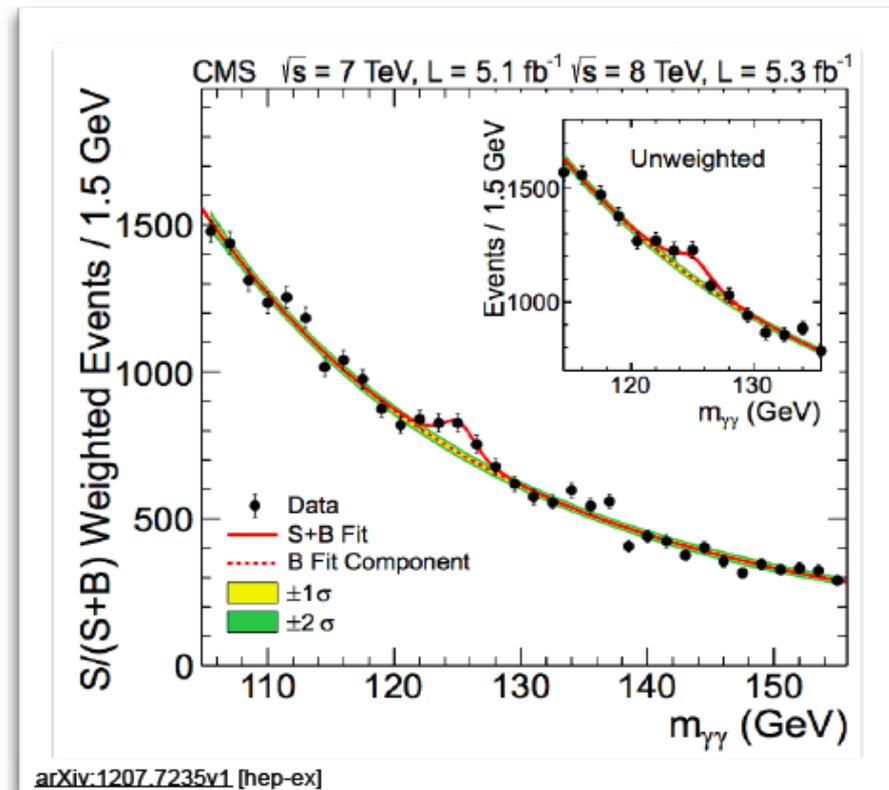
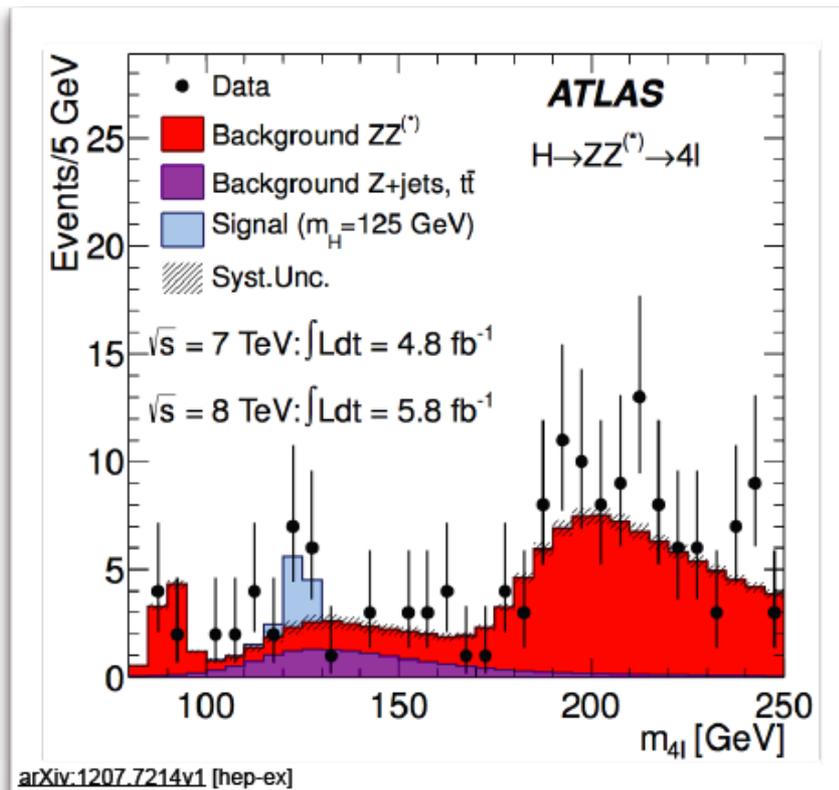
## Introduction

LHC experiments have confirmed once more that we understand very well gauge interactions...



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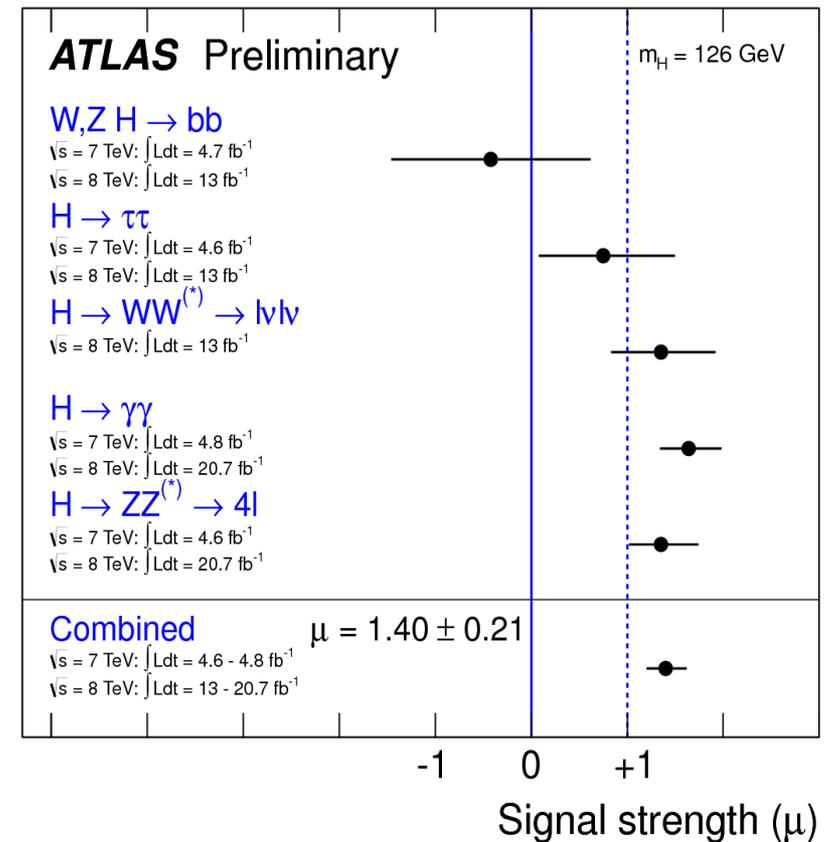
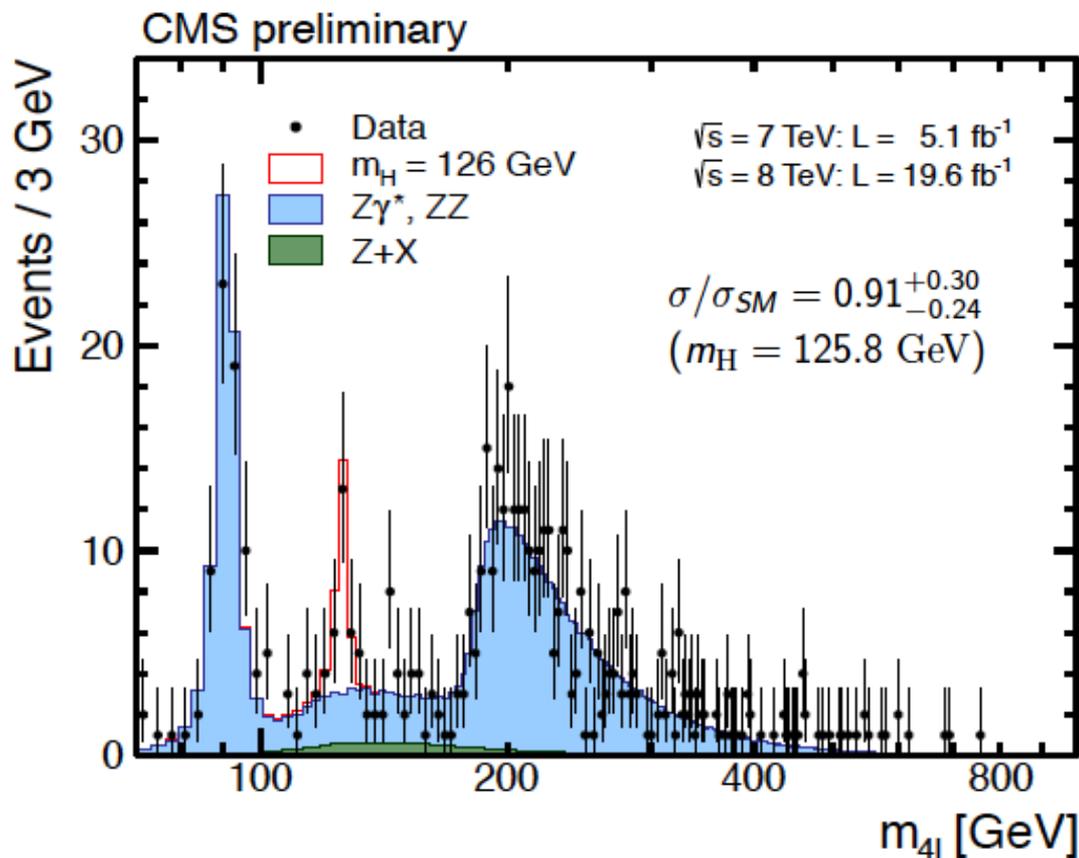
LHC experiments have confirmed once more that we understand very well gauge interactions, but the “*breaking-news*” announced July 4<sup>th</sup> 2012 is about the symmetry breaking sector of the theory:



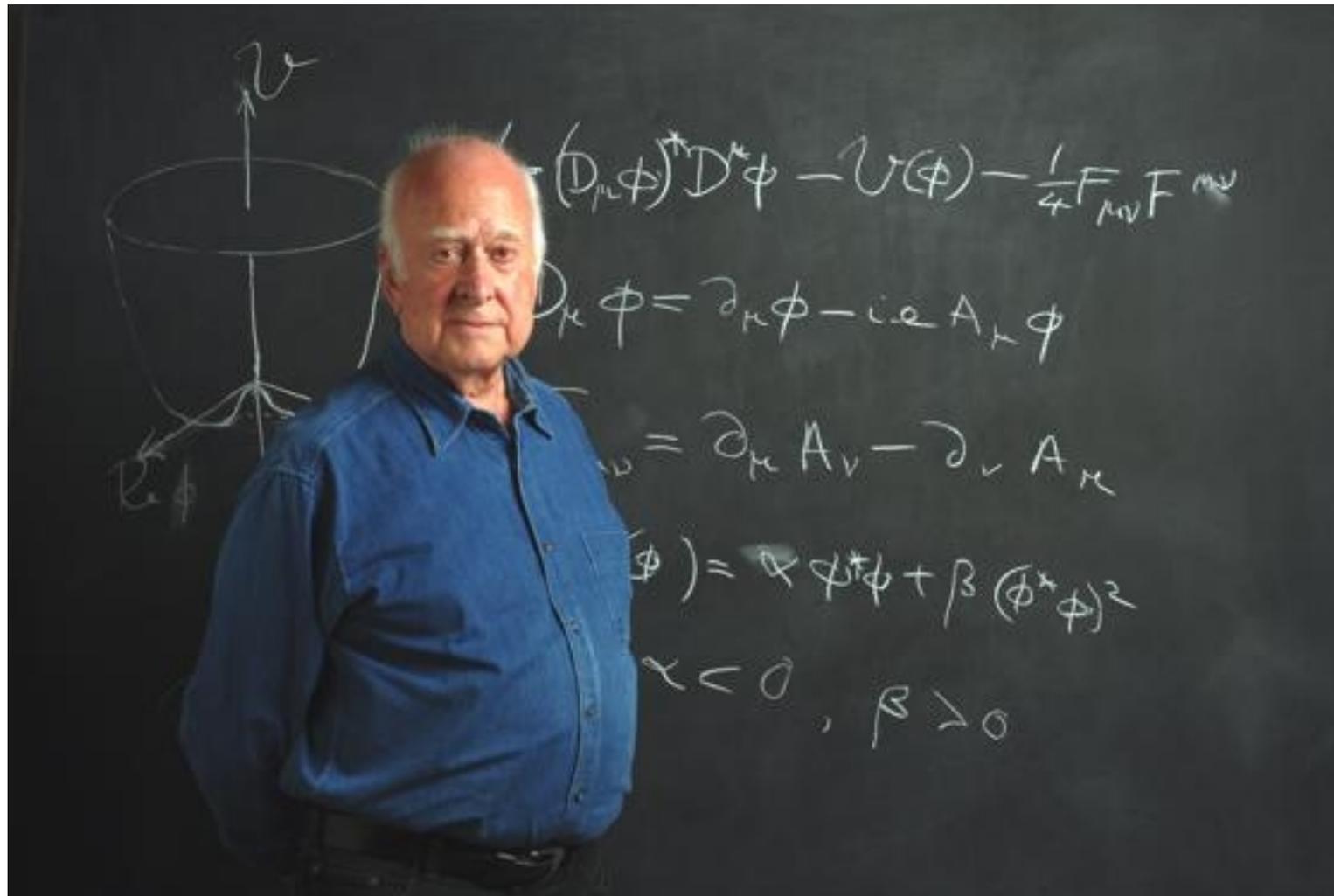
*Clear evidence of a new particle compatible with the properties of the Higgs boson*

## ► Introduction

*The more we a look at it, the more this particle looks like the “standard” Higgs boson:*

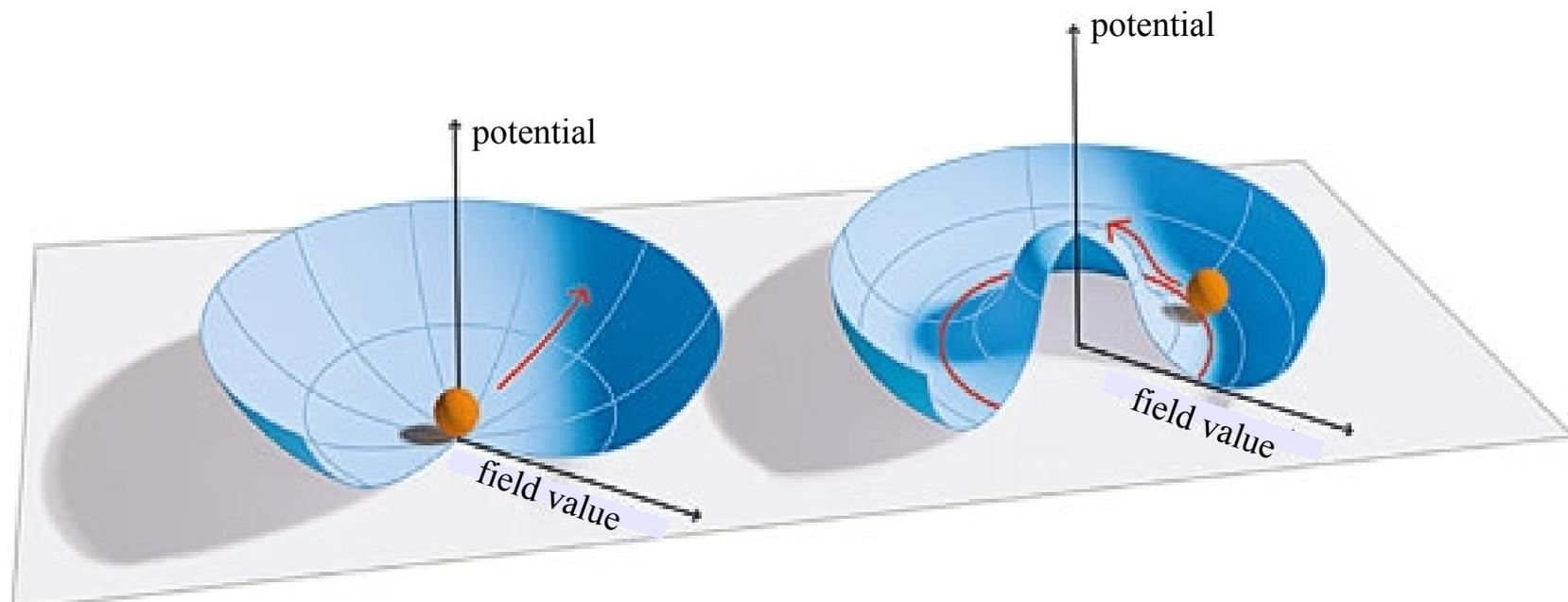


► The SM Higgs sector



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The Higgs mechanism, namely the introduction of an elementary  $SU(2)_L$  scalar doublet, with  $\phi^4$  potential, is the most economical & simple choice to achieve the spontaneous symmetry breaking of both gauge [  $SU(2)_L \times U(1)_Y \rightarrow U(1)_Q$  ] and flavor symmetries that we observe in nature.



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$$\mathcal{L}_{\text{higgs}}(\phi, A_a, \psi_i) = D\phi^\dagger D\phi - V(\phi)$$

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2 + Y^{ij} \psi_L^i \psi_R^j \phi$$

Till very recently only the ground state determined by this potential (*and the corresponding Goldstone boson structure*) was tested with good accuracy:

$$v = \langle \phi^\dagger \phi \rangle^{1/2} \sim 246 \text{ GeV} \quad [ m_W = \frac{1}{2} g v ]$$

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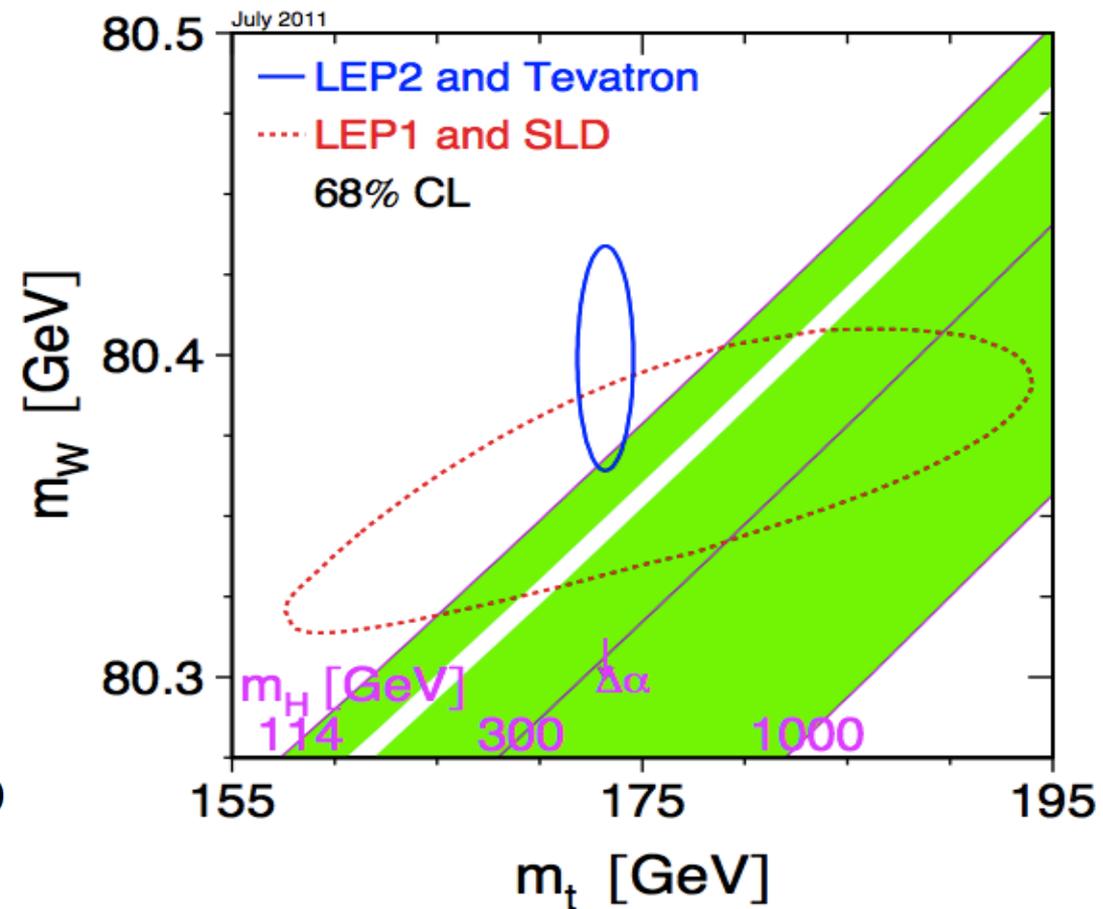
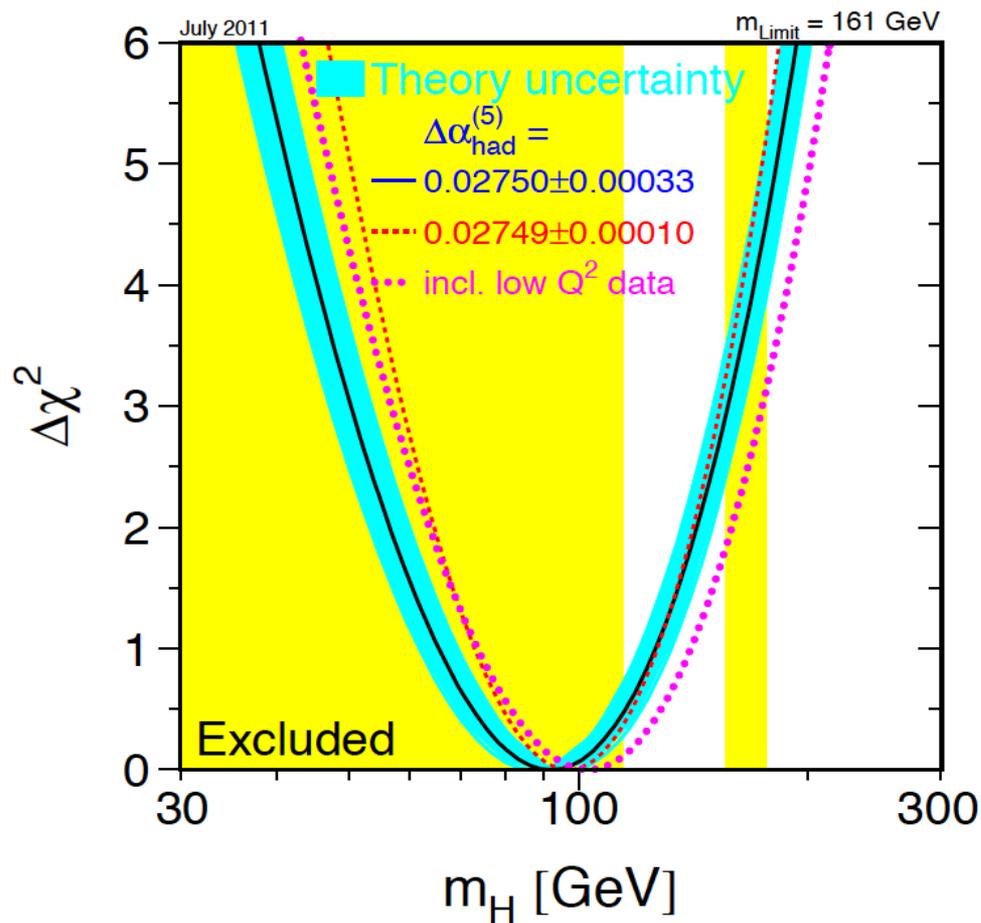
$$v = \langle \phi^\dagger \phi \rangle^{1/2} \sim 246 \text{ GeV} \quad [ m_W = 1/2 g v ]$$

The situation has substantially changed a few weeks ago, with the observation of the 4<sup>th</sup> degree of freedom of the Higgs field (or its *massive excitation*):

$$\lambda_{(\text{tree})} = 1/2 m_h^2 / v^2 \sim 0.13$$

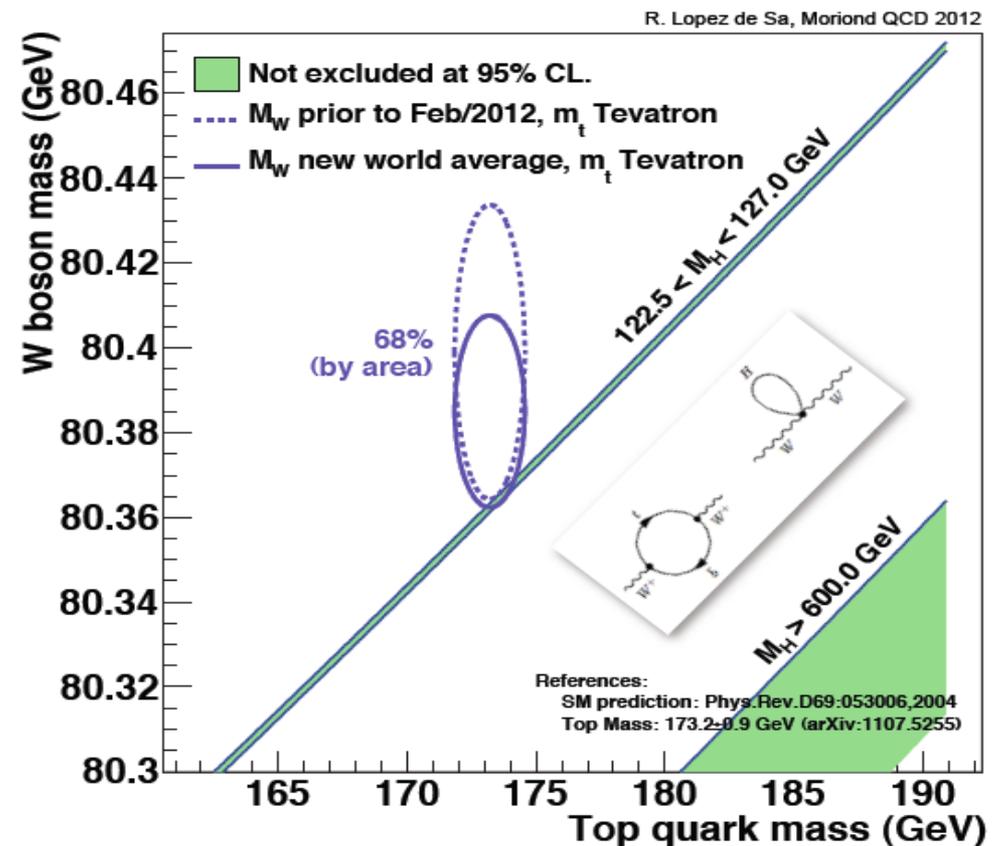
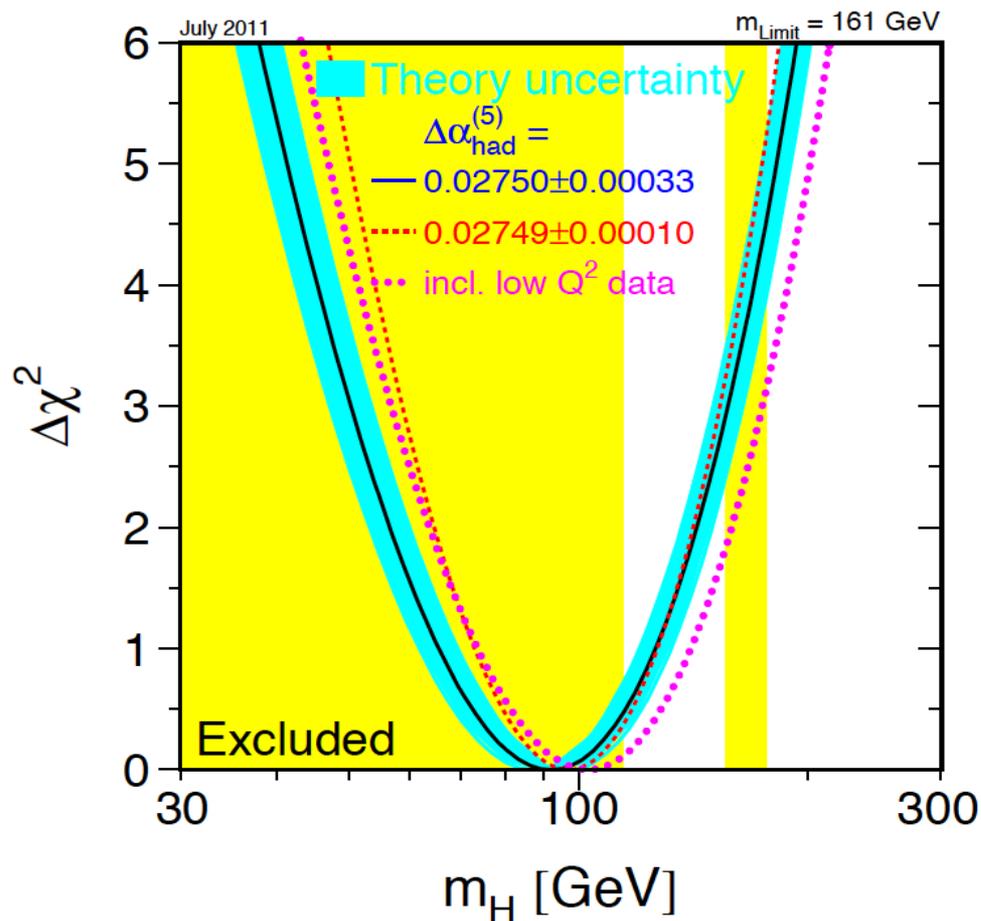
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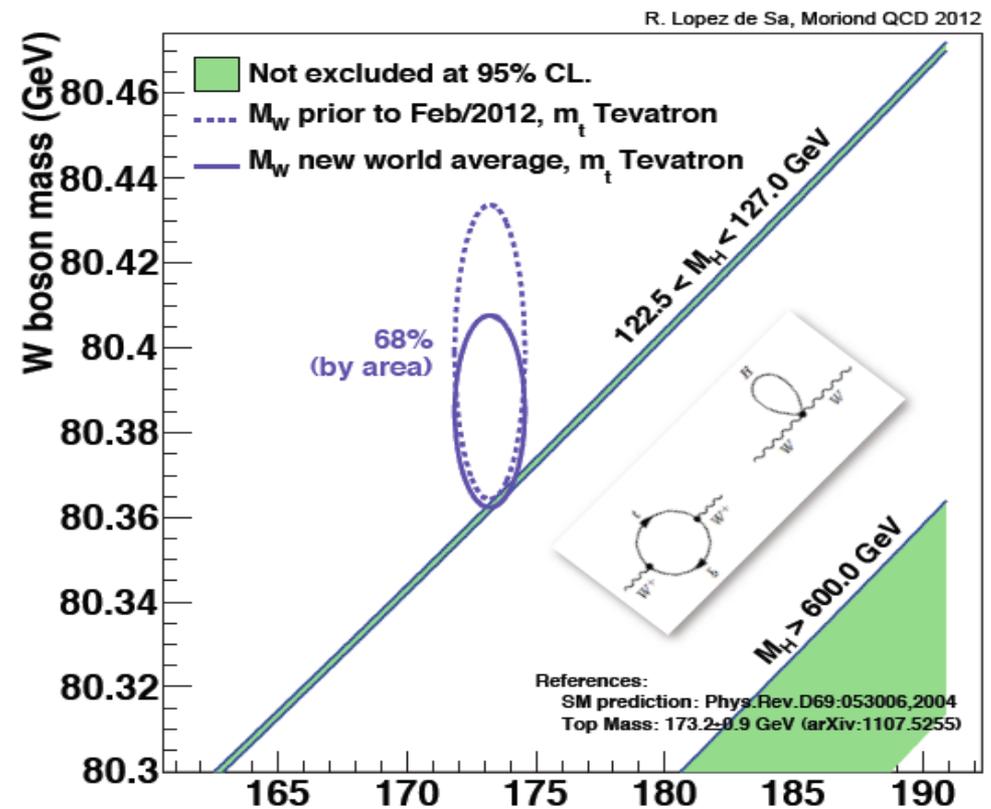
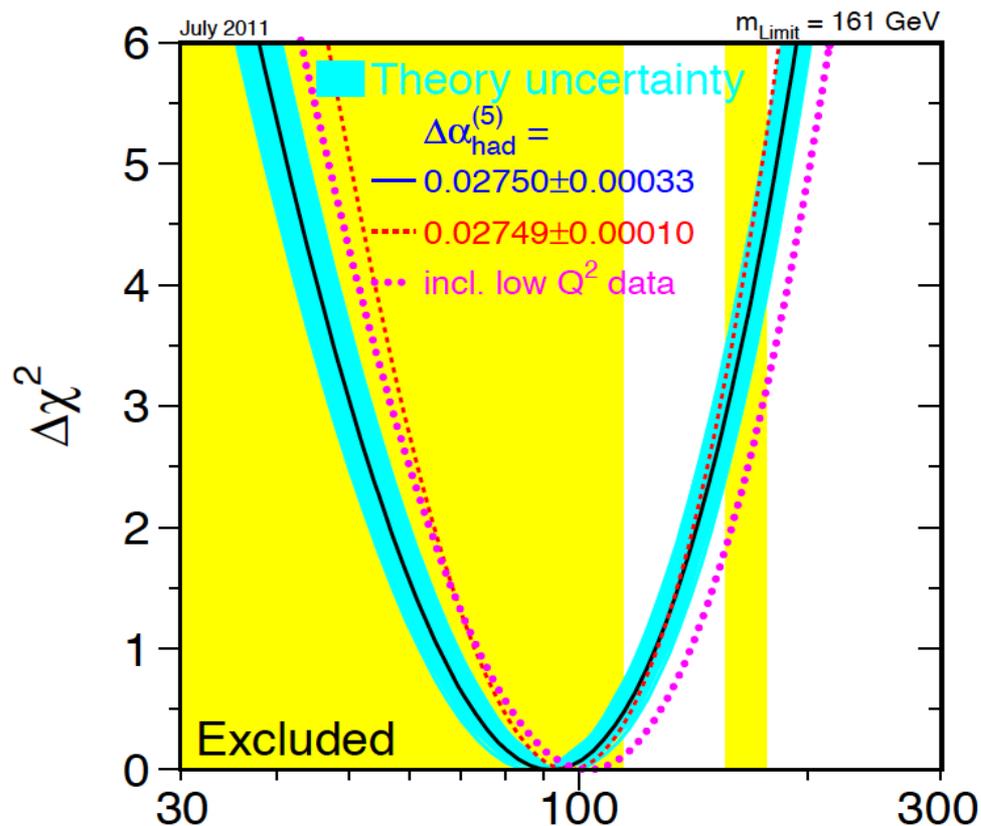
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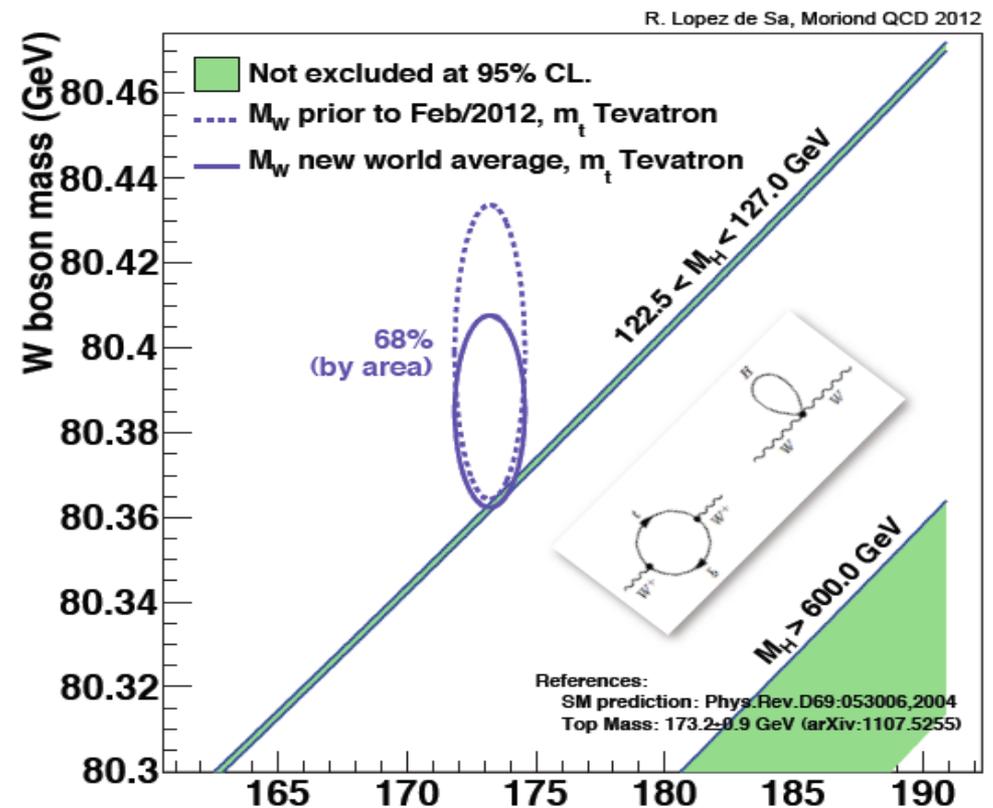
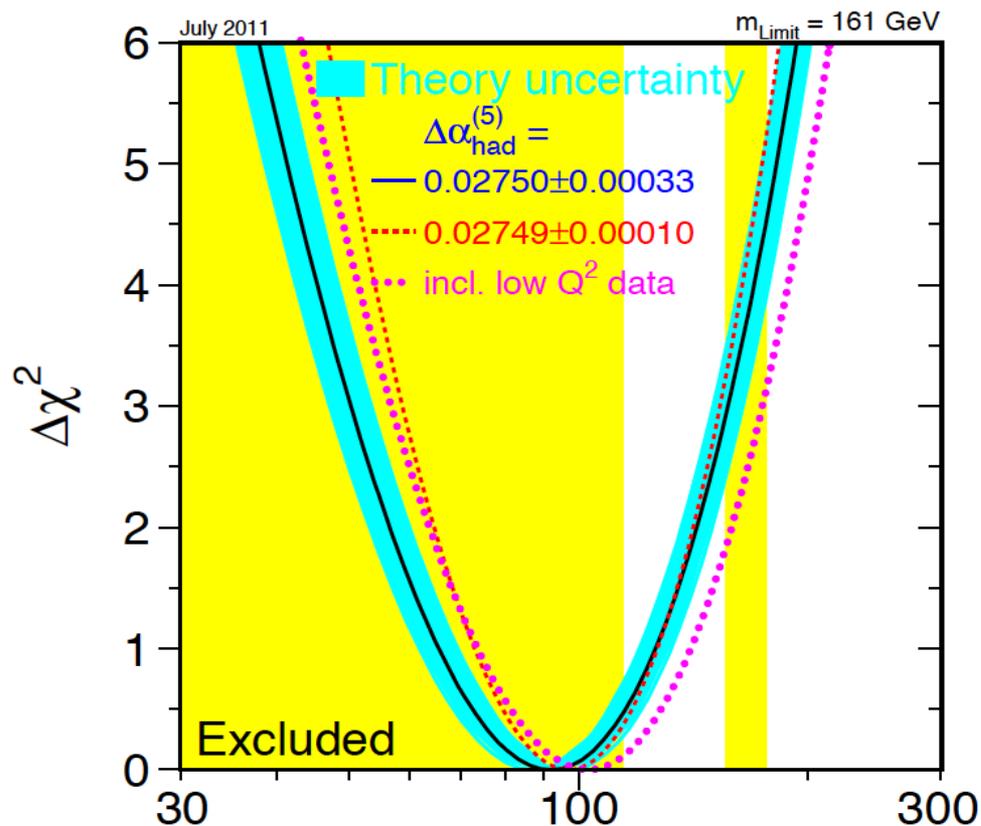
Actually some information about the Higgs mass was already present in the e.w. precision tests (assuming the validity of the SM up to high scales):



Message n.1: The observation of the physical Higgs boson with  $m_h$  well consistent with the (indirect) prediction of the e.w. precision tests is a great success of the SM!

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More generally, we have a strong indication that the symmetry breaking sector of the theory has a minimal and weakly coupled structure (at least around the TeV scale)

► The SM Higgs sector

Still, the SM Higgs potential is “ugly” and hides the most serious *theoretical problems* of this highly successful theory:

$$V(\phi) = -\mu^2 \phi^+ \phi + \lambda (\phi^+ \phi)^2 + Y^{ij} \psi_L^i \psi_R^j \phi$$

vacuum instability

possible internal inconsistency of the model ( $\lambda < 0$ ) at large energies  
[ *key dependence on  $m_h$*  ]

Quadratic sensitivity to the cut-off

$$\Delta\mu^2 \sim \Delta m_h^2 \sim \Lambda^2$$

(indication of *new physics* close to the electroweak scale ?)

SM flavor problem

(unexplained span over several orders of magnitude and strongly hierarchical structure of the Yukawa coupl.)

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neutrino  
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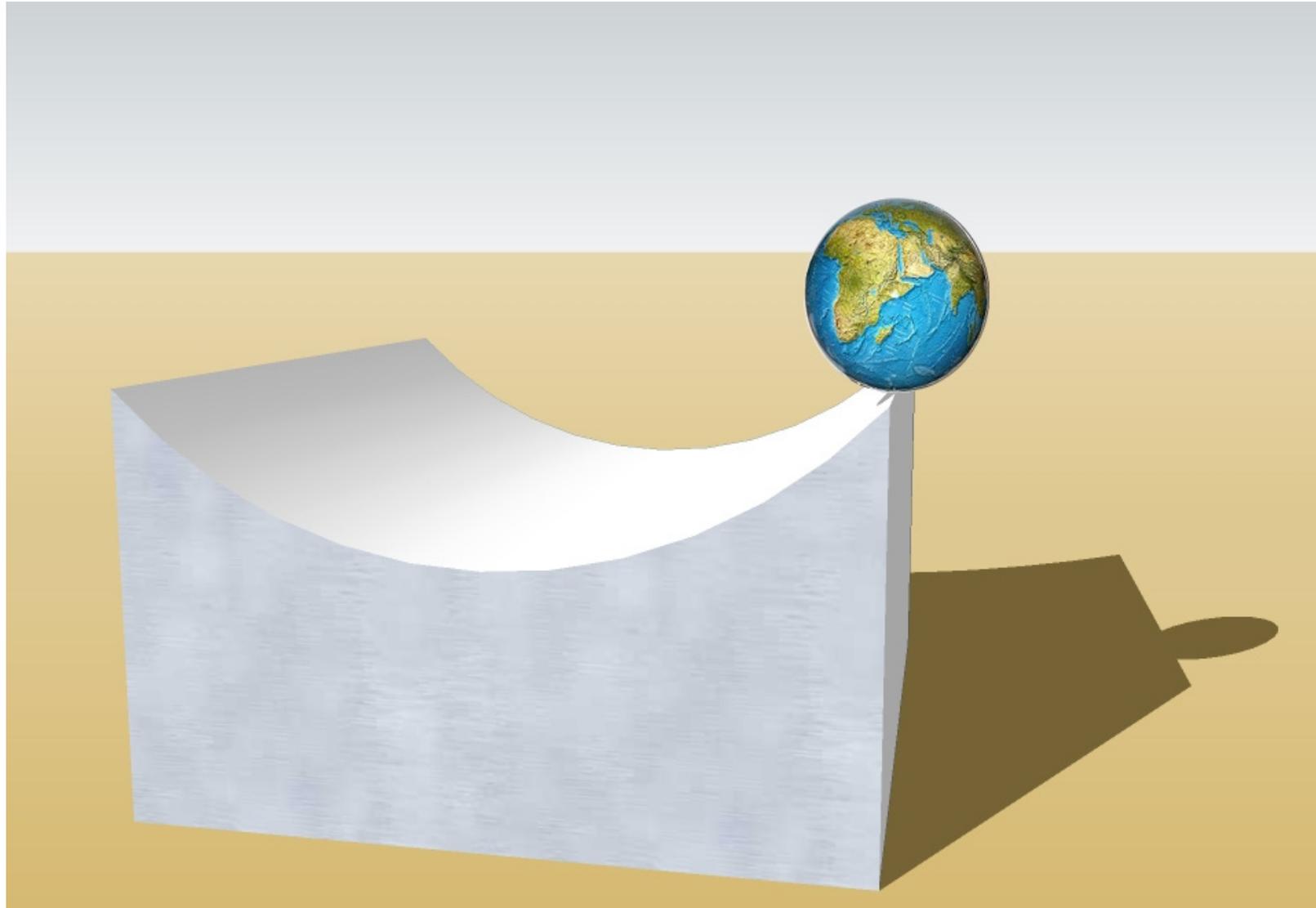
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► Stability and metastability bounds

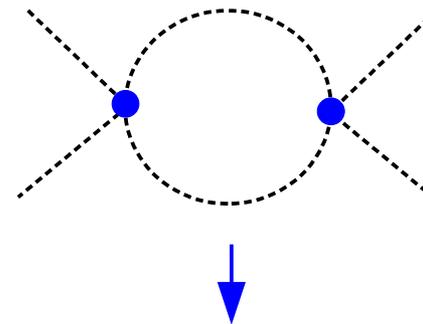


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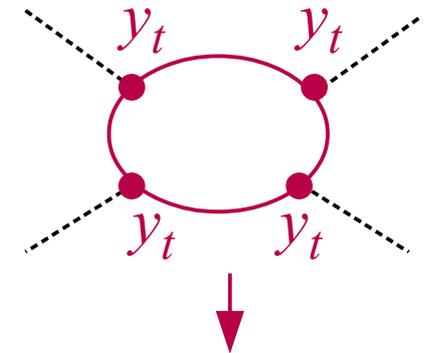
At large field values the shape of the Higgs potential is determined by the RGE evolution of the Higgs self coupling:

$$V_{\text{eff}}(|\phi| \gg v) \approx \lambda(|\phi|) \times |\phi|^4 + \mathcal{O}(v^2|\phi|^2)$$

The evolution of  $\lambda$  is determined by two main effects:



growing  $\lambda$  at large energies



decreasing  $\lambda$

$$\lambda(v) \propto \frac{m_h^2}{v^2}$$

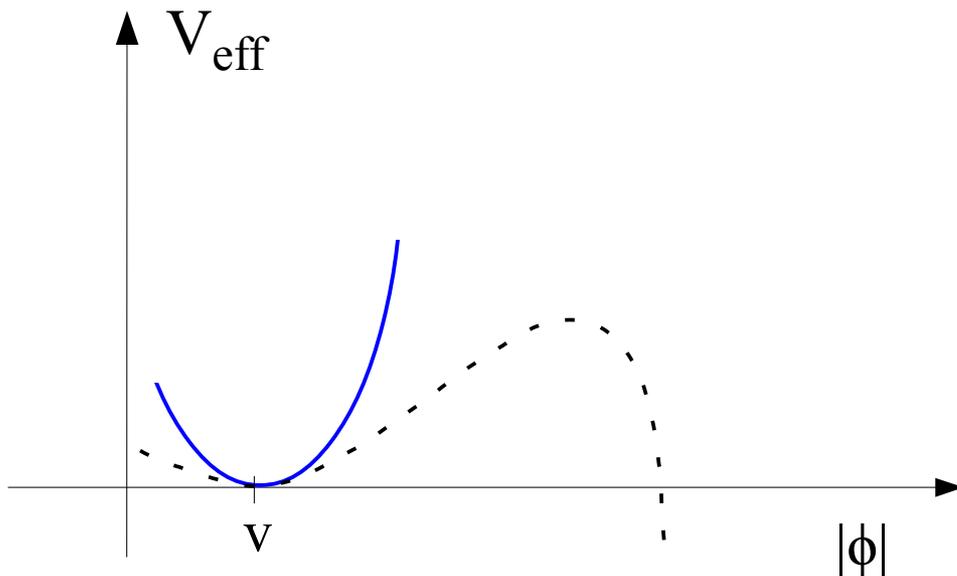
$$y_t(v) \propto \frac{m_t}{v}$$

Given the large value of  $y_t$ , the destabilization due to top-quark loops is quite relevant

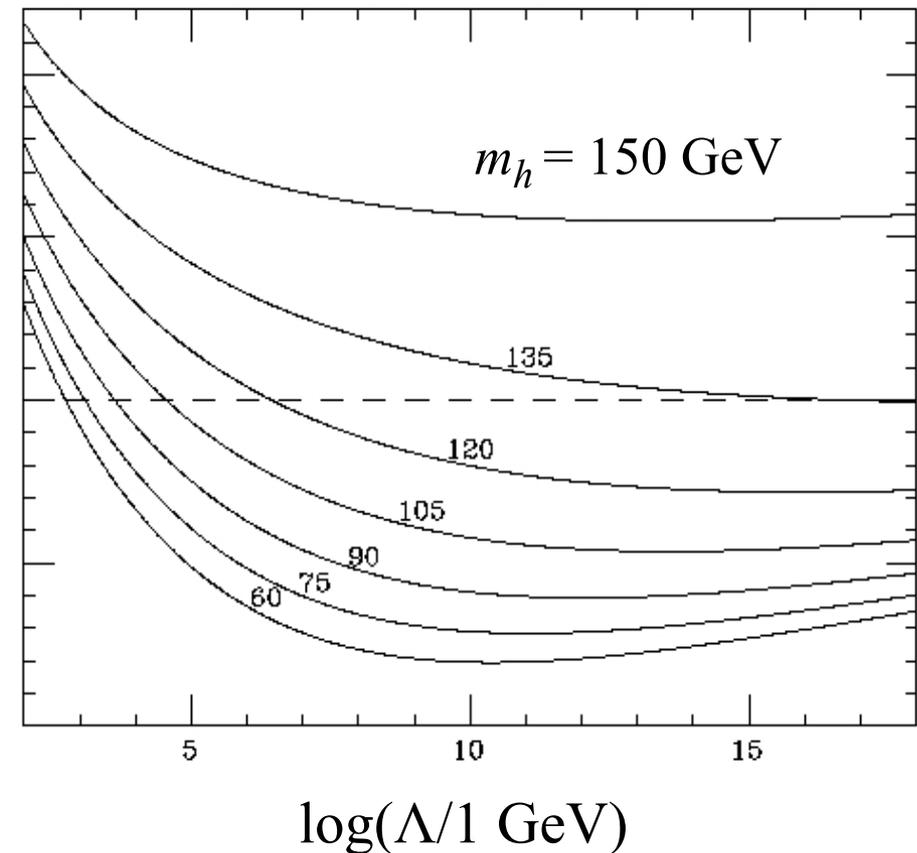
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$\lambda(\Lambda)$

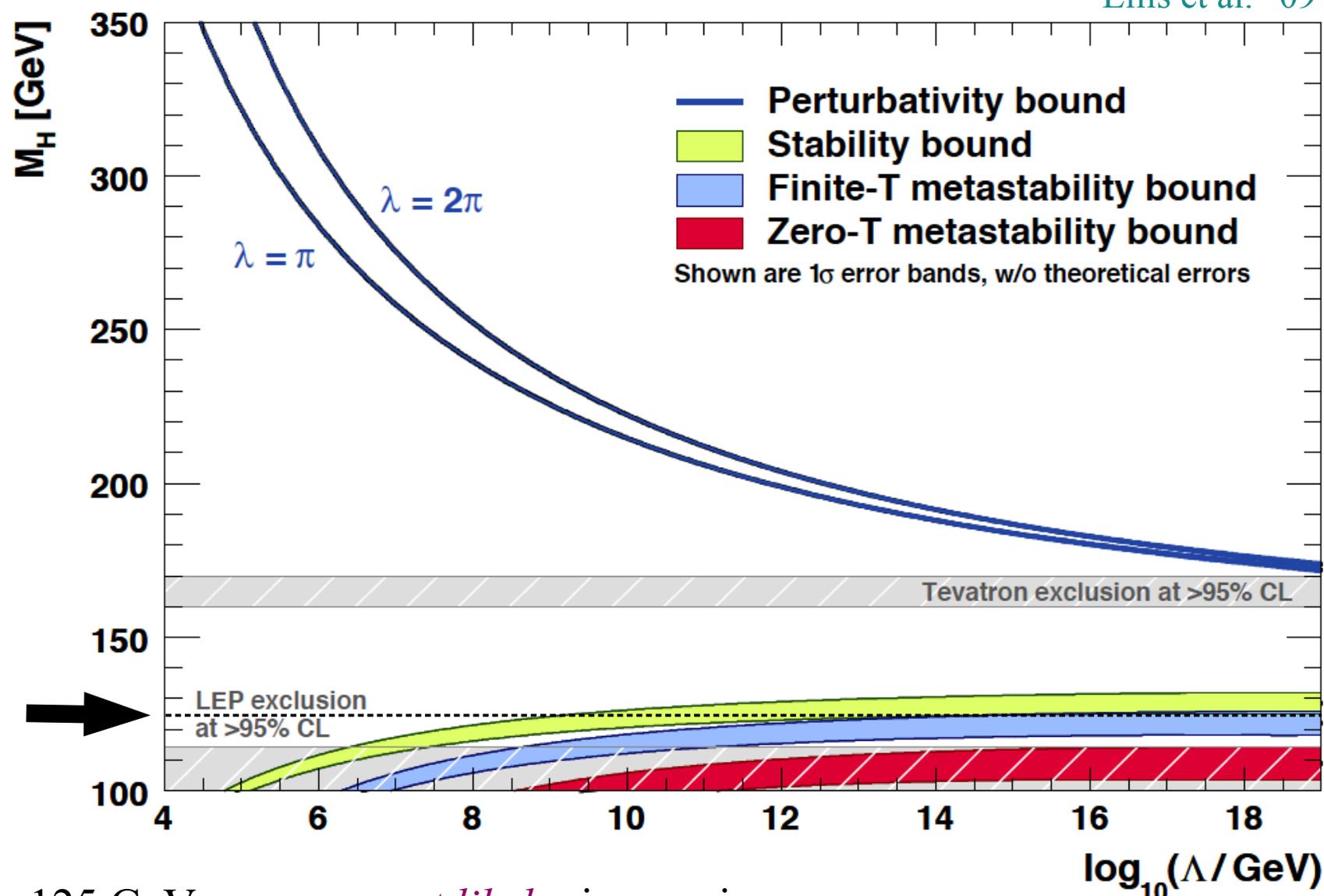


The problem was well-known since a long time, but now for the first time we can “quantify it”, knowing the Higgs mass

Cabibbo, Maiani, Parisi, Petronzio, '79;  
Hung '79; Lindner 86; Sher '89; ....

► Stability and metastability bounds

Ellis et al. '09



For  $m_h \sim 125$  GeV we are *-most likely-* in a region where the Higgs potential is not absolutely stable

★ The metastability condition:

Can we rule out the model (and determine an upper bound on the new-physics scale  $\Lambda$ ) if there is a second (deeper) minimum at large field values ?

Not really: The model could still be consistent if the lifetime of the (unstable) e.w. minimum is sufficiently long (i.e. longer than the age of the Universe)

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The e.w. minimum is destabilized by:

quantum fluctuations (at  $T=0$ )

computable in a  
model-independent way

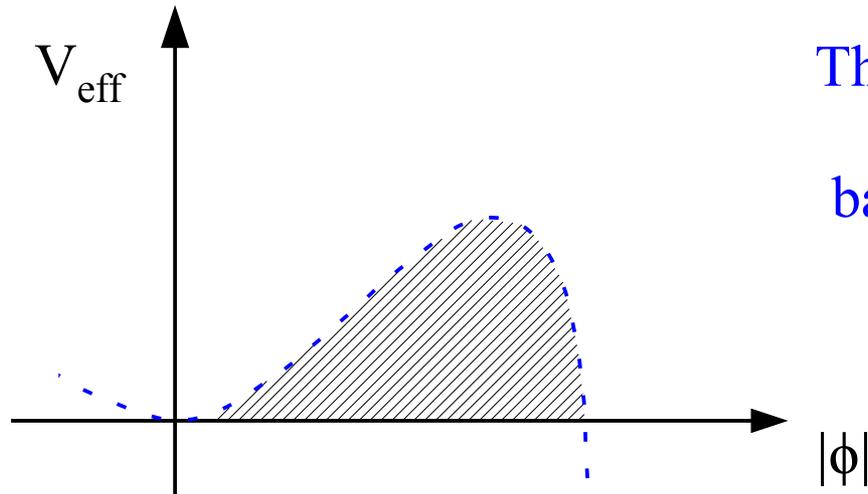
thermal fluctuations

strongly dependent on the thermal history  
of the universe & competing with quant.  
fluctuations only for very high  $T$



The most conservative bound  
is obtained by considering the stability under  
quantum fluctuations at zero temperature

★ The quantum-tunneling rate:



The quantum tunneling occurs via bubble formation in the homogeneous background of the false (e.w.) minimum

Coleman '79

At the semi-classical level, the tunneling probability can be written as:

$$p \approx K e^{-S_0[h]}$$

Volume factor

$K \propto T_U^4$  not exactly calculable within the semi-classical approx.

Euclidean action

$$\int \frac{1}{2} (\partial_\mu h)^2 + V(h)$$

*Bounce*

solution of the e.o.m. that interpolates between the false and the true vacuum

**N.B.:** within a QFT (system with infinite d.o.f.) the tunneling is suppressed even in absence of a potential barrier (kinematic barrier due to the boundary conditions)

★ The quantum-tunneling rate:

If we neglect the mass term, the tree-level Higgs potential is scale invariant & its bounces have a rather simple form:

$$h(r) = \left( \frac{2}{|\lambda|} \right)^{1/2} \frac{2R}{r^2 + R^2} \quad r = x_\mu x_\mu \quad O(4) \text{ invariant bounces minimize the action}$$

$R =$  arbitrary scale parameter

$$S_0[h] = \frac{8\pi^2}{3|\lambda|} \quad \rightarrow \quad p_{\text{semicl.}} \approx (T_U/R)^4 e^{-8\pi^2/3|\lambda|}$$

If  $|\lambda|$  remains sufficiently small, the tunneling rate can be very suppressed

**N.B.:** the tunneling rate is a pure non-perturbative phenomenon - cannot be computed to any finite order in “ordinary” perturbation theory [*wrong choice of the vacuum*]

★ The quantum-tunneling rate:

To go beyond the semi-classical level we need to take into account the quantum fluctuations around the (non-constant) bounce solution

Callan, Coleman '79

Non-trivial problem which has been solved (semi-analytically) in the SM case:

G.I., Ridolfi, Strumia '01



- Quantum corrections break scale invariance
- The tunneling is dominated by bounces of size  $R$ , such that  $\lambda(1/R)$  reaches its minimum value:

$$p = \max_R \frac{V_U}{R^4} \exp \left[ -\frac{8\pi^2}{3|\lambda(\mu)|} - \Delta S(\mu R) \right]$$

$\mu$  independent

$\Delta S \approx 0$  if we set  $\mu = 1/R$

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- The tunneling is dominated by bounces of size  $R$ , such that  $\lambda(1/R)$  reaches its minimum value
- The critical  $R$  determine the reference scale of the volume pre-factor:

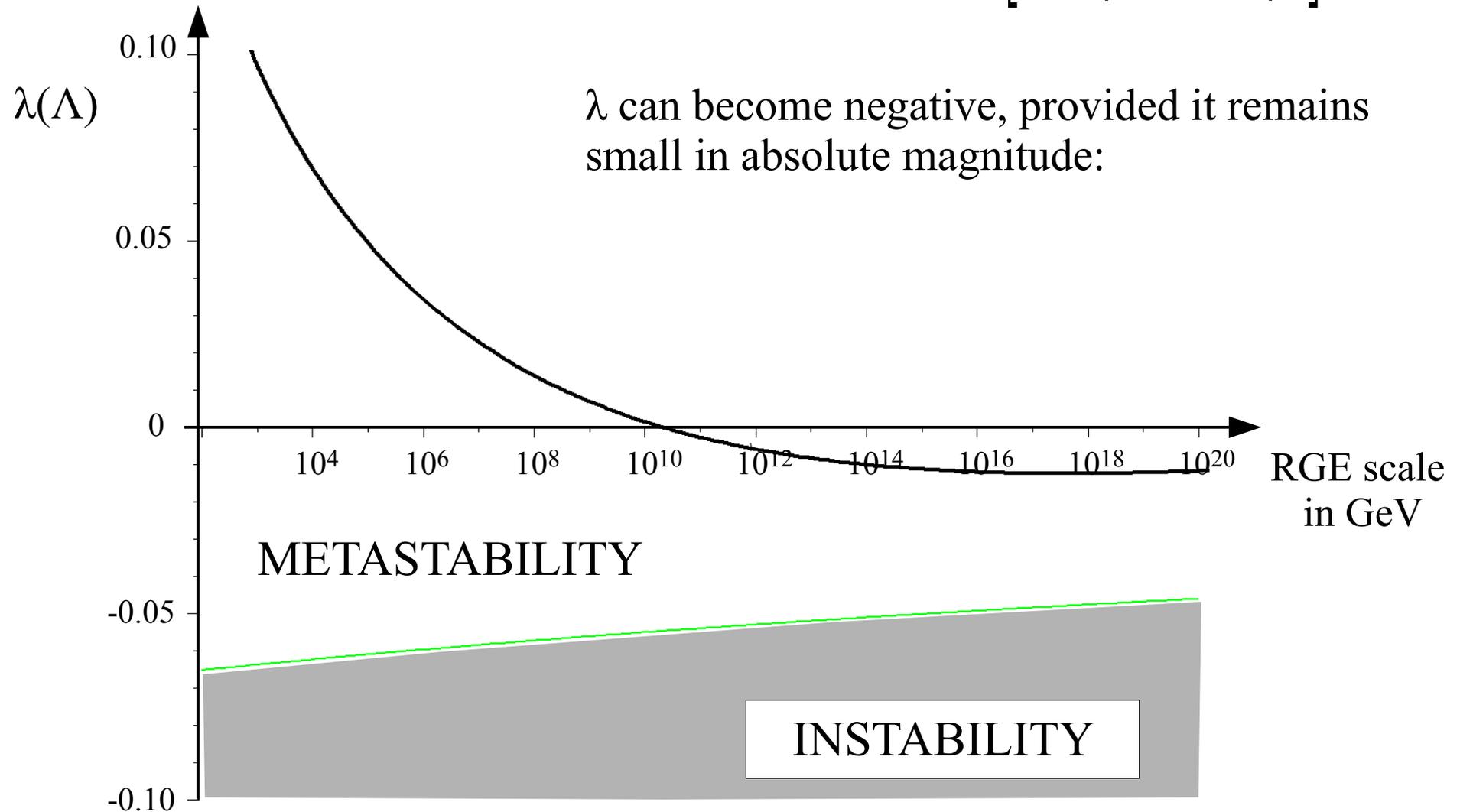
$$p \approx \max_R \frac{V_U}{R^4} \exp \left[ - \frac{8 \pi^2}{3 |\lambda(1/R)|} \right]$$

The leading gravitational effects are also calculable when  $1/R$  is not far from (but below)  $M_{\text{pl}}$

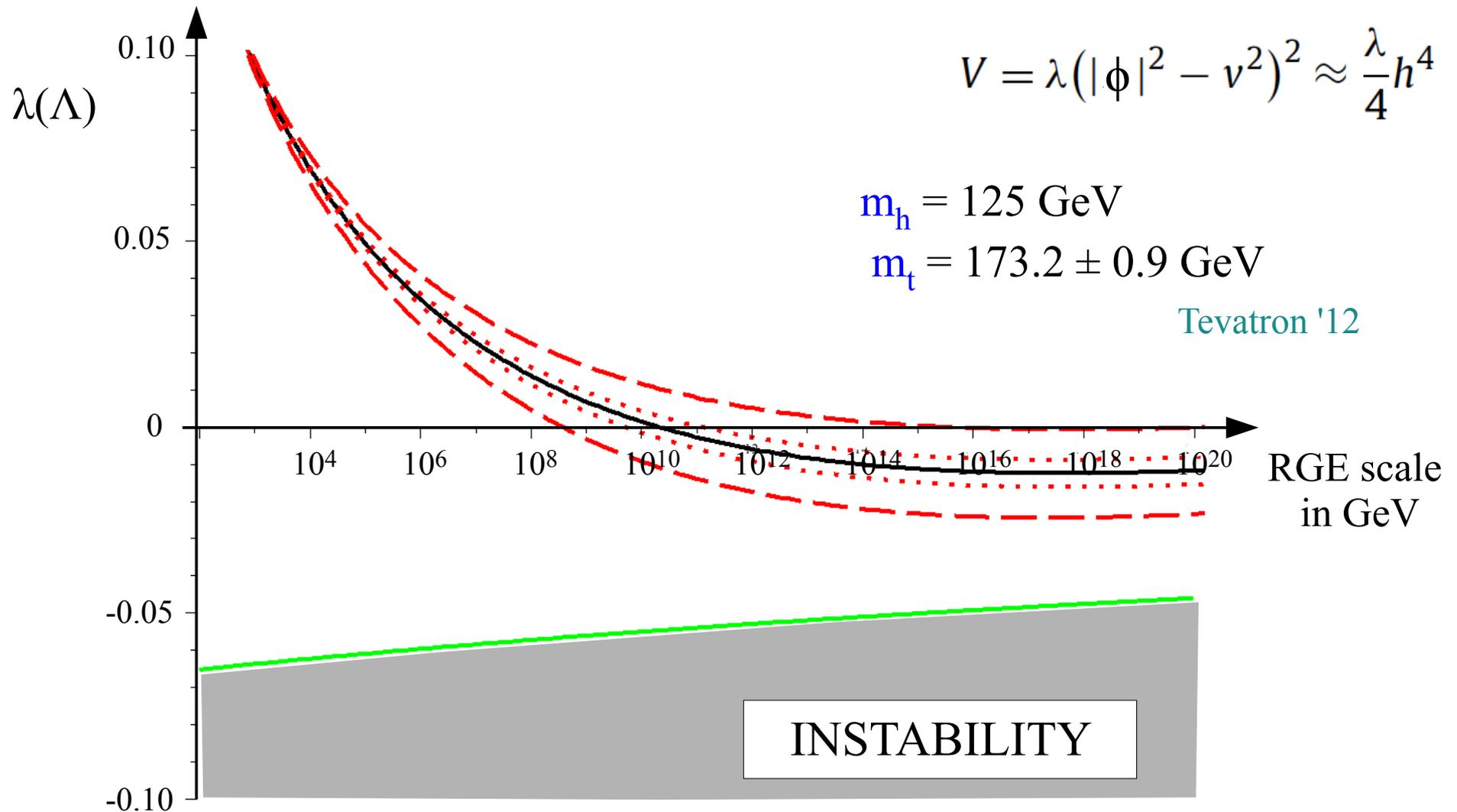
G.I., Rychkov, Strumia, Tetradis '08

★ The metastability condition:

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**Message n.2:** For  $m_h = 125$  GeV and the present central value of  $m_{top}$ , the SM vacuum is unstable but sufficiently long-lived, compared to the age of the Universe



► Vacuum stability at NNLO (for  $m_h \sim 125$  GeV)

For  $m_h = 125$  GeV and the present central value of  $m_{\text{top}}$ , the SM vacuum is unstable but sufficiently long-lived, compared to the age of the Universe

How “precise” is this statement?

A full NNLO analysis has recently become possible:

- Two-loop potential Ford, Jack, Jones '92, '01
- Three-loop beta functions Mihaila, Salomon, Steinhauser 1201.5868  
Chetyrkin, Zoller, 1205.2892
- Two-loop threshold corrections in relating  $\lambda(v)$  to the Higgs mass:

$$\lambda(\mu) = \frac{G_F m_h^2}{\sqrt{2}} + \Delta\lambda(\mu)$$

*(dominant uncertainty)*

Yukawa×QCD

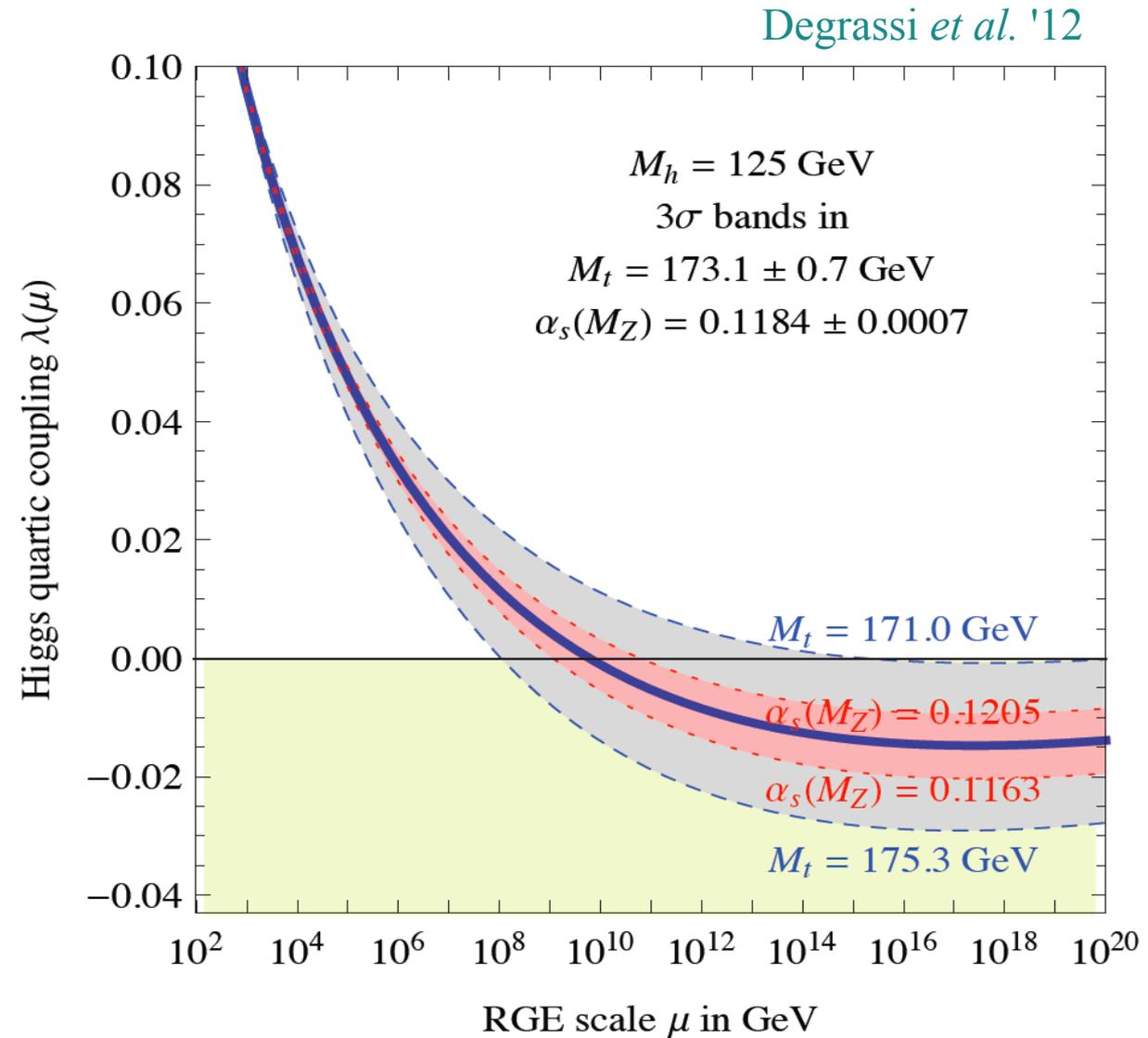
Yukawa×QCD  
Yuk.×Yuk.

Bezrukov, Kalmykov, Kniehl,  
Shaposhnikov, 1205.2893

Degrassi, Di Vita, Elias-Miro', Espinosa,  
Giudice, G.I., Strumia 1205.6497

Given the fast running of  $\lambda$  close to the e.w. scale, the dominant uncertainty comes from threshold (non-log enhanced) corrections at the electroweak scale (or in the precise evaluation of the initial condition).

While the smallness of  $\lambda$  (and the other couplings) at high energies imply that the 3-loop terms in the beta functions play a very minor role (useful to control the error).



With the NNLO calculation we are able to derive a very precise relation between Higgs and top masses from vacuum stability:

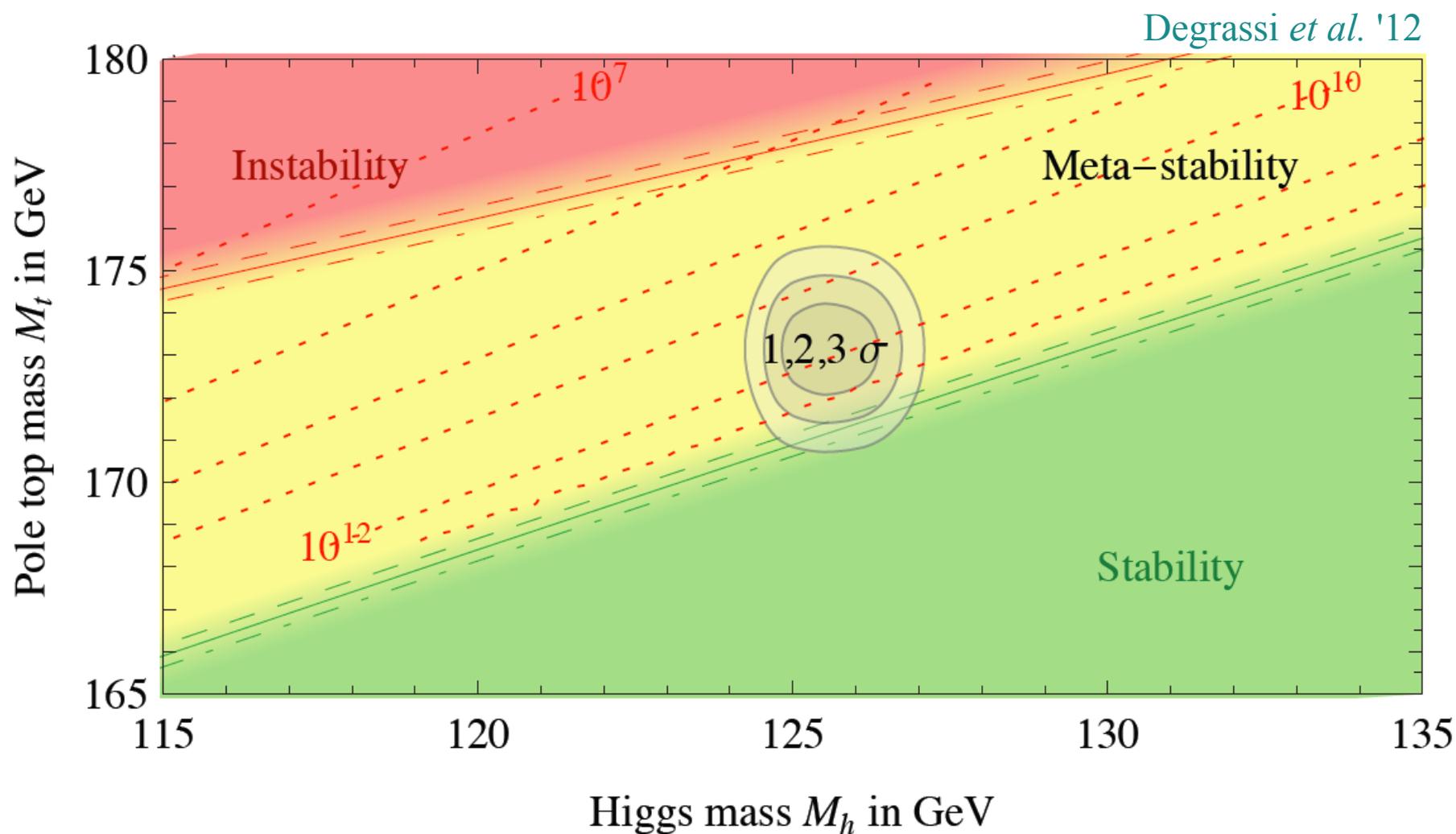
*Absolute stability:*

$$M_h \text{ [GeV]} > 129.4 + 2.0 \left( \frac{M_t \text{ [GeV]} - 173.1}{1.0} \right) - 0.5 \left( \frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 1.0_{\text{th}}$$

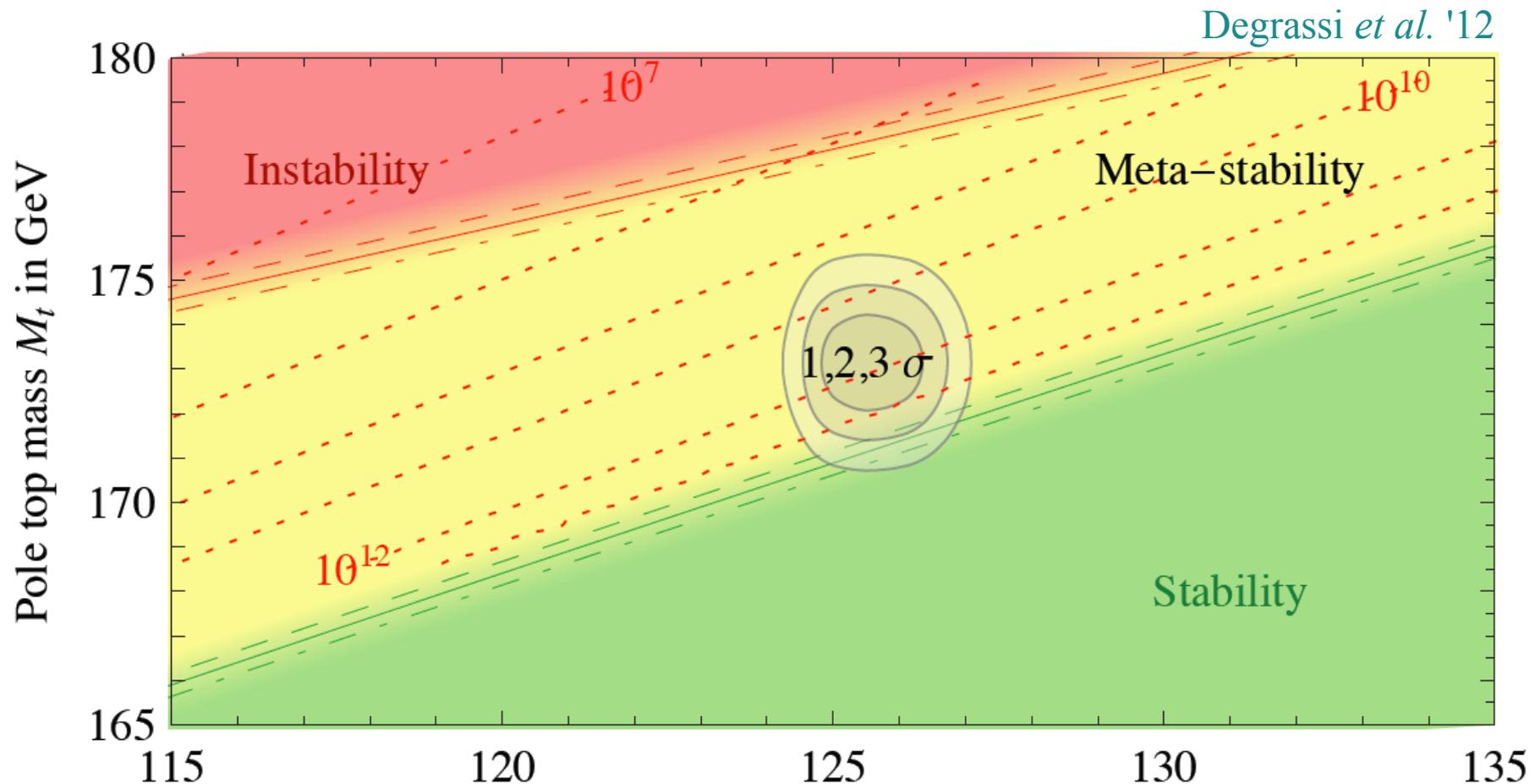
Conservative th. error given the size of the shifts from NLO to NNLO:

- + 0.6 GeV due to the QCD threshold corrections to  $\lambda$
- + 0.2 GeV due to the Yukawa threshold corrections to  $\lambda$
- 0.2 GeV from RG equation at 3 loops
- 0.1 GeV from the effective potential at 2 loops.

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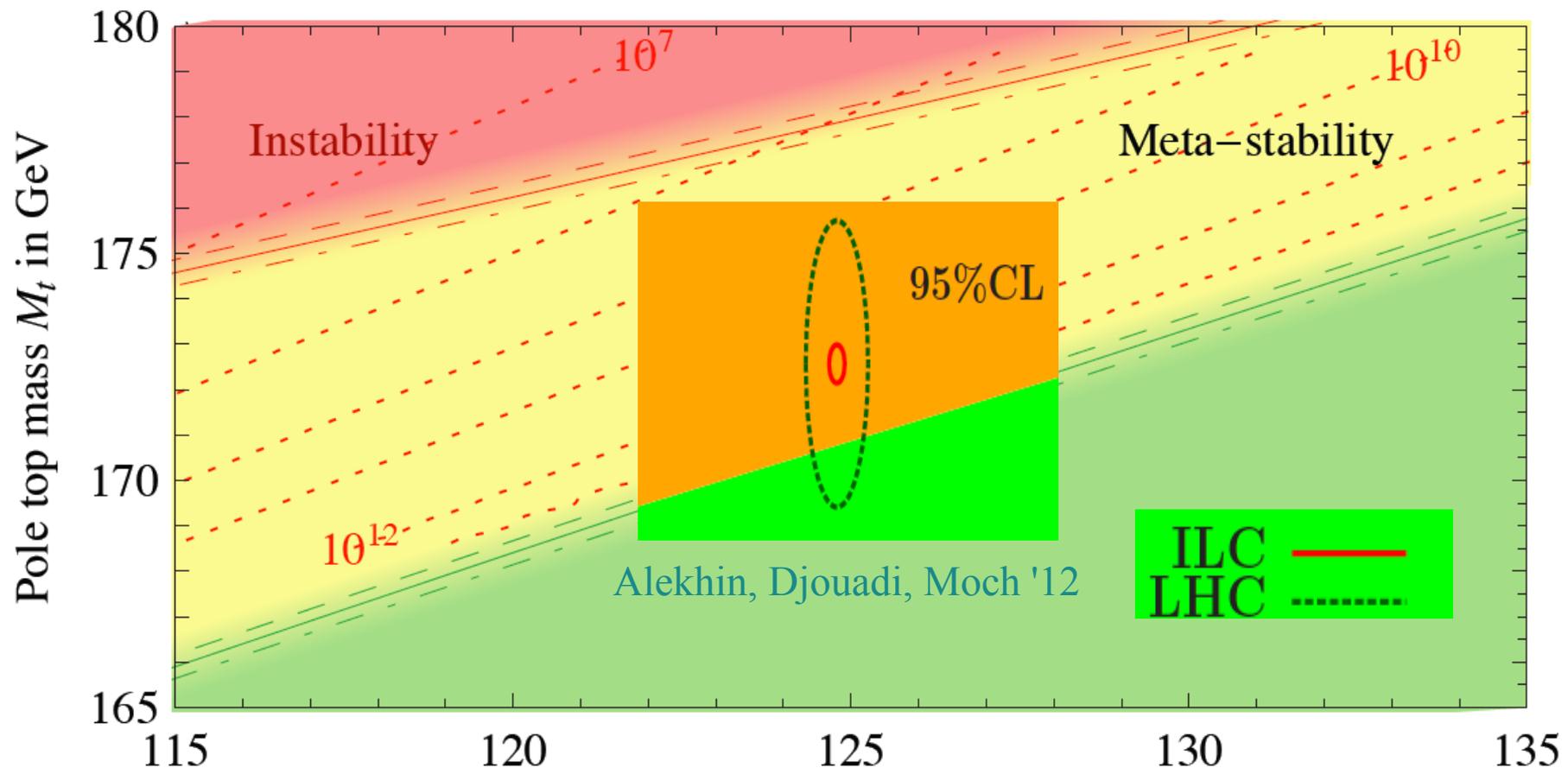
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Assuming a precise determination of  $m_h$  by ATLAS & CMS in a short time, the main uncertainty will remain the top mass.

Note also that the  $m_t$  measured by Tevatron is not really the pole mass (*possible larger error...* Alekhin, Djouadi, Moch '12, Hoang & Stewart, '07-'08)

With the NNLO calculation we are able to derive a very precise relation between Higgs and top masses from vacuum stability:



A linear collider would be the ideal machine to bring down this uncertainty, determining more precisely the fate of the SM vacuum (*if in the meanwhile we have not found anything else...!*)

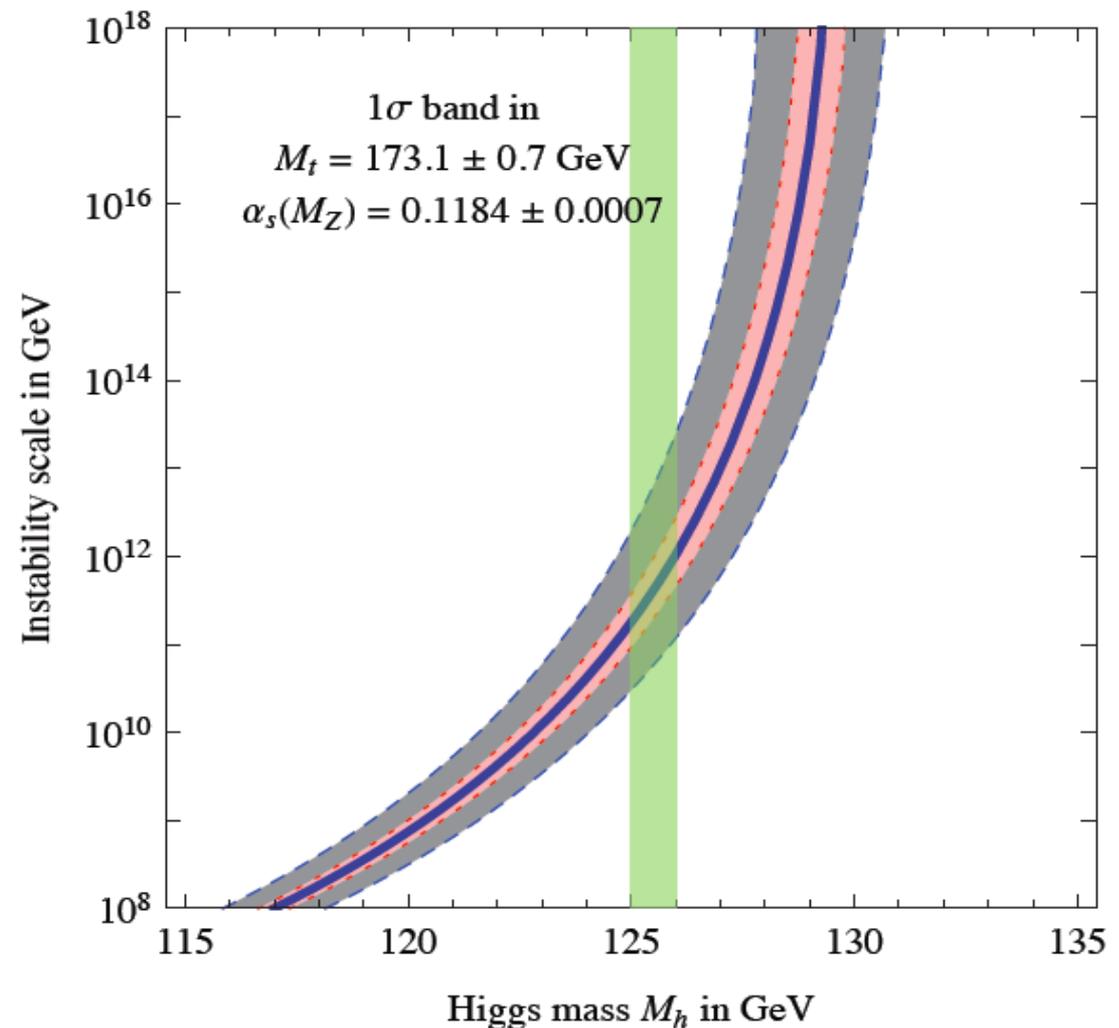
Two additional remarks about the instability of the SM potential:

- I. What about the instability because of thermal fluctuations?
- II. What about adding to the model heavy right-handed neutrinos?

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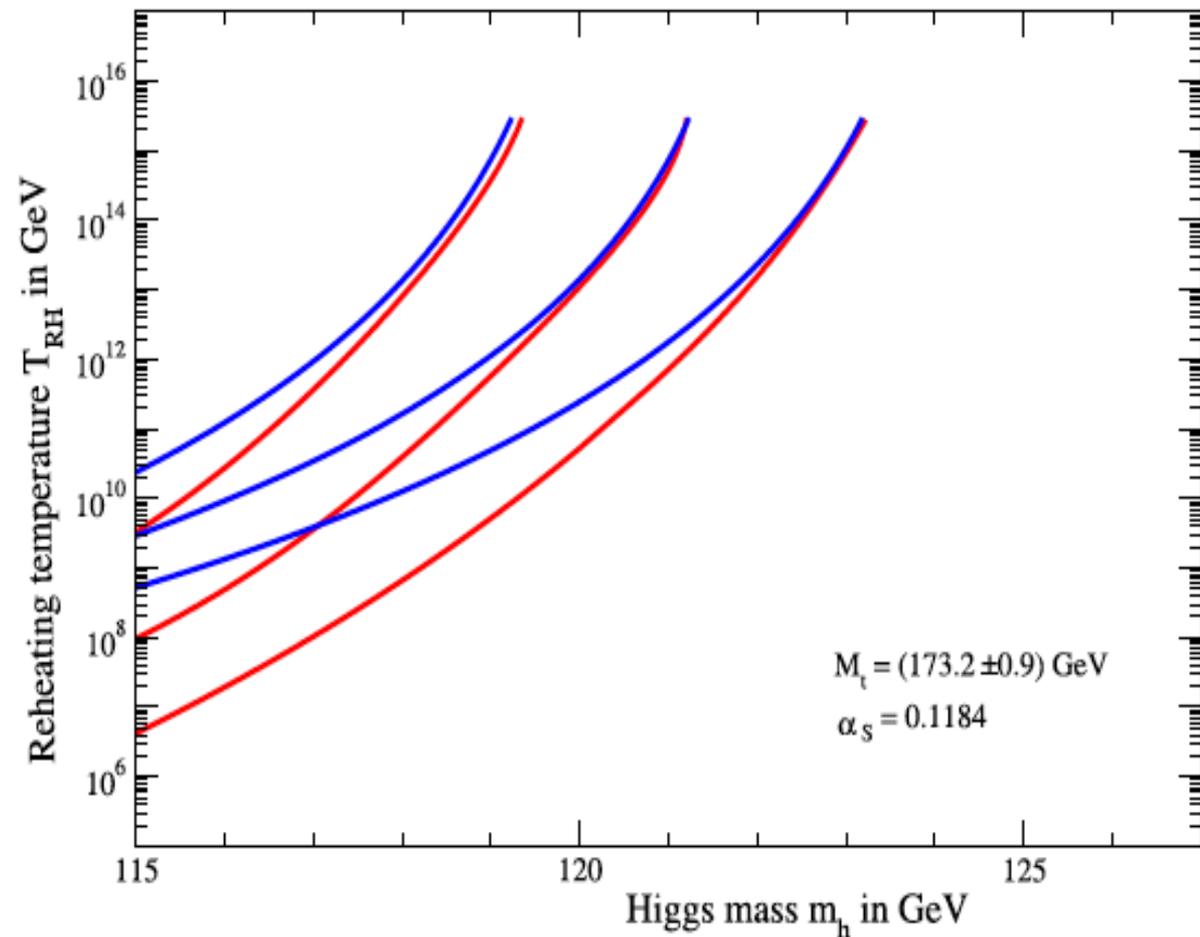
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II. What about adding to the model heavy right-handed neutrinos?

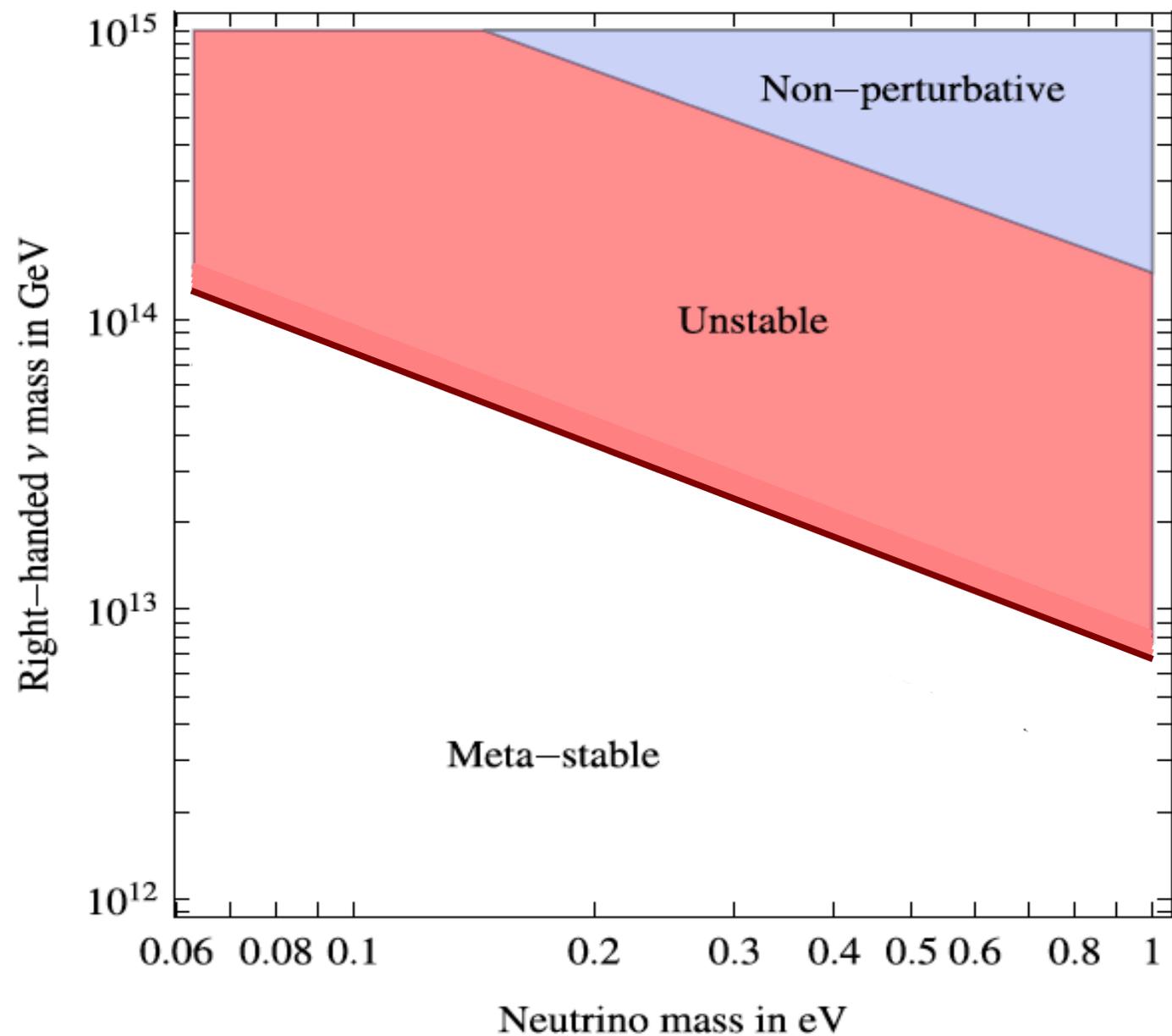
On general ground, adding new fermions may induce a further destabilization of the potential. However, the effect depend on the size of the new Yukawa couplings:

$$m_\nu \sim Y_n^T \frac{v^2}{M_R} Y_n$$



Requiring a sufficiently stable Higgs potential allow us to derive an upper bound on  $M_R$

$$m_\nu \sim Y_n^T \frac{v^2}{M_R} Y_n$$

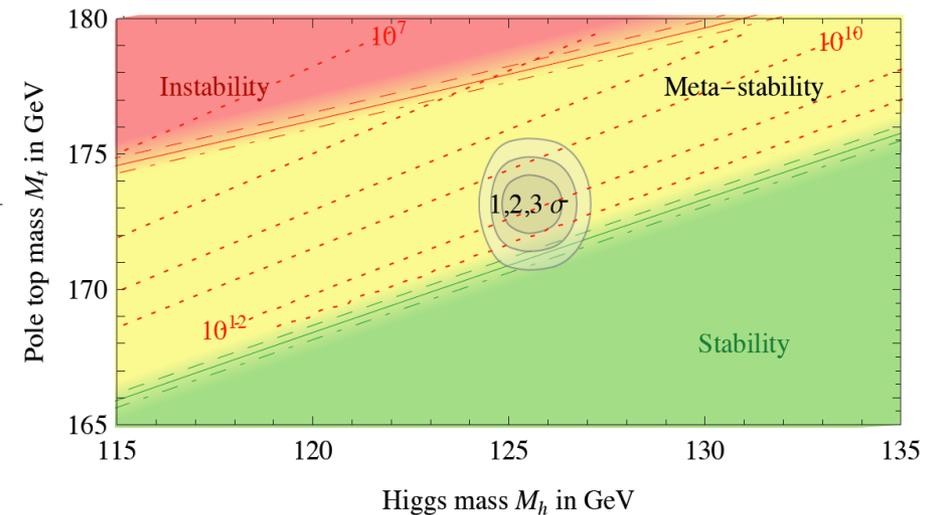
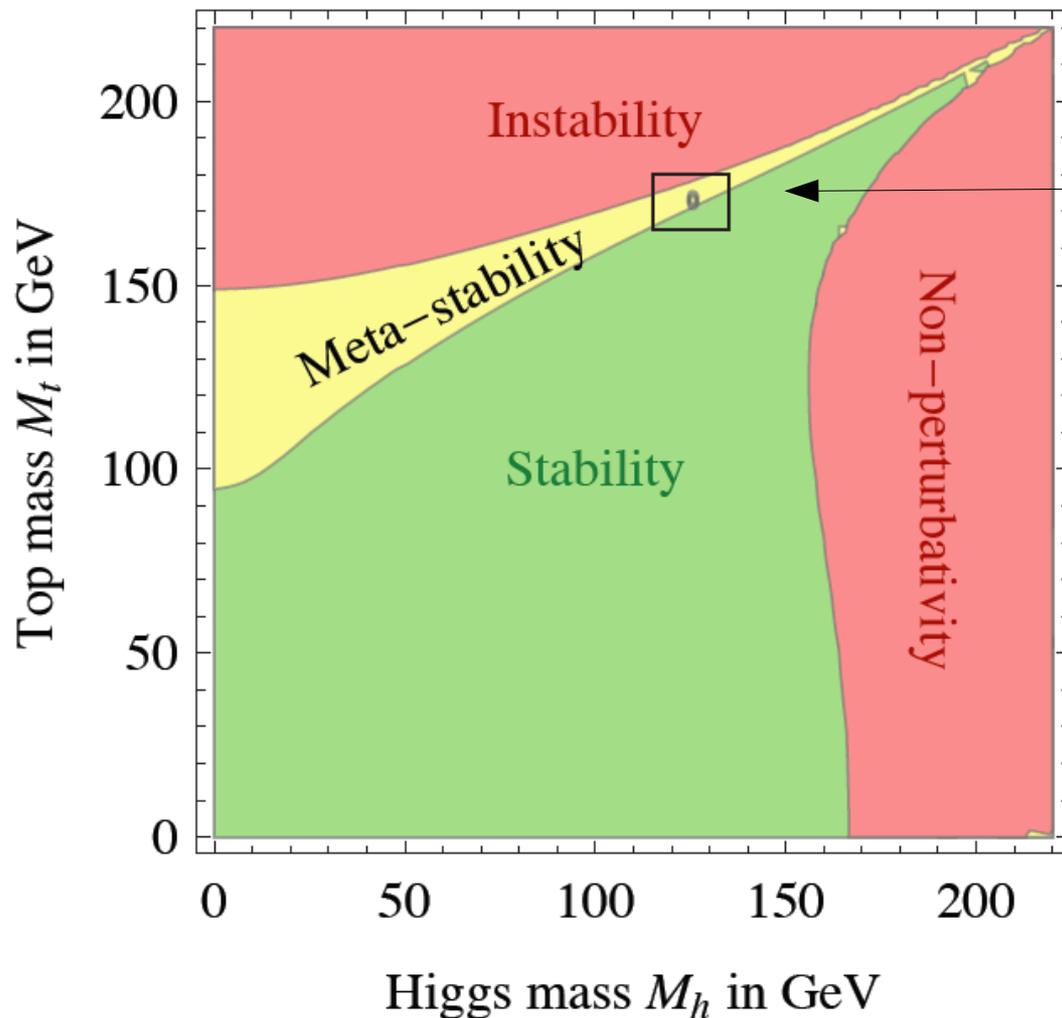
Elias-Miro *et al.* '11

Still enough room  
for leptogenesis to  
take place.

► Speculations on Planck-scale dynamics



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Looking at the plane from a more distant perspective, it appears more clearly that “we live” in a quite “peculiar” region...

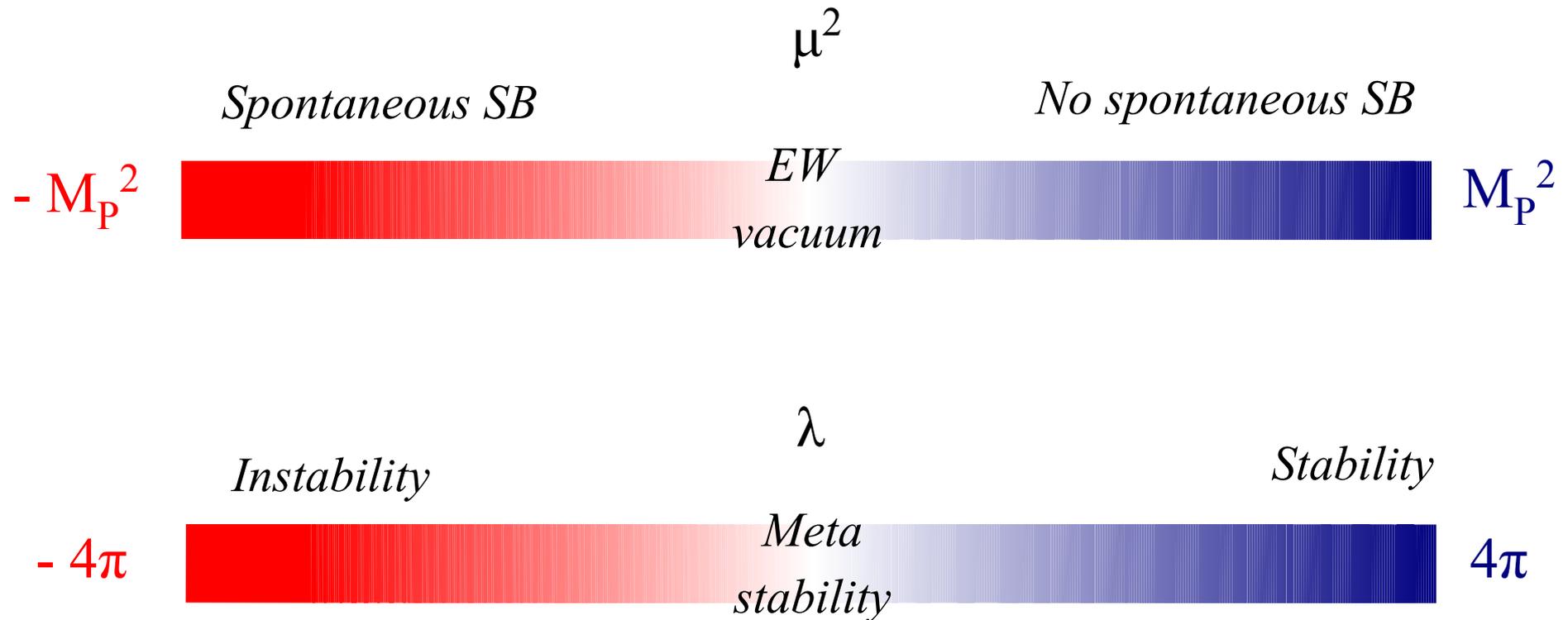
Moving  $m_t$  down by  $\sim 2$  GeV, we reach the even more peculiar configuration where  $\lambda(M_{pl})=0$

Froggatt, Nielsen, Takanishi, '01  
 Arkani-Hamed *et al.*, '08  
 Shaposhnikov, Wetterich, '10  
 ...

► Speculations on Planck-scale dynamics

It seems that the Higgs potential is “doubly tuned” around two “critical values”:

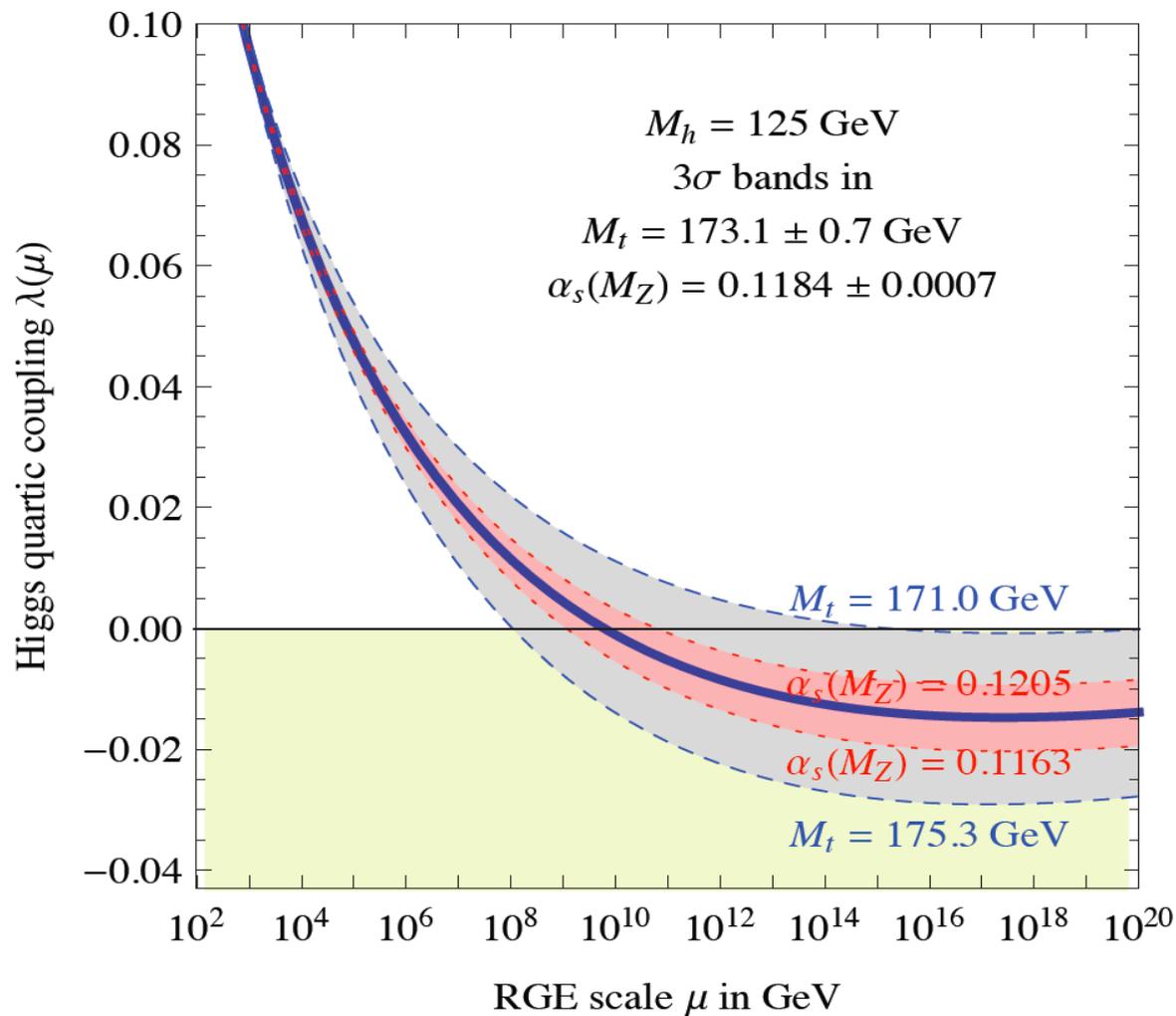
$$V(\phi) = -\mu^2 \phi^+ \phi + \lambda (\phi^+ \phi)^2$$



► Speculations on Planck-scale dynamics

What's special about  $\lambda(M_{\text{pl}})=0$ ?

Despite also the beta function vanishes, is not a true fixed point (other coupl.  $\neq 0$ )



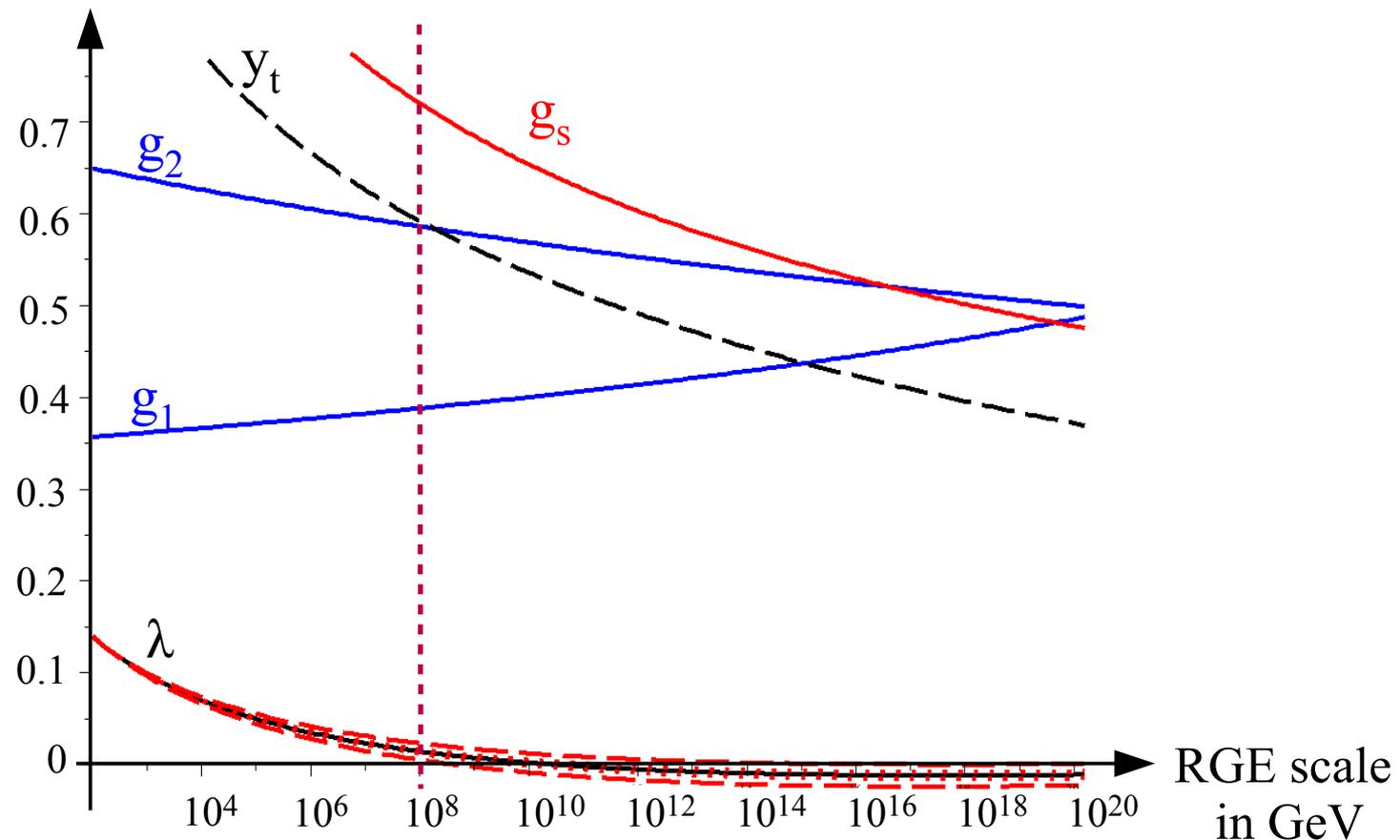
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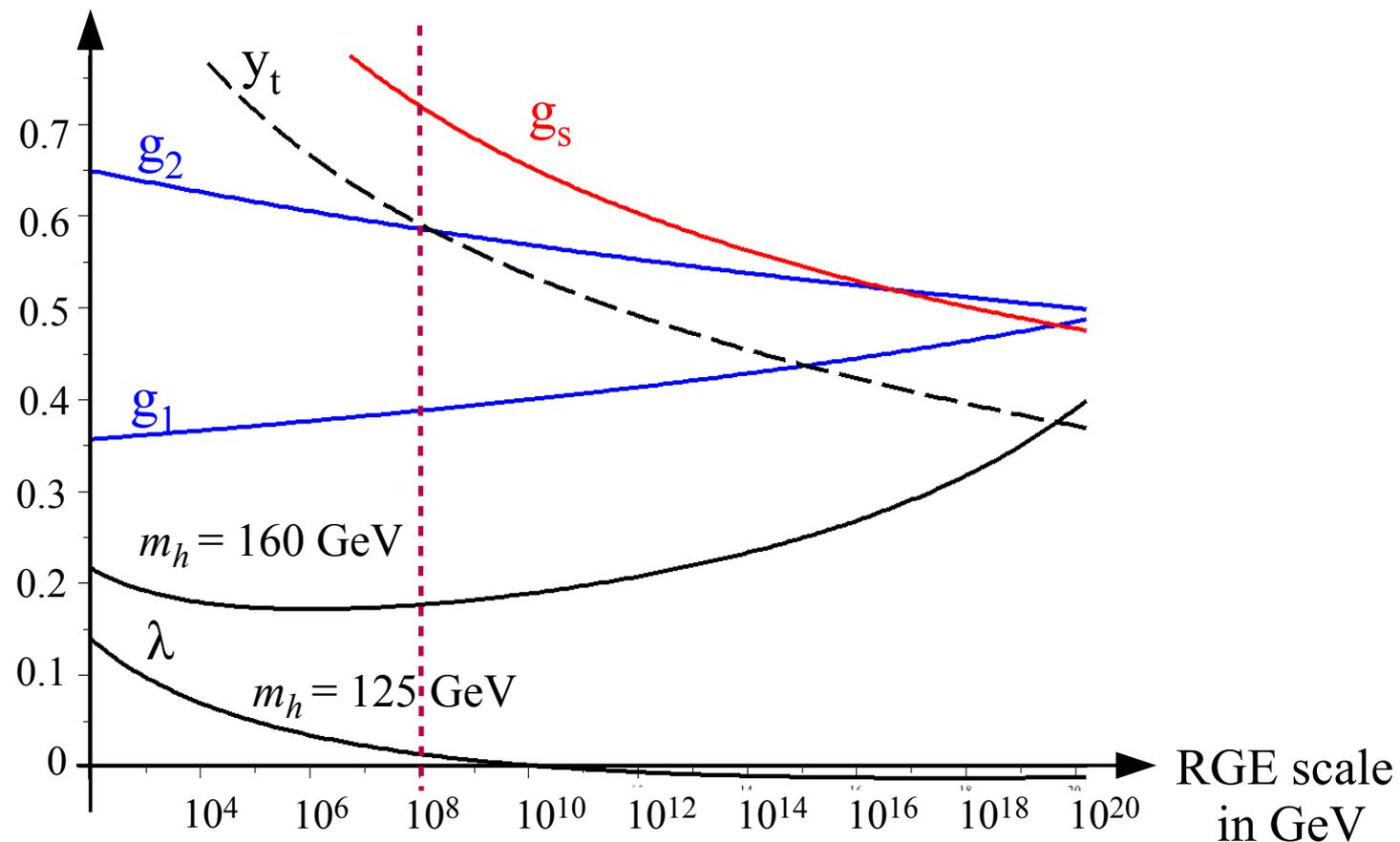
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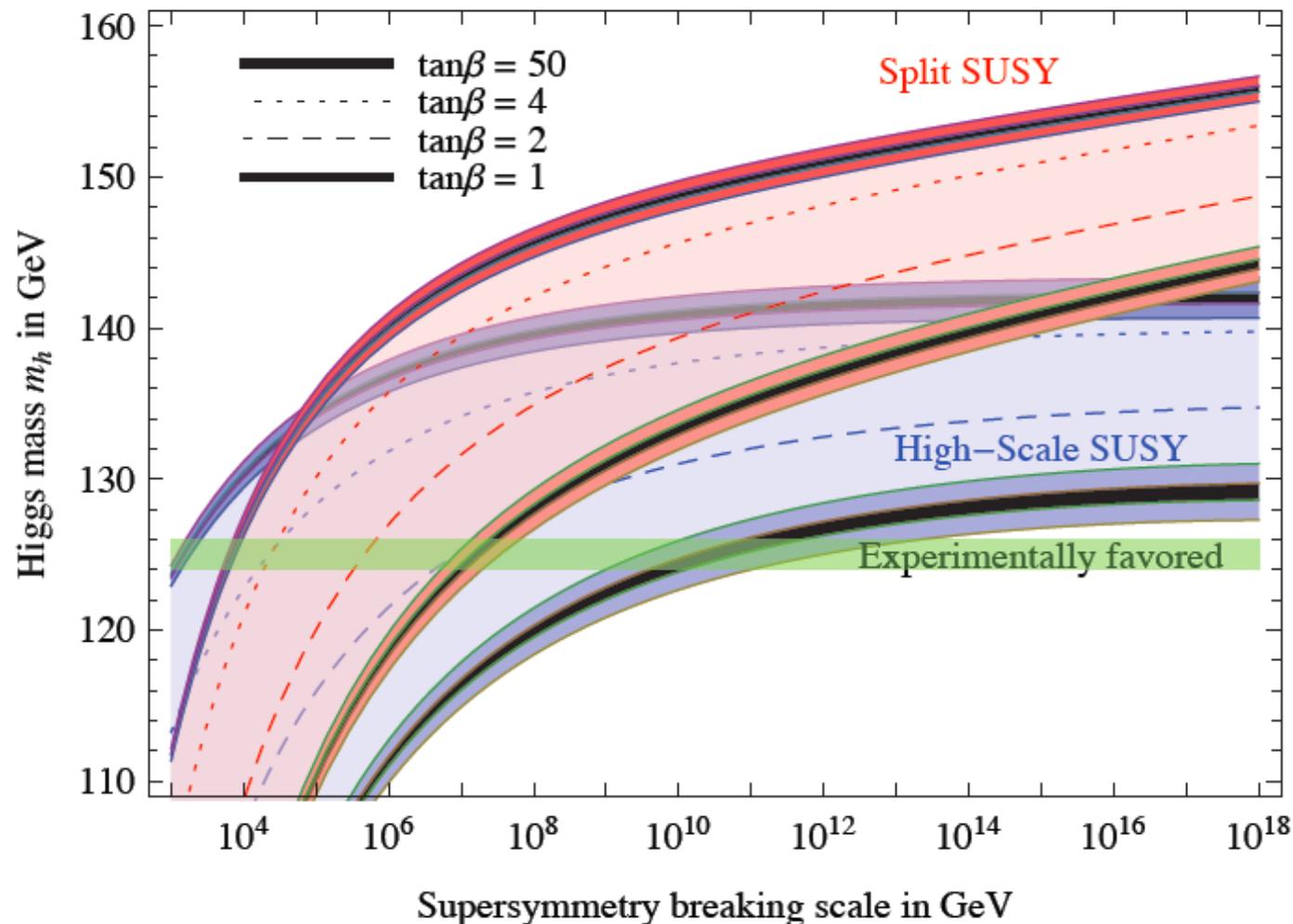
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► Speculations on Planck-scale dynamics

The smallness of  $\lambda$  certainly fits well with the possibility of a high-scale matching with a weakly coupled theory

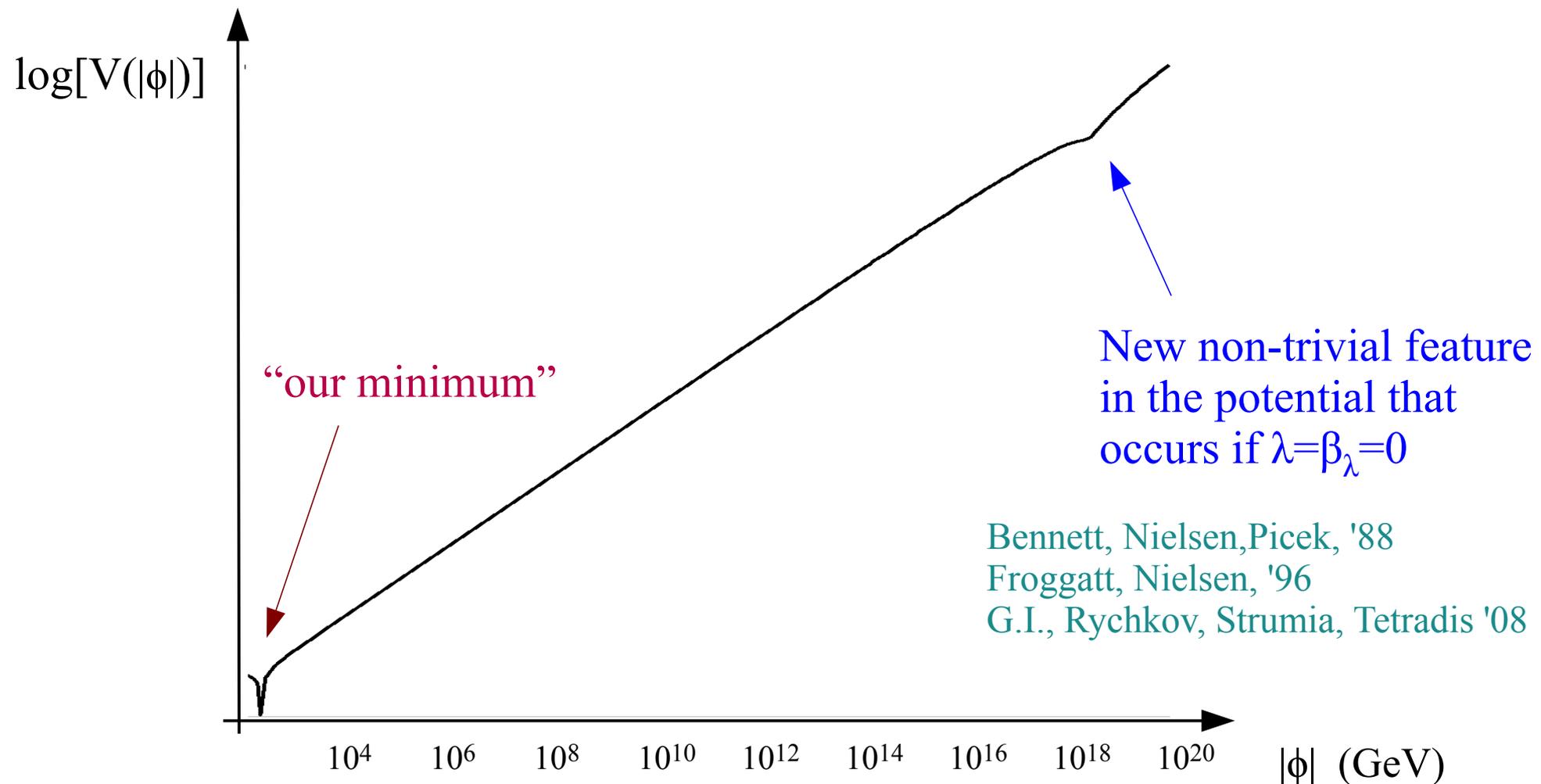
Giudice & Strumia '11-'12



► Speculations on Planck-scale dynamics

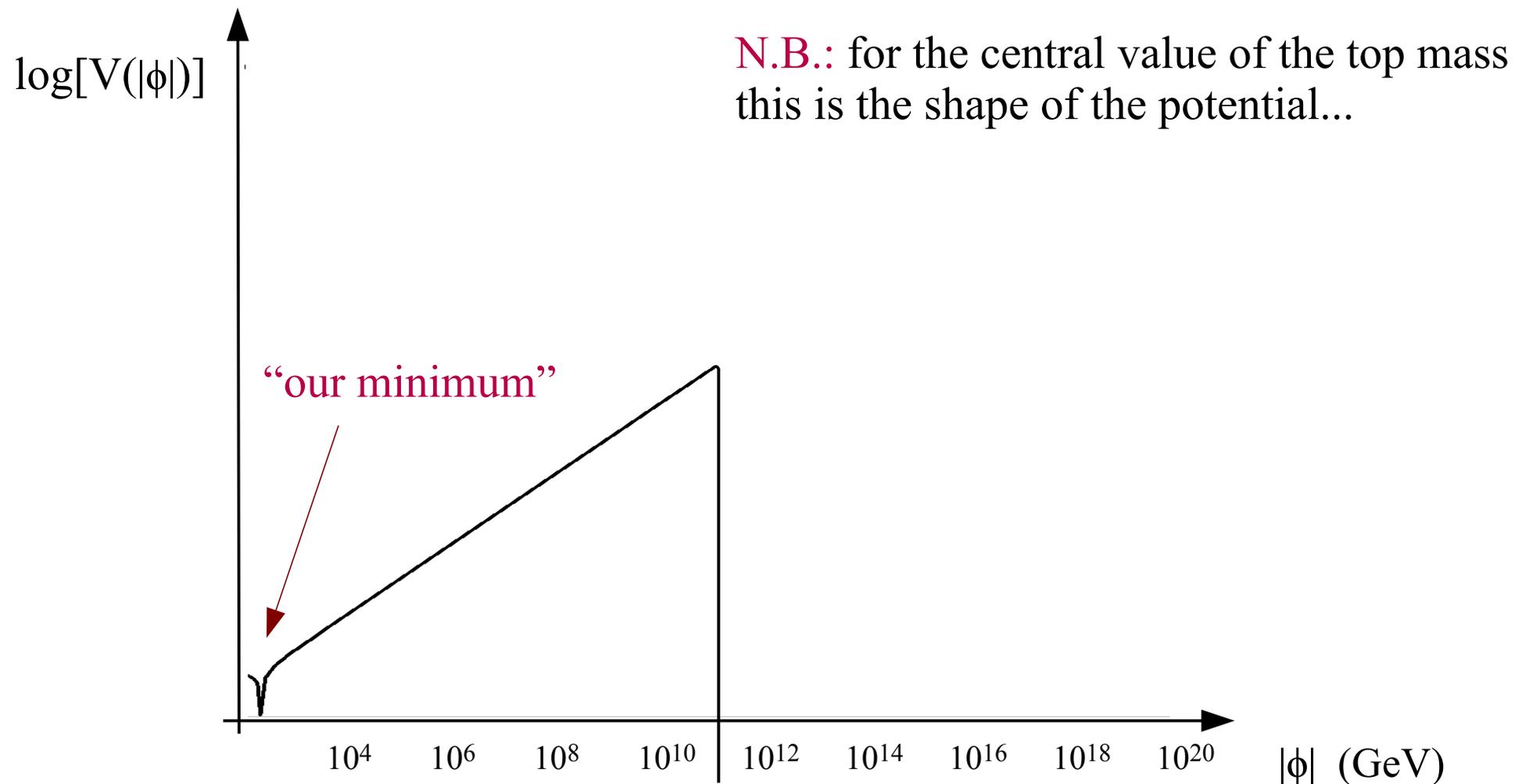
Probably the most attractive feature of having  $\lambda=0$  close to  $M_{\text{pl}}$  (*assuming no new physics below such scale*) is the possibility that the Higgs field has played some role in the early Universe, during inflation.

Bezrukov & Shaposhnikov, '08  
Notari & Masina '11-'12



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## ► Speculations on Planck-scale dynamics

Probably the most attractive feature of having  $\lambda=0$  close to  $M_{\text{pl}}$  (*assuming no new physics below such scale*) is the possibility that the Higgs field has played some role in the early Universe, during inflation.

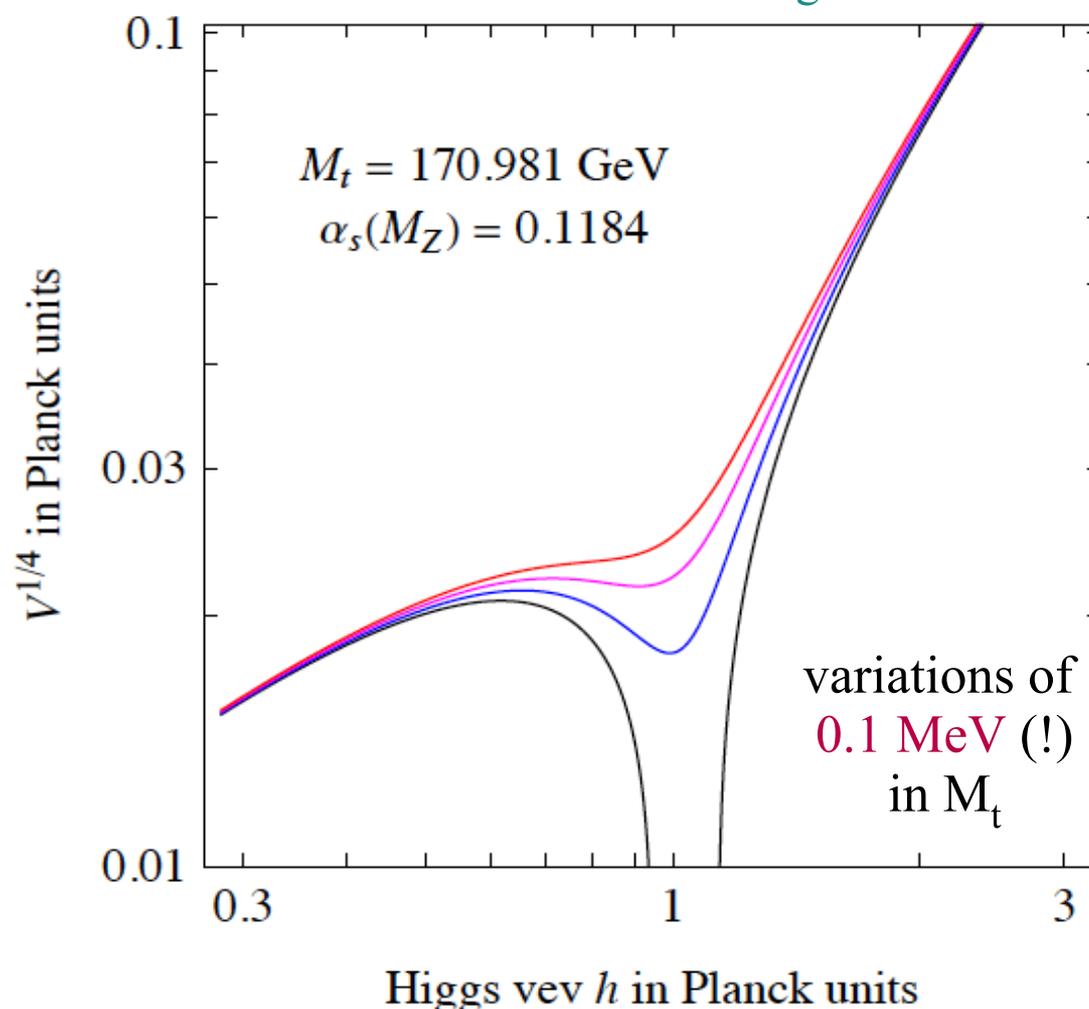
The minimal set-up (SM only) does not work (*field trapped into the new minimum or too large fluctuations*)

But the problem can be solved with non-minimal couplings of the Higgs field to gravity and/or to other fields

Bezrukov & Shaposhnikov, '08  
Notari & Masina '11-'12

The minimality of the scheme is lost, but it remains an intriguing possibility.

Degrassi et al. '12



## ► Conclusions

- A SM-like Higgs with  $m_h \sim 125$  GeV does not allow us to derive model-independent conclusions about the scale of New Physics: the Higgs potential is most likely unstable, but the e.w. vacuum is certainly sufficiently long-lived.

## ► Conclusions

- A SM-like Higgs with  $m_h \sim 125$  GeV does not allow us to derive model-independent conclusions about the scale of New Physics: the Higgs potential is most likely unstable, but the e.w. vacuum is certainly sufficiently long-lived.
- Clear indication of a small, or even vanishing, Higgs self-coupling at high energies: if the SM is only an effective theory, we have to match it into a model where the Higgs
  - is a weakly interacting particle, if the matching occurs close to the e.w. scale [*as indicated by naturalness*]
  - may have a vanishing intrinsic self-coupling (trivial  $\lambda\phi^4$ , with gauge & Yukawa), if the matching occurs above  $\sim 10^8$  GeV
- More precise determinations of both  $m_h$  &  $m_t$  would be very useful, especially in absence of other NP signals, to better investigate the structure of the Higgs potential at high energies