Perturbative QCD calculations for the LHC

Massimiliano Grazzini*

University of Zurich

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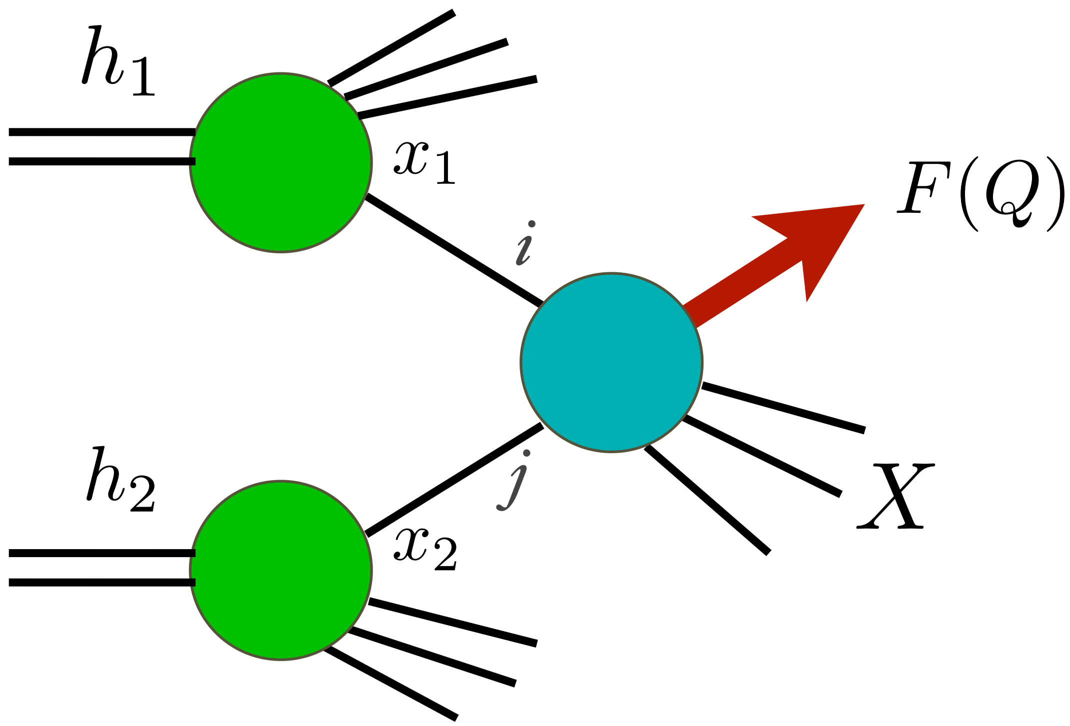
*On leave of absence from INFN, Sezione di Firenze

Outline

- Introduction
- The NLO revolution
- NNLO: techniques and new results
- Beyond NNLO
- Summary and Outlook

Hadron colliders



High-p_T interactions are characterized by the presence of a hard scale Q

They can be controlled through the factorization theorem

The corresponding cross section can be written as

Partonic cross section

$$\sigma(P_1,P_2)=\sum_{i,j}\int dx_1 dx_2 f_{i/h_1}(x_1,\mu_F^2) f_{j/h_2}(x_2,\mu_F^2)\hat{\sigma}_{ij}(p_1,p_2,\alpha_S(\mu_R),Q^2;\mu_F^2,\mu_R^2)$$

Parton distribution functions (PDFs)

renormalization scale

Predictions for hadronic cross section depend on knowledge of both $\hat{\sigma}_{ij}$ and $f_{i,h}(x,\mu_F^2)$

factorization scale

PDFs

Determined by global fits to different data sets

Standard procedure:

- Parametrize at input scale $Q_0 = 1 - 4\ \mathrm{GeV}$

$$xf(x, Q_0^2) = Ax^\alpha (1-x)^\beta (1 + \epsilon\sqrt{x} + \gamma x + ...)$$

- Impose momentum sum rule: $\sum_a \int_0^1 dx\, x f_a(x, Q_0^2) = 1$

- Evolve to desired Q^2 through DGLAP equation

$$Q^2 \frac{\partial f_a(x, Q^2)}{\partial Q^2} = \int_x^1 \frac{dz}{z} P_{ab}(\alpha_{\mathrm{S}}(Q^2), z) f_b(x/z, Q^2)$$

$$P_{ab}(\alpha_S, z) = \frac{\alpha_S}{2\pi} P_{ab}^{(0)}(z) + \left(\frac{\alpha_S}{2\pi}\right)^2 P_{ab}^{(1)}(z) + \left(\frac{\alpha_S}{2\pi}\right)^3 P_{ab}^{(2)}(z) + ...$$

LO (1974) NLO (1980) NNLO (2004: Moch et al.)

- Compute observables and then fit to data to obtain the parameters

PDFs

Main groups: MSTW, CTEQ, NNPDF, ABM, JR, HERAPDF......

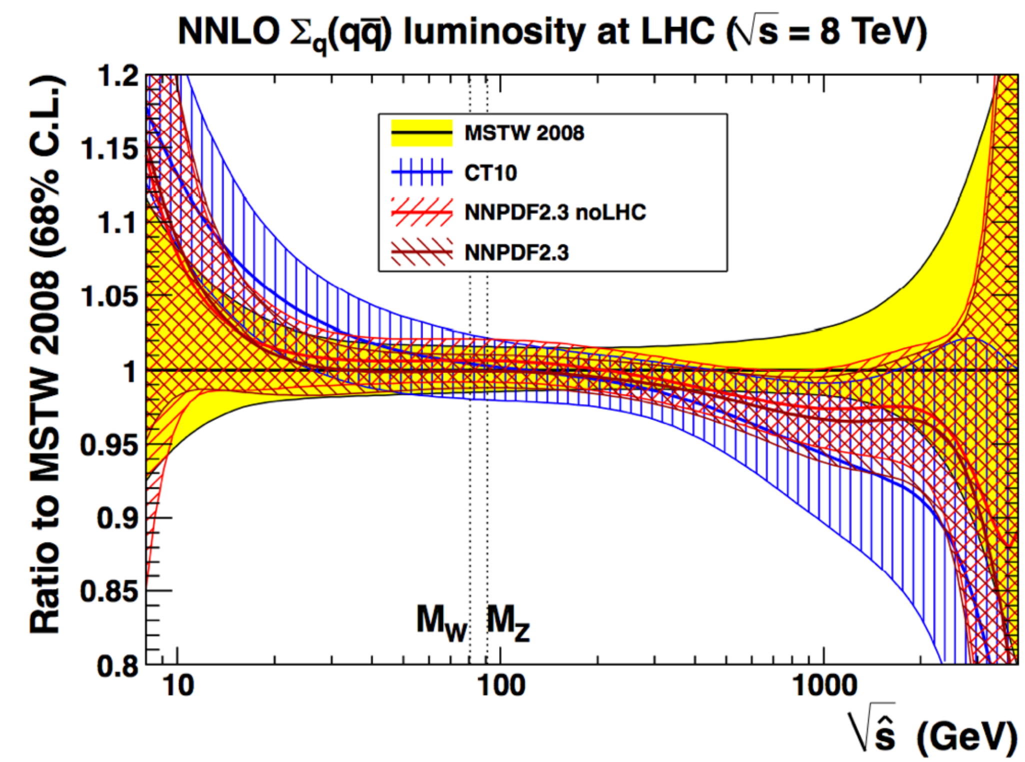
Broad agreement but differences due to:
- choice of data sets
- treatment of errors
- treatment of heavy quarks
- initial parametrization
- theoretical assumptions

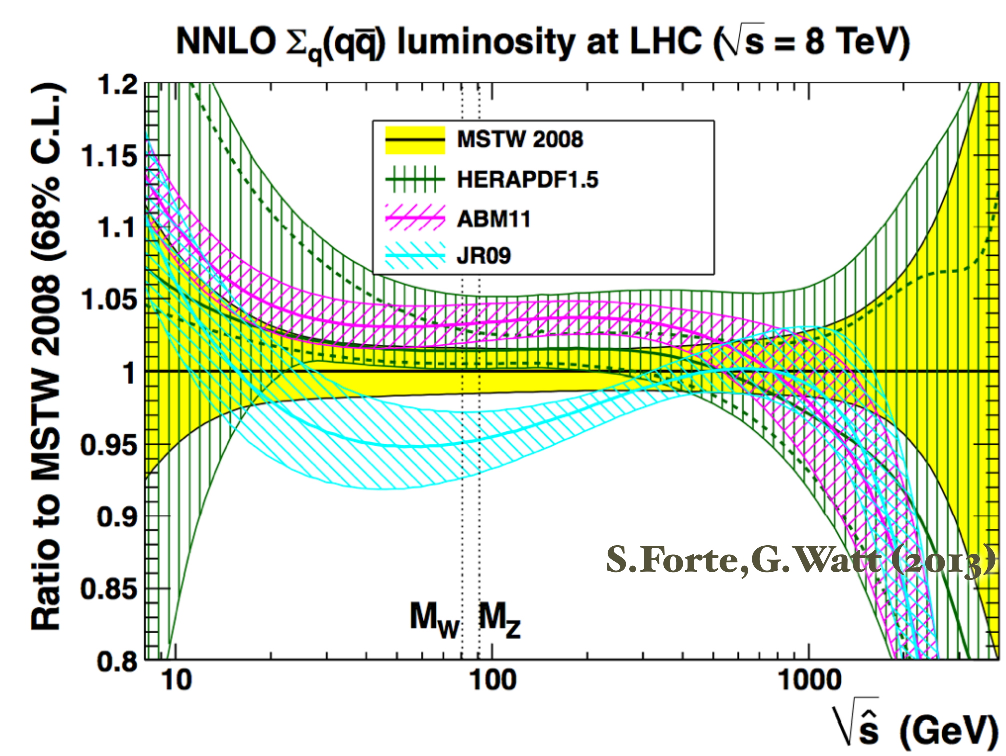
....

All groups provide PDFs with 'errors'

Such errors come from the experimental uncertainties in the data used in the fit

Theoretical assumptions in the way the fit is set up and performed are more difficult to assess





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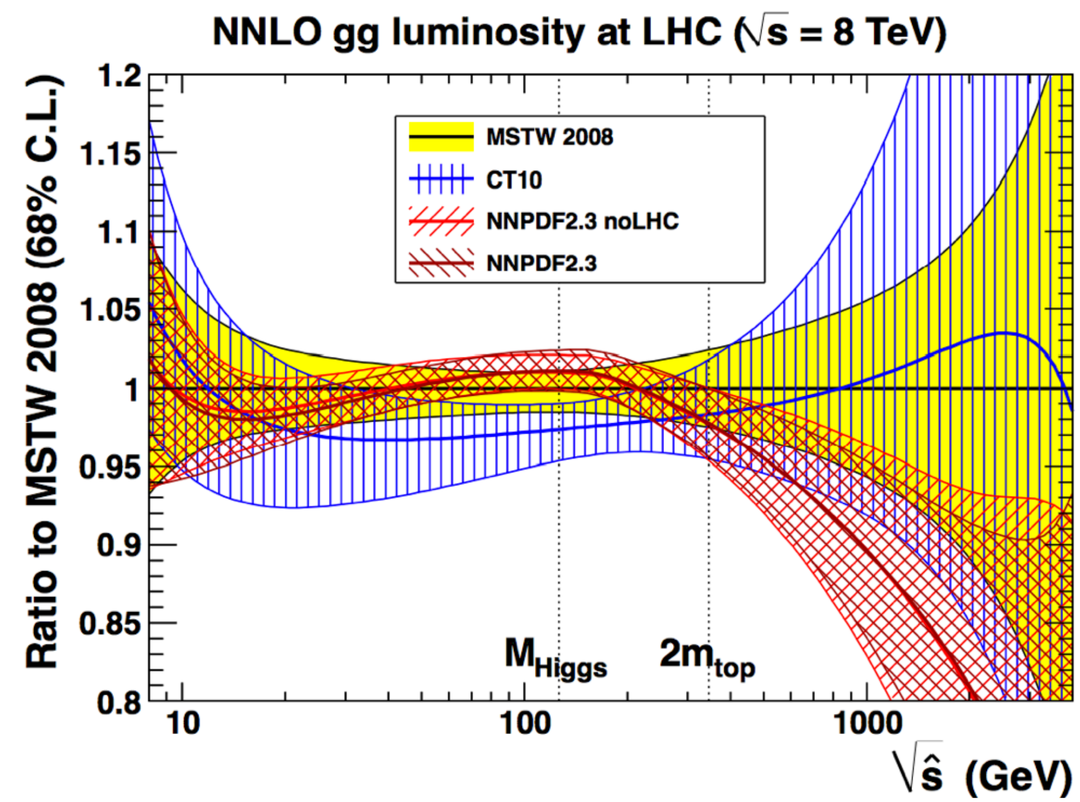
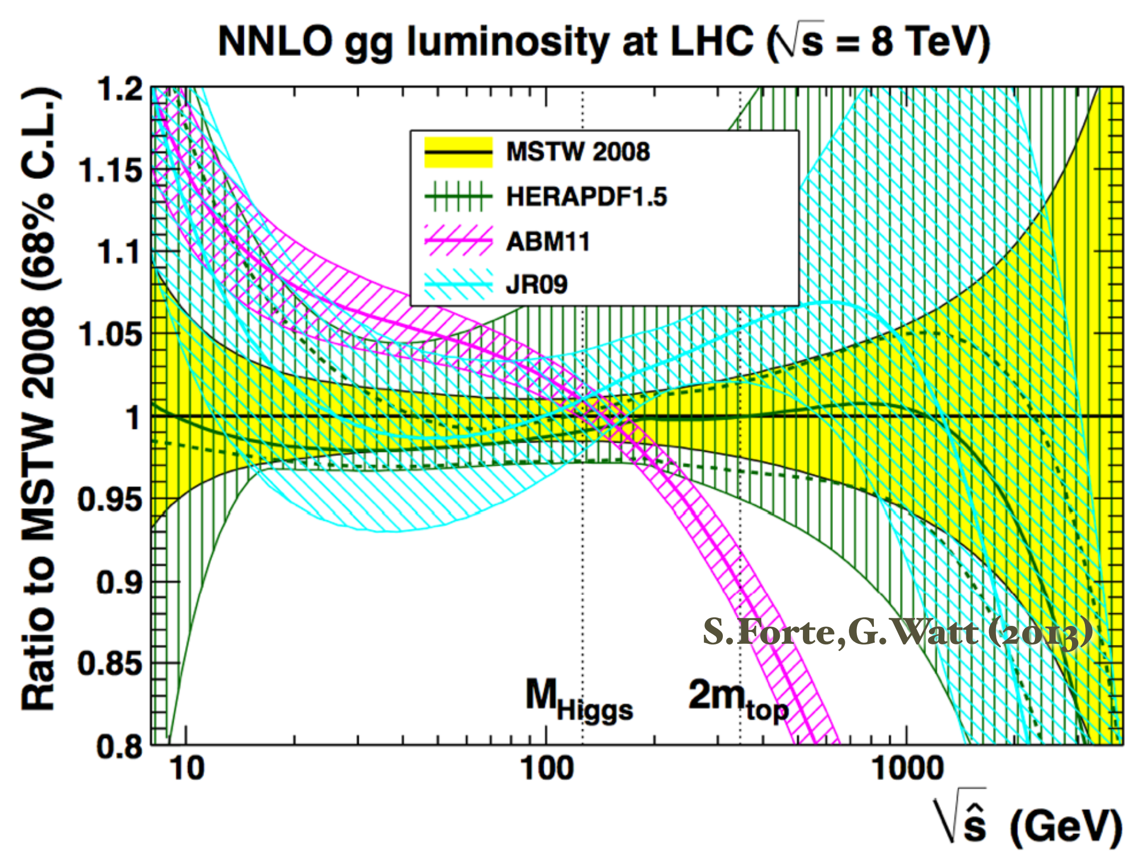




The NNPDF approach

The fitting procedure relies on the choice of the functional form, which introduces a bias in the fit

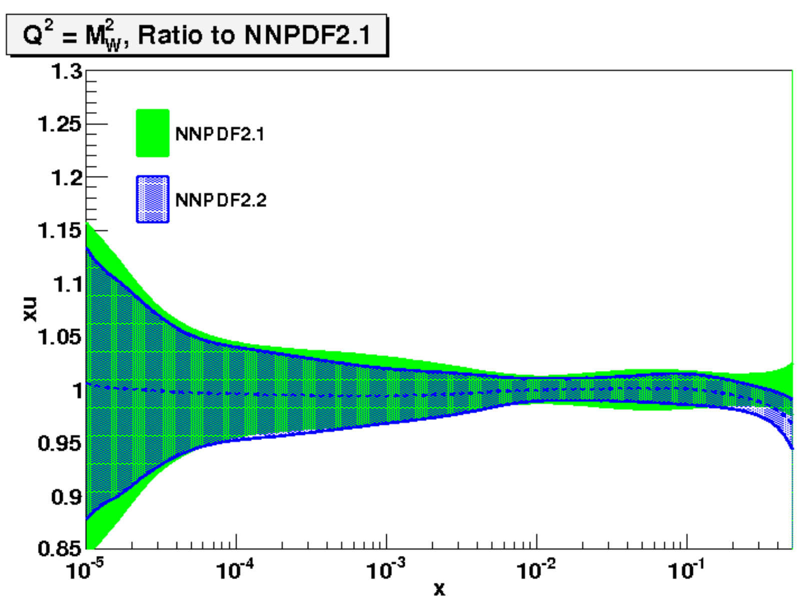
The classical approach to PDF fitting is based on the choice of a (relatively) simple parametrization

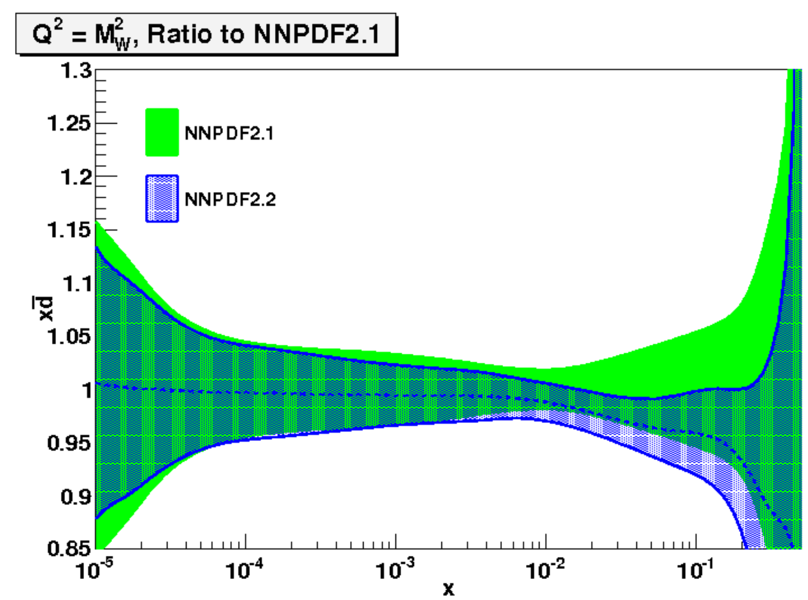
The NNPDF approach generates Monte Carlo replicas of the experimental data

Fit PDFs by using a set of neural networks on each replica

No need to rely on standard error propagation

More realistic error estimate





Effect of inclusion of D0, ATLAS and CMS lepton charge asymmetry data

nnpdf.hepforge.org

Nice reweighting technique to include newly available data

Partonic cross section

The partonic cross section for high-p_T processes can be computed as a series expansion in the QCD coupling α_S

$$\hat{\sigma} = \alpha_S^k \left(\underbrace{\hat{\sigma}^{(0)}}_{\text{LO}} + \frac{\alpha_S}{\pi} \underbrace{\hat{\sigma}^{(1)}}_{\text{NLO}} + \left(\frac{\alpha_S}{\pi}\right)^2 \underbrace{\hat{\sigma}^{(2)}}_{\text{NNLO}} + \ldots \right)$$

Leading order (LO) calculations typically give only the order of magnitude of cross sections and distributions

- the scale of α_S is not defined
- jets ⟺ partons: jet structure starts to appear only beyond LO

To obtain reliable predictions next-to leading order (NLO) is needed

The bottleneck has been for many years the evaluation of the 1 loop correction

This is a field that has seen the most significant advances in the last few years

Partonic cross section: NLO

NLO calculations have been carried out over a period of about 30 years

Main difficulties: one has to consider virtual and real corrections

They are affected by different kinds of singularities:

- UV singularities affect only virtual corrections: removed by renormalization
- IR (soft and collinear) singularities: present in both virtual and real corrections

The observable has to be properly defined according to the KLN theorem: it must be infrared and collinear safe or at least collinear factorizable

Such quantities are finite order by order in perturbation theory

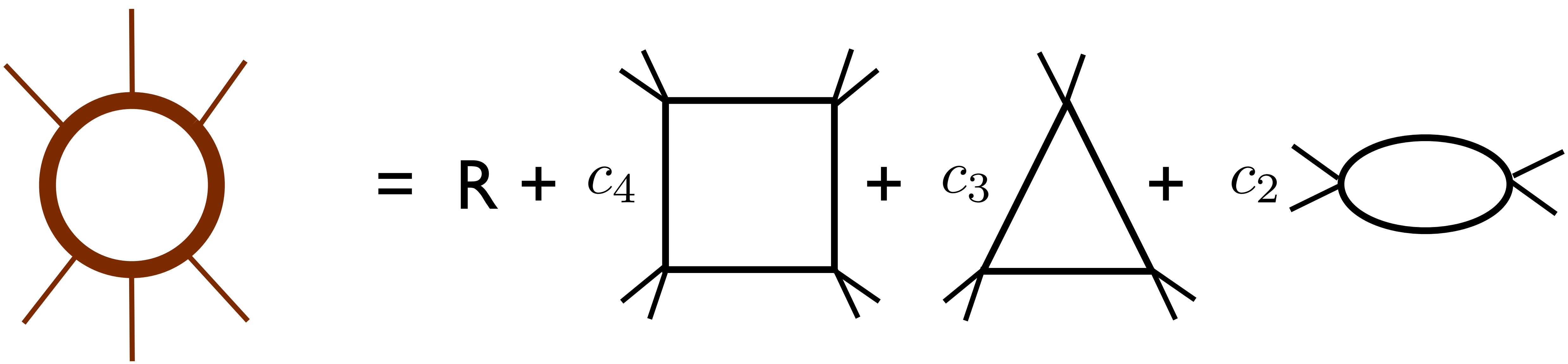
General methods exist to handle and cancel IR singularities

S.Frixione, Z.Kunszt, A. Signer (1995)
S.Catani, M. Seymour (1996)

The NLO revolution

For many years the bottleneck has been the computation of the relevant one-loop amplitudes → Enormous progress in the last years

The traditional approach based on Feynman diagrams is now complemented with new powerful methods based on recursion relations and unitarity



General one-loop amplitude expressed as a sum of known boxes, triangles and bubble integrals plus a remainder term

Coefficient of these integrals can be computed by taking suitable multiple cuts

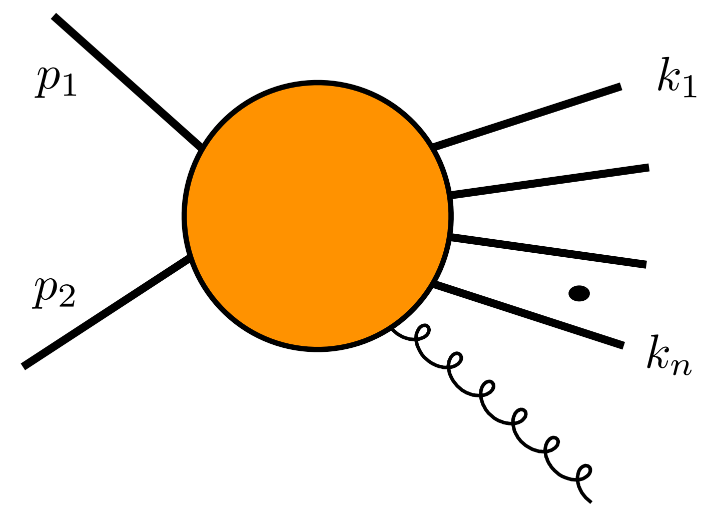
For example c_4 → Simple product of four tree-level amplitudes evaluated at complex momenta

R.Britto, F.Cachazo, B.Feng (2004)

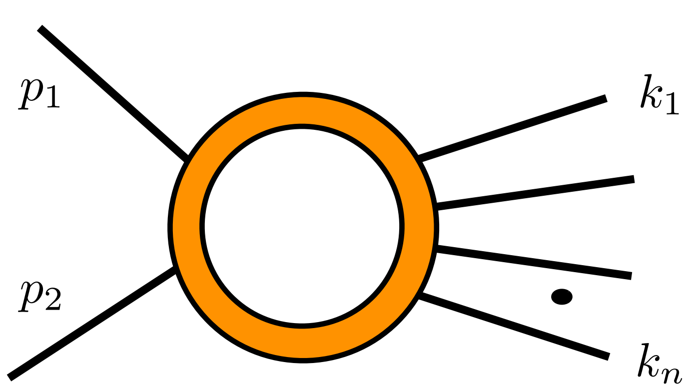
The NLO revolution

Automation of NLO corrections

Combine available methods to compute real corrections.....



with most efficient techniques for virtual corrections:



- "traditional" methods to evaluate tensor and scalar integrals

A.Denner, S.Dittmaier (2006,2011)

- fully numerical evaluation based on reduction at the integrand level

G.Ossola, C.Papadopoulos, R.Pittau (2007)
K.Ellis, W.Giele, Z.Kunszt (2007)

The NLO revolution

New general tools have been developed to compute one-loop amplitudes

GoSam

OpenLoops

Helac-NLO

NJet

BlackHat

MadGraph5_aMC@NLO

In the following I give a selection of the most recent interesting results

H+multijets

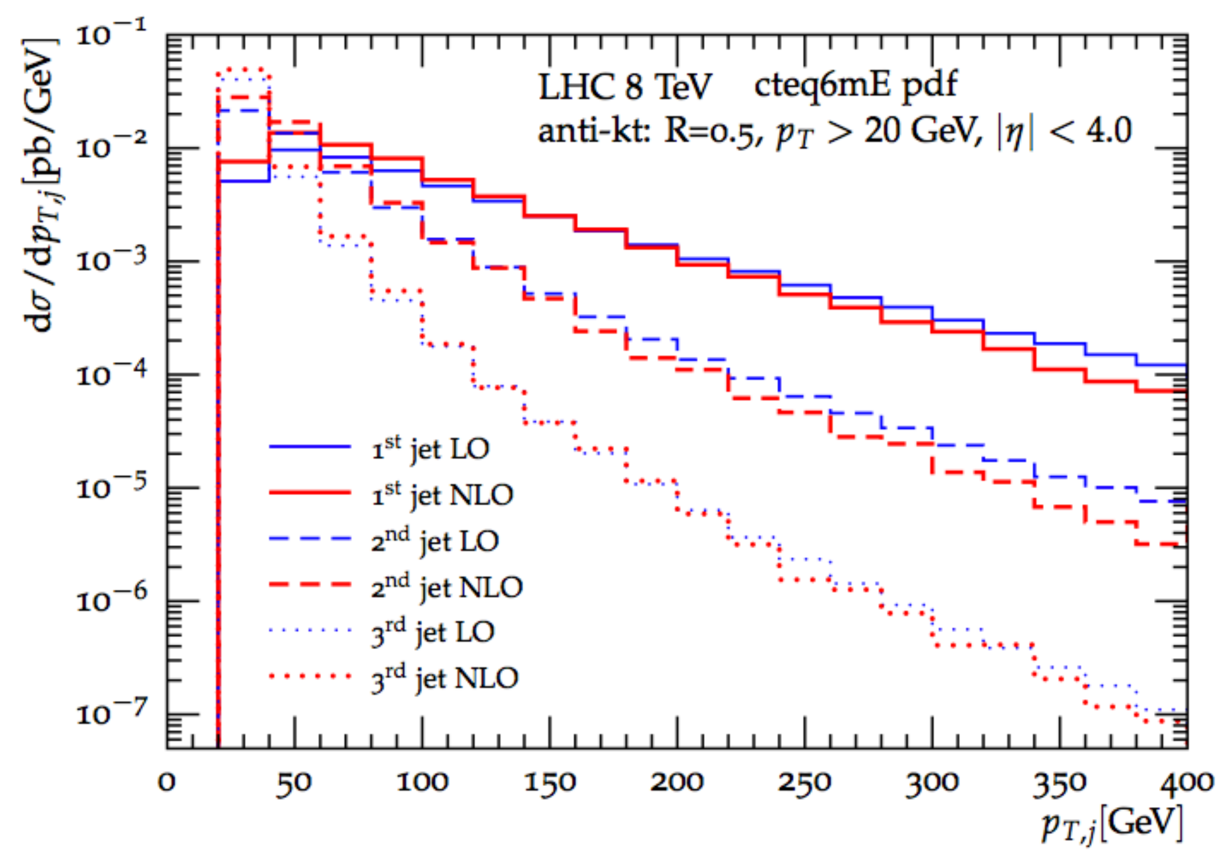
G.Cullen, H.van Deurzen, N.Greiner, G.Luisoni, P.Mastrolia, E.Mirabella, G.Ossola, T.Peraro, F.Tramontano (2013)

gluon fusion is the dominant Higgs production channel

H+jets acts as a contamination to VBF

H+up to 3 jets computed at NLO

Figure 1. Sample hexagon diagrams which enter in the six-parton one-loop amplitudes for $q\bar{q} \to Hq\bar{q}g$ and $gg \to Hggg$. The dot represents the effective ggH vertex.



Born+virtual with GoSam +SHERPA

Real+dipoles with Madgraph+MadDipole

Significant change in shape in the jet pt spectra

pp→5 jets

S.Badger, B.Biedermann, P.Uwer, V.Yundin (2013)

Multijet production particularly important ➡ NLO needed

- 2 jet production

K.Ellis, Z.Kunszt, D.Soper (1992)
W.Giele,N.Glover,D.Kosower (1993)

- 3 jet production

Z.Trocsanyi (1996);W.Giele, R.Kilgore (1997) (gluon only)
Z.Nagy (2002)

- 4 jet production

Z.Bern et al. (2011)
S.Badger et al (2012)

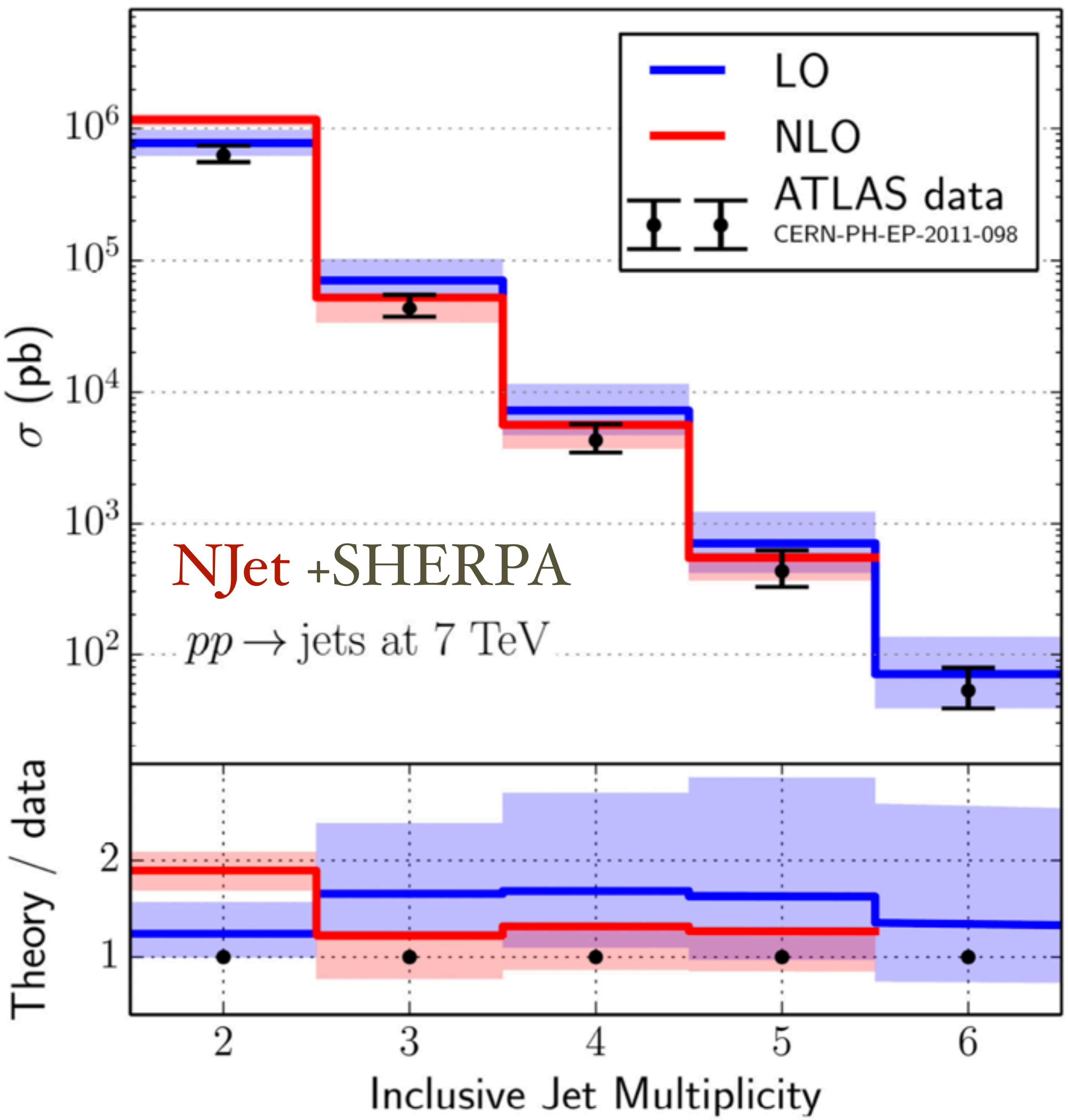
- 5 jet production

S.Badger et al (2013)

10 years to go from 2 to 3 and to 3 to 4: 1 year only to go from 4 to 5 jets !

pp→5 jets

B.Biederman (ZPW 2014)



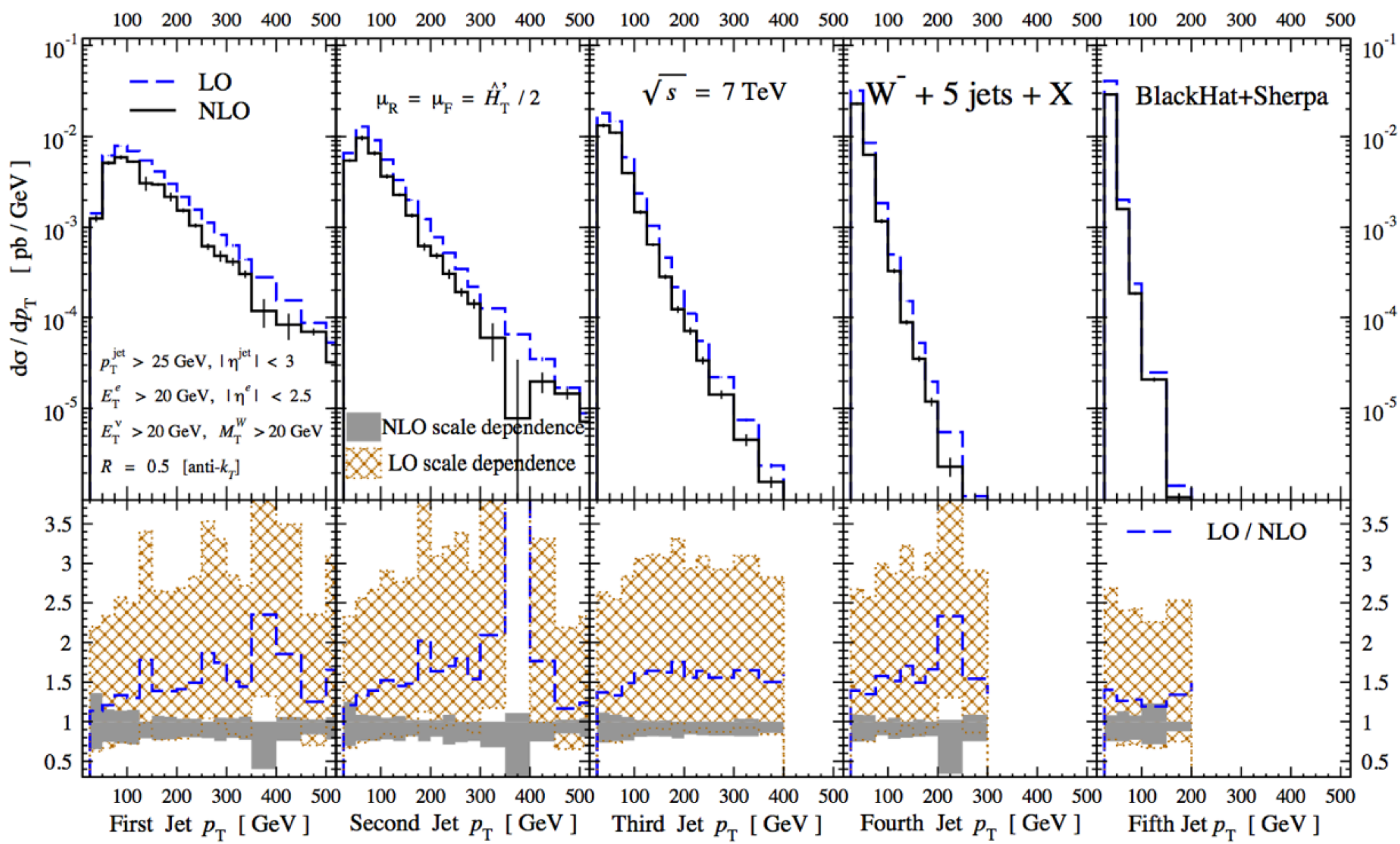
$\sigma_2^{\text{7TeV-NLO}}$	$1175(3)^{1046(+)}_{1295(-)}$ nb
$\sigma_3^{\text{7TeV-NLO}}$	$52.5(0.3)^{54.4(+)}_{33.2(-)}$ nb
$\sigma_4^{\text{7TeV-NLO}}$	$5.65(0.07)^{5.36(+)}_{3.72(-)}$ nb
$\sigma_5^{\text{7TeV-NLO}}$	$0.544(0.016)^{0.479(+)}_{0.367(-)}$ nb
$\sigma_6^{\text{7TeV-LO}}$	$0.0496(0.0005)^{0.0263(+)}_{0.0992(-)}$ nb

- ratio between theory and data about 1.2 – 1.3
- reduction of the cross section from LO -> NLO (except for 2-jets)

W+5 jets

Z.Bern et al. (2013)

Blackhat+Sherpa



ttbar and Wt at NLO

OpenLoops+Sherpa

F.Cascioli, S.Kallweit,
S.Pozzorini,P.Maierhofer (2013);
see also R.Frederix (2013)

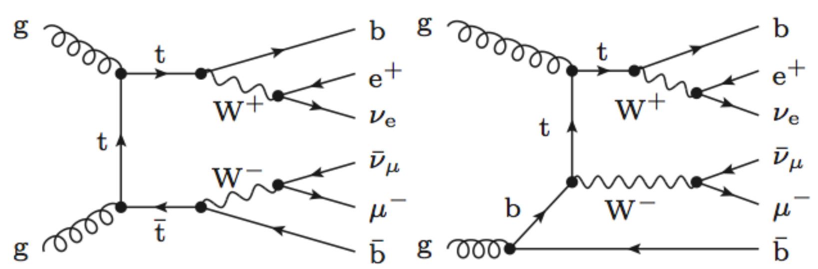


Fig. 1 Representative $t\bar{t}$-like (left) and Wt-like (right) tree diagrams.

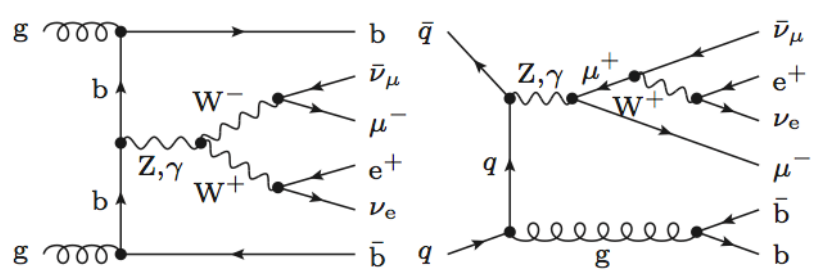
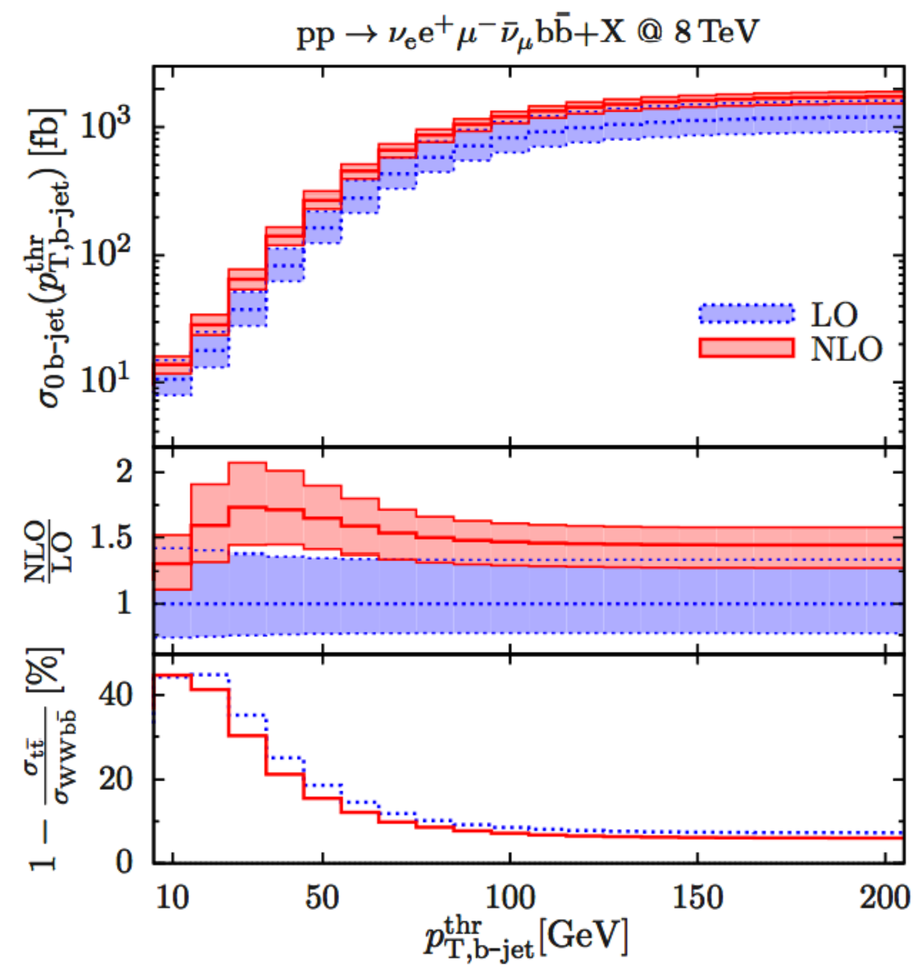


Fig. 2 Representative tree topologies without top resonances and with two (left) or only one (right) resonant W-boson.

The separation of the ttbar and Wt processes is quite subtle

Use of 4F and massive b-quarks allows a unified description of the two processes



It allows a consistent study of the 0 and 1 jet bin relevant as a background to Higgs production

Finite width effects in the 0-jet bin grow to up to 40% at low p_T threshold

MadGraph5_aMC@NLO

J.Alwall, R.Frederix, S.Frixione,
V.Hirschi,F.Maltoni,
O.Mattelaer,R.Pittau,T.Stelzer,P.Torrielli,
M.Zaro (to appear)

MadGraph and aMC@NLO replaced by MadGraph_aMC@NLO

A single framework, which inherits the features of the two codes and aims at computing NLO corrections for any user defined theory and matching to parton shower within the MC@NLO framework

- very simple usage
- no external dependences
- automatic evaluation of scale and PDFs uncertainties
- can compute all processes that have up to $2 \to 4$ at Born level

MadGraph5_aMC@NLO

	Process	Syntax	Cross section (pb)					
Heavy quarks+vector bosons			LO 13 TeV			NLO 13 TeV		
e.1	$pp \to W^{\pm} b\bar{b}$	p p > wpm b b~	$3.074 \pm 0.002 \cdot 10^{2}$	$^{+42.3\%}_{-29.2\%}$	$^{+2.0\%}_{-1.6\%}$	$8.162 \pm 0.034 \cdot 10^{2}$	$^{+29.8\%}_{-23.6\%}$	$^{+1.5\%}_{-1.2\%}$
e.2	$pp \to Z b\bar{b}$	p p > z b b~	$6.993 \pm 0.003 \cdot 10^{2}$	$^{+33.5\%}_{-24.4\%}$	$^{+1.0\%}_{-1.4\%}$	$1.235 \pm 0.004 \cdot 10^{3}$	$^{+19.9\%}_{-17.4\%}$	$^{+1.0\%}_{-1.4\%}$
e.3	$pp \to \gamma b\bar{b}$	p p > a b b~	$1.731 \pm 0.001 \cdot 10^{3}$	$^{+51.9\%}_{-34.8\%}$	$^{+1.6\%}_{-2.1\%}$	$4.171 \pm 0.015 \cdot 10^{3}$	$^{+33.7\%}_{-27.1\%}$	$^{+1.4\%}_{-1.9\%}$
e.4*	$pp \to W^{\pm} b\bar{b} j$	p p > wpm b b~ j	$1.861 \pm 0.003 \cdot 10^{2}$	$^{+42.5\%}_{-27.7\%}$	$^{+0.7\%}_{-0.7\%}$	$3.957 \pm 0.013 \cdot 10^{2}$	$^{+27.0\%}_{-21.0\%}$	$^{+0.7\%}_{-0.6\%}$
e.5*	$pp \to Z b\bar{b} j$	p p > z b b~ j	$1.604 \pm 0.001 \cdot 10^{2}$	$^{+42.4\%}_{-27.6\%}$	$^{+0.9\%}_{-1.1\%}$	$2.805 \pm 0.009 \cdot 10^{2}$	$^{+21.0\%}_{-17.6\%}$	$^{+0.8\%}_{-1.0\%}$
e.6*	$pp \to \gamma b\bar{b} j$	p p > a b b~ j	$7.812 \pm 0.017 \cdot 10^{2}$	$^{+51.2\%}_{-32.0\%}$	$^{+1.0\%}_{-1.5\%}$	$1.233 \pm 0.004 \cdot 10^{3}$	$^{+18.9\%}_{-19.9\%}$	$^{+1.0\%}_{-1.5\%}$
e.7	$pp \to t\bar{t} W^{\pm}$	p p > t t~ wpm	$3.777 \pm 0.003 \cdot 10^{-1}$	$^{+23.9\%}_{-18.0\%}$	$^{+2.1\%}_{-1.6\%}$	$5.662 \pm 0.021 \cdot 10^{-1}$	$^{+11.2\%}_{-10.6\%}$	$^{+1.7\%}_{-1.3\%}$
e.8	$pp \to t\bar{t} Z$	p p > t t~ z	$5.273 \pm 0.004 \cdot 10^{-1}$	$^{+30.5\%}_{-21.8\%}$	$^{+1.8\%}_{-2.1\%}$	$7.598 \pm 0.026 \cdot 10^{-1}$	$^{+9.7\%}_{-11.1\%}$	$^{+1.9\%}_{-2.2\%}$
e.9	$pp \to t\bar{t} \gamma$	p p > t t~ a	$1.204 \pm 0.001 \cdot 10^{0}$	$^{+29.6\%}_{-21.3\%}$	$^{+1.6\%}_{-1.8\%}$	$1.744 \pm 0.005 \cdot 10^{0}$	$^{+9.8\%}_{-11.0\%}$	$^{+1.7\%}_{-2.0\%}$
e.10*	$pp \to t\bar{t} W^{\pm} j$	p p > t t~ wpm j	$2.352 \pm 0.002 \cdot 10^{-1}$	$^{+40.9\%}_{-27.1\%}$	$^{+1.3\%}_{-1.0\%}$	$3.404 \pm 0.011 \cdot 10^{-1}$	$^{+11.2\%}_{-14.0\%}$	$^{+1.2\%}_{-0.9\%}$
e.11*	$pp \to t\bar{t} Z j$	p p > t t~ z j	$3.953 \pm 0.004 \cdot 10^{-1}$	$^{+46.2\%}_{-29.5\%}$	$^{+2.7\%}_{-3.0\%}$	$5.074 \pm 0.016 \cdot 10^{-1}$	$^{+7.0\%}_{-12.3\%}$	$^{+2.5\%}_{-2.9\%}$
e.12*	$pp \to t\bar{t} \gamma j$	p p > t t~ a j	$8.726 \pm 0.010 \cdot 10^{-1}$	$^{+45.4\%}_{-29.1\%}$	$^{+2.3\%}_{-2.6\%}$	$1.135 \pm 0.004 \cdot 10^{0}$	$^{+7.5\%}_{-12.2\%}$	$^{+2.2\%}_{-2.5\%}$
e.13*	$pp \to t\bar{t} W^{-} W^{+}$ (4f)	p p > t t~ w+ w-	$6.675 \pm 0.006 \cdot 10^{-3}$	$^{+30.9\%}_{-21.9\%}$	$^{+2.1\%}_{-2.0\%}$	$9.904 \pm 0.026 \cdot 10^{-3}$	$^{+10.9\%}_{-11.8\%}$	$^{+2.1\%}_{-2.1\%}$
e.14*	$pp \to t\bar{t} W^{\pm} Z$	p p > t t~ wpm z	$2.404 \pm 0.002 \cdot 10^{-3}$	$^{+26.6\%}_{-19.6\%}$	$^{+2.5\%}_{-1.8\%}$	$3.525 \pm 0.010 \cdot 10^{-3}$	$^{+10.6\%}_{-10.8\%}$	$^{+2.3\%}_{-1.6\%}$
e.15*	$pp \to t\bar{t} W^{\pm} \gamma$	p p > t t~ wpm a	$2.718 \pm 0.003 \cdot 10^{-3}$	$^{+25.4\%}_{-18.9\%}$	$^{+2.3\%}_{-1.8\%}$	$3.927 \pm 0.013 \cdot 10^{-3}$	$^{+10.3\%}_{-10.4\%}$	$^{+2.0\%}_{-1.5\%}$
e.16*	$pp \to t\bar{t} Z Z$	p p > t t~ z z	$1.349 \pm 0.014 \cdot 10^{-3}$	$^{+29.3\%}_{-21.1\%}$	$^{+1.7\%}_{-1.5\%}$	$1.840 \pm 0.007 \cdot 10^{-3}$	$^{+7.9\%}_{-9.9\%}$	$^{+1.7\%}_{-1.5\%}$
e.17*	$pp \to t\bar{t} Z \gamma$	p p > t t~ z a	$2.548 \pm 0.003 \cdot 10^{-3}$	$^{+30.1\%}_{-21.5\%}$	$^{+1.7\%}_{-1.6\%}$	$3.656 \pm 0.012 \cdot 10^{-3}$	$^{+9.7\%}_{-11.0\%}$	$^{+1.8\%}_{-1.9\%}$
e.18*	$pp \to t\bar{t} \gamma\gamma$	p p > t t~ a a	$3.272 \pm 0.006 \cdot 10^{-3}$	$^{+28.4\%}_{-20.6\%}$	$^{+1.3\%}_{-1.1\%}$	$4.402 \pm 0.015 \cdot 10^{-3}$	$^{+7.8\%}_{-9.7\%}$	$^{+1.4\%}_{-1.4\%}$

courtesy of S.Frixione

Do we need NNLO ?

Well, we can say that NNLO predictions are useful at least in the following cases:

- For those processes whose NLO corrections are comparable to the LO contributions

  ➡ Higgs production at hadron colliders

- For those benchmark processes measured with high experimental accuracy

  - α_S measurements from e^+e^- event shape variables

  ➡ - W and Z production

  - heavy quark hadroproduction

- Processes relevant to determine PDFs or that can hide new physics signal

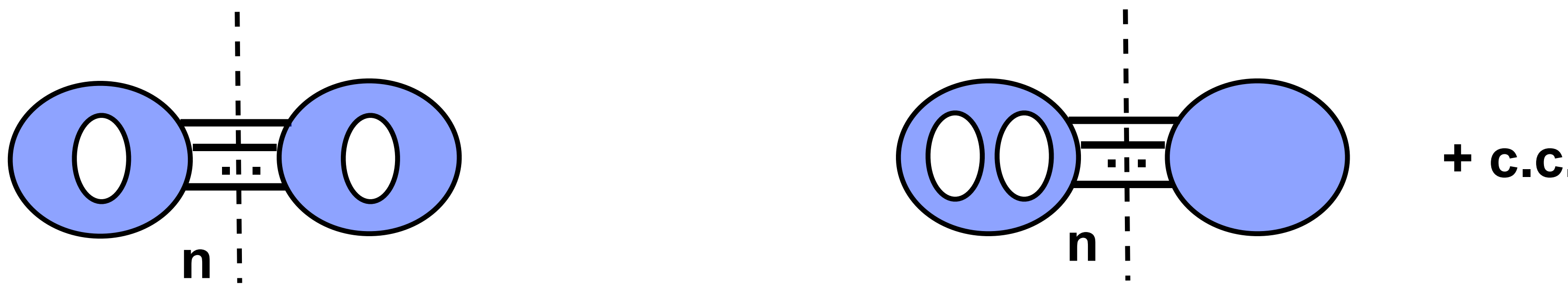
  ➡ high E_T jet hadroproduction

- For important background processes (eg. vector boson pair production)

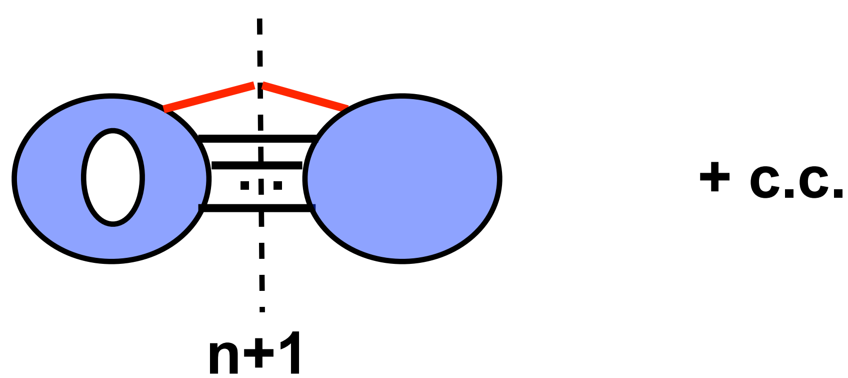
Ingredients of NNLO calculations

Let us assume that the process involves n partons at LO ➡ we need:

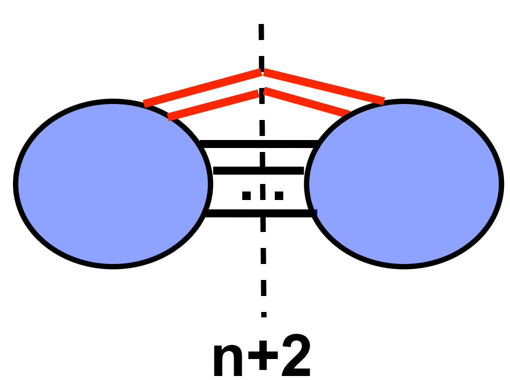
- Double virtual contribution with n resolved partons



- Real-virtual contribution with 1 unresolved parton



- Double-real contribution with 2 unresolved partons



Ingredients of NNLO calculations

Same difficulty appearing at NLO: these ingredients are affected by different kinds of singularities

- UV sing. affect only virtual corrections ➡ removed by renormalization

- IR singularities present in all the three contributions

IR singularities cancel out in IR safe quantities

IR safe quantities are those that are independent of the presence of arbitrarily soft partons and independent on the individual momenta of a bunch of collinear partons

Unfortunately the pattern of the cancellation of IR singularities is much more involved than at NLO !

(Fully) inclusive processes

In the case of one-scale quantities double real, real virtual and double virtual contributions can be analytically computed and the singularities explicitly cancelled

- DIS structure functions

E. Zijlstra, W. Van Neerven (1992)

- Single hadron production

P.J.Rijken, W.L.Van Neerven (1997)
A.Mitov, S.Moch (2006)

- DY lepton pair production

R.Hamberg, W.Van Neerven, T.Matsuura (1991)

- Higgs boson production

R.Harlander, W.B. Kilgore (2002)

C. Anastasiou, K. Melnikov (2002)
V. Ravindran, J. Smith, W.L.Van Neerven (2003)

....

+

Vector boson rapidity distribution

➡ modelling the phase space constraint with an effective "propagator"

C.Anastasiou, K.Melnikov, L.Dixon,F.Petriello (2003)

But real experiments have finite acceptances !

What about more exclusive processes?

They are essential to avoid extrapolations from the fiducial region where measurements are carried out

Many of the ingredients for NNLO corrections available since long time

Example: pp→ 2 jets

- Tree level six-parton amplitudes
- One-loop 5-parton amplitudes

Z.Bern,L.Dixon,D.Kosower (1993)

- Two-loop 4-parton amplitudes

C.Anastasiou, N.Glover,C.Oleari,M.Tejeda-Yeomans (2001)
N.Glover,C.Oleari,M.Tejeda-Yeomans (2001)

Despite this fact until recently the computation of the corresponding NNLO corrections could not be performed

The IR singularity structure of the double-real, real virtual and double virtual contributions has now been understood

S. Catani (1998); J.Campbell, N. Glover (1998)
S. Catani, MG (1999); Z.Bern, V. Del Duca, W. Kilgore, C. Schmidt (1999), D. Kosower, P. Uwer (1999), S. Catani, MG (2000)
G.Sterman, M. Tejeda-Yeomans (2002)

However the organization of the calculation into finite pieces that can be integrated numerically is still a formidable task

Two main strategies have been followed:

- **Sector decomposition**
- **Subtraction method**

Sector decomposition

K. Hepp (1966)
T. Binoth, G.Heinrich (2000,2004)
C.Anastasiou, K.Melnikov, F.Petriello (2004)

Sector decomposition as implemented by Anastasiou and collaborators works by dividing the integration region into sectors each containing a single singularity that can be made explicit by expansion into distributions

➡ This leads to a fully automated procedure by which the coefficients of the poles as well as finite terms can be computed numerically

The method has been successfully applied to a number of important fully exclusive NNLO computations

- Higgs and vector boson production in hadron collisions

C.Anastasiou, K.Melnikov, F.Petriello (2005)
K.Melnikov, F.Petriello (2006)

- NNLO QED computation of muon decay

C.Anastasiou, K.Melnikov, F.Petriello (2005)

- Semileptonic decay $b \longrightarrow c\, l\, \bar{\nu}_l$

K.Melnikov (2008)

Subtraction method

R.K. Ellis, D.A.Ross, A.E.Terrano (1981)

S.Frixione, Z.Kunszt, A. Signer (1995)
S.Catani, M. Seymour (1996)

$$d\sigma = \int_{n+1} r d\Phi_{n+1} + \int_n v d\Phi_n$$

$$d\sigma = \int_{n+1} \left(r d\Phi_{n+1} - \tilde{r} d\tilde{\Phi}_{n+1} \right) + \int_{n+1} \tilde{r} d\tilde{\Phi}_{n+1} + \int_n v d\Phi_n$$

Add and subtract a local counterterm with the same singularity structure of the real contribution that can be integrated analytically over the phase space of the unresolved parton

How to extend this procedure to NNLO ?

This absolutely non trivial issue has attracted quite an amount of work

D. Kosower (1998,2003,2005)

S. Weinzierl (2003)

S. Frixione, MG (2004)

A. & T. Gehrmann, N. Glover (2005)

G, Somogyi, Z. Trocsanyi, V. Del Duca (2005, 2007)

Goal ➡ Formulate a general scheme that can be possibly applied to any process

Antenna subtraction

A&T.Gehrmann, N.Glover (2005)

Counterterms constructed from antennae extracted from physical matrix elements

Successfully applied to e+ e- → 3 jets

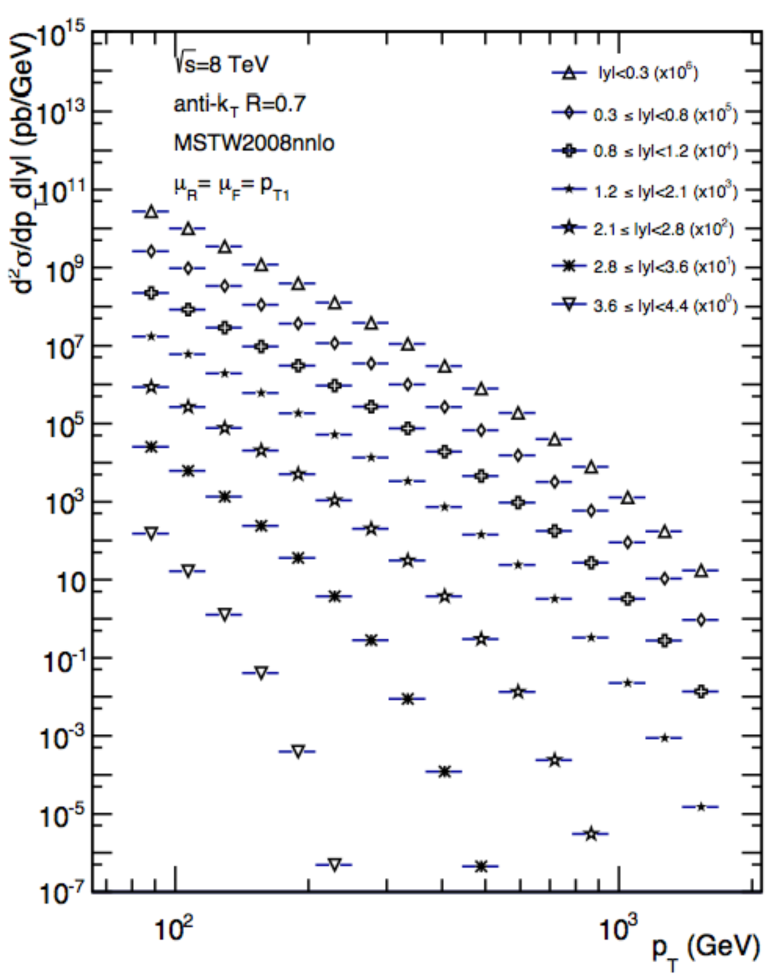
A. & T. Gehrmann, N. Glover, G. Heinrich (2007)
S.Weinzierl (2008)

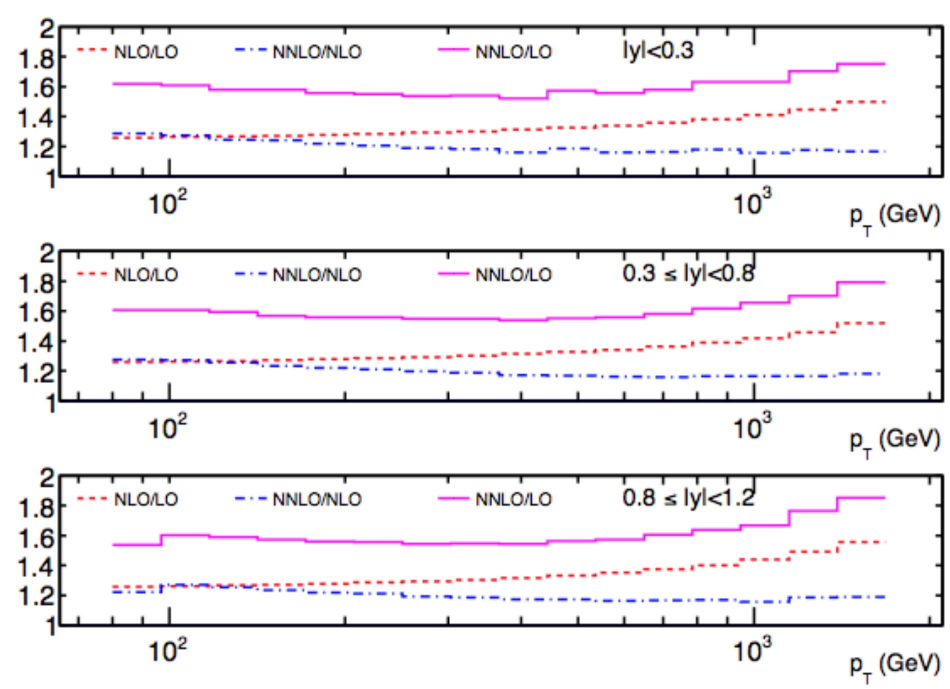
Now extended to hadronic collisions

R. Boughezal, A.Gehrmann, M.Ritzmann (2010)
T.Gehrmann et al. (2010)
N.Glover, J.Pires (2010)
A.Gehrmann, G.Abelof (2011)
T.Gehrmann, P.F.Monni (2011)

First results for pp→ 2 jets (gluons only)

A. & T. Gehrmann, N. Glover, J.Currie,J.Pires (2013,2014)





Two new developments

- Improvement on sector decomposition proposed to reduce the number of integrals based on non-linear transformations

C.Anastasiou, A. Lazopoulos, F. Herzog (2010)

Aims at reducing large number of terms obtained in sector decomposition

First application: computation of fully exclusive $H \to b\bar{b}$ decay at NNLO

C.Anastasiou, A. Lazopoulos, F. Herzog (2011)

- Joint use of subtraction and sector decomposition

M.Czakon (2010,2011)
R.Boughezal, K.Melnikov, F.Petriello (2011)

Separate singularities as Frixione-Kunszt-Signer and make singular contributions explicit

Now use known universal structure of the singular behavior of the amplitude

- Calculation of the total cross section for $t\bar{t}$ production completed

P.Bernreuther, M.Czakon,A.Mitov (2012)
M.Czakon, A.Mitov; M.Czakon, P.Fielder, A.Mitov (2013)

- First results for H+jet(s)

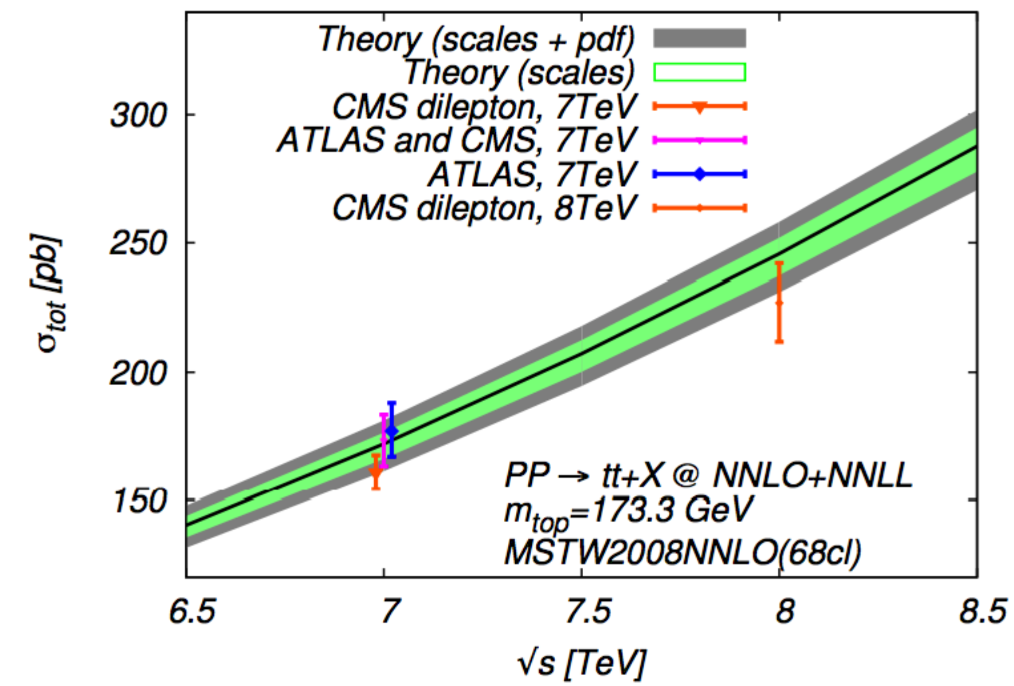
R.Boughezal, K.Melnikov, F.Petriello (2011)

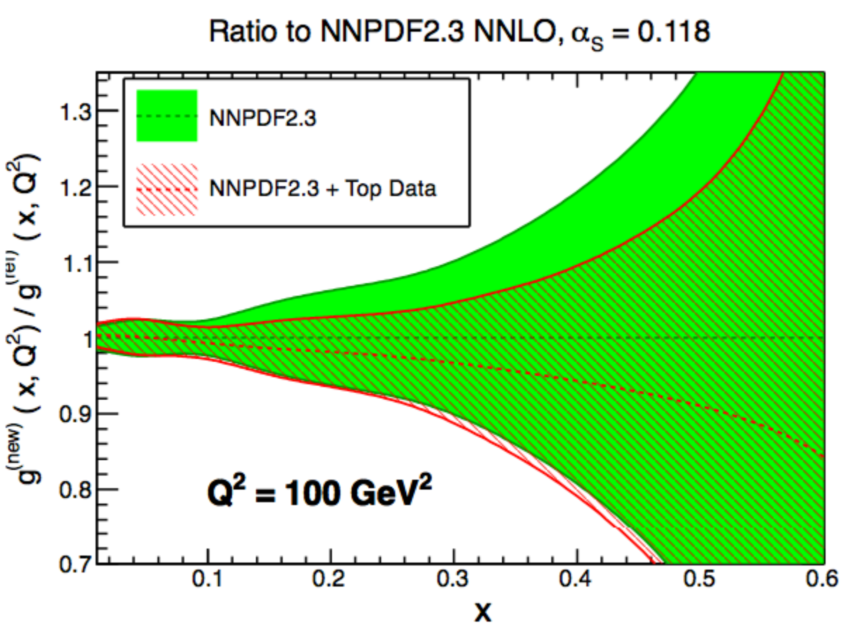
ttbar@NNLO

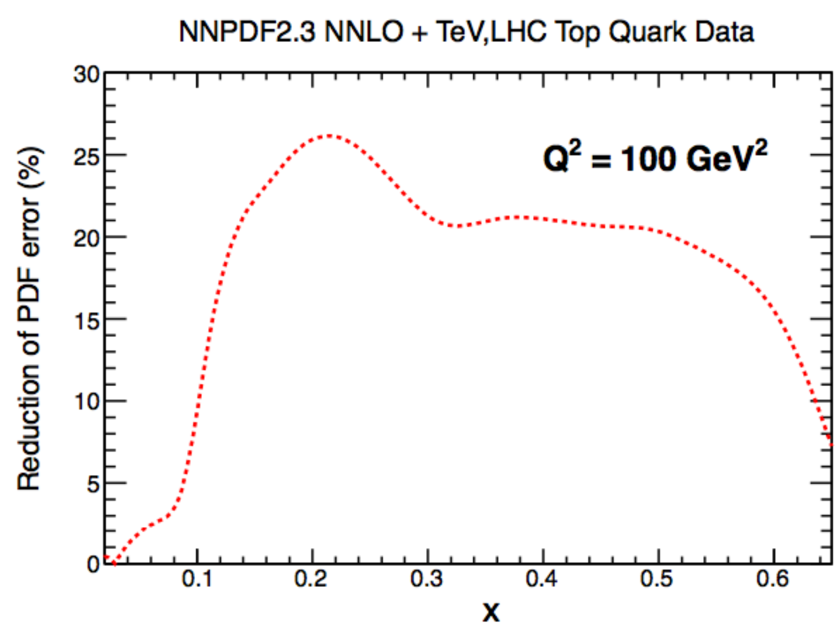
New NNLO calculation supplemented with soft-gluon resummation provides a rather accurate prediction (residual scale uncertainties at the few % level)

P.Bernreuther, M.Czakon,A.Mitov (2012)
M.Czakon, A.Mitov; M.Czakon, P.Fielder, A.Mitov (2013)

Collider	$\sigma_{\rm tot}$ [pb]	scales [pb]	pdf [pb]
Tevatron	7.164	$^{+0.110(1.5\%)}_{-0.200(2.8\%)}$	$^{+0.169(2.4\%)}_{-0.122(1.7\%)}$
LHC 7 TeV	172.0	$^{+4.4(2.6\%)}_{-5.8(3.4\%)}$	$^{+4.7(2.7\%)}_{-4.8(2.8\%)}$
LHC 8 TeV	245.8	$^{+6.2(2.5\%)}_{-8.4(3.4\%)}$	$^{+6.2(2.5\%)}_{-6.4(2.6\%)}$
LHC 14 TeV	953.6	$^{+22.7(2.4\%)}_{-33.9(3.6\%)}$	$^{+16.2(1.7\%)}_{-17.8(1.9\%)}$







Impact of top data in the PDF fits is to reduce the uncertainty of the gluon

M.Czakon, M.Mangano, A.Mitov, J.Rojo (2013)

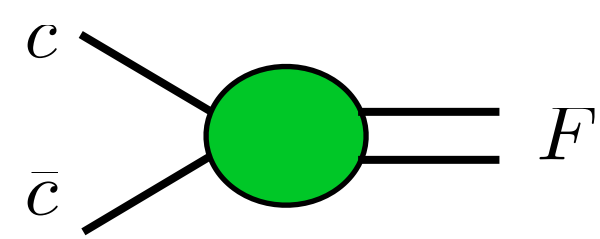
Differential predictions have been anticipated but are not available yet

q_T subtraction

S. Catani, MG (2007)

Let us consider a specific, though important class of processes: the production of colourless high-mass systems F in hadron collisions (F may consist of lepton pairs, vector bosons, Higgs bosons......)

At LO it starts with $c\bar{c} \longrightarrow F$



Strategy: start from NLO calculation of F+jet(s) and observe that as soon as the transverse momentum of the F $q_T \neq 0$ one can write:

$$d\sigma^F_{(N)NLO}|_{q_T\neq 0} = d\sigma^{F+\text{jets}}_{(N)LO}$$

Define a counterterm to deal with singular behaviour at $q_T \to 0$

But.....

the singular behaviour of $d\sigma^{F+\text{jets}}_{(N)LO}$ is well known from the resummation program of large logarithmic contributions at small transverse momenta

G. Parisi, R. Petronzio (1979)
J. Collins, D.E. Soper, G. Sterman (1985)
S. Catani, D. de Florian, MG (2000)

choose $d\sigma^{CT} \sim d\sigma^{(LO)} \otimes \Sigma^F(q_T/Q)$

where $\Sigma^F(q_T/Q) \sim \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \sum_{k=1}^{2n} \Sigma^{F(n;k)} \frac{Q^2}{q_T^2} \ln^{k-1} \frac{Q^2}{q_T^2}$

Then the calculation can be extended to include the q_T contribution:

$$d\sigma^F_{(N)NLO} = \mathcal{H}^F_{(N)NLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+\text{jets}}_{(N)LO} - d\sigma^{CT}_{(N)LO} \right]$$

where I have subtracted the truncation of the counterterm at (N)LO and added a contribution at $q_T=0$ to restore the correct normalization

The hard-collinear function $\mathcal{H}^F$ has been explicitly computed up to NNLO for vector and Higgs boson production

S. Catani, MG (2010)
S. Catani, L.Cieri, D. de Florian, G.Ferrera, MG (2013)

Recently its general form in terms of the relevant virtual amplitudes for an arbitrary colour singlet F has been provided up to NNLO

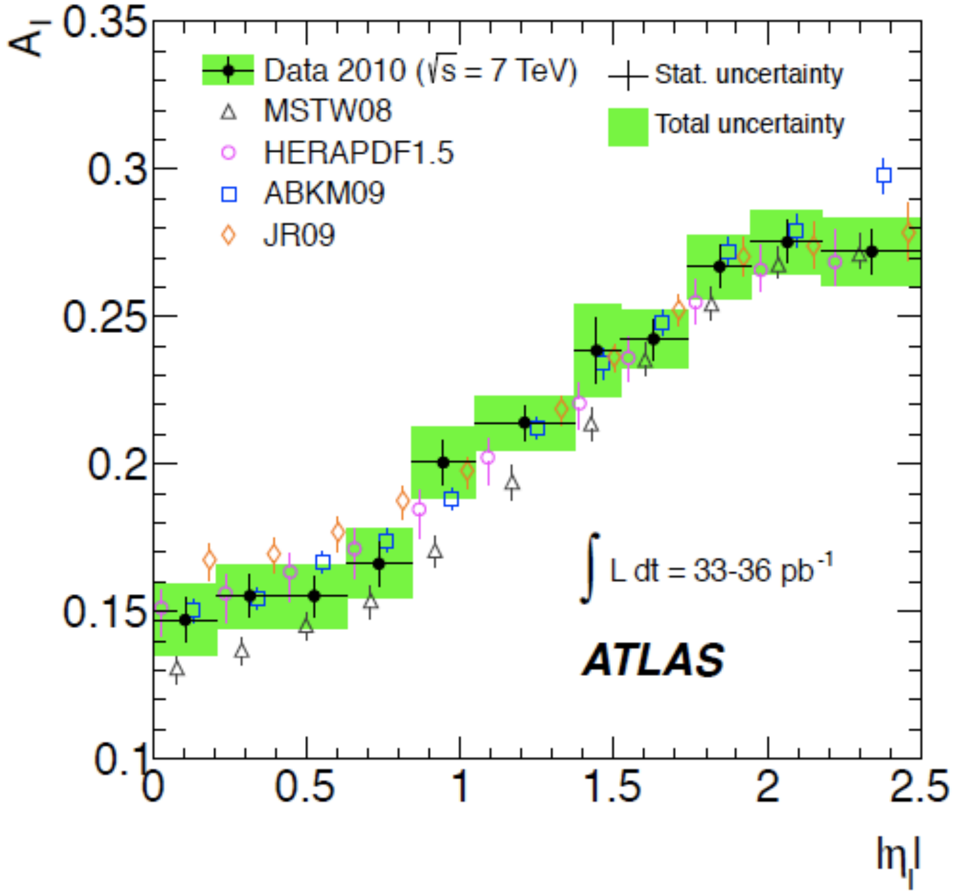
S. Catani, L.Cieri, D. de Florian, G.Ferrera, MG (2013)
T. Gehrmann, T.Lubbert,L.Yang (2014)

the method can be applied also to vector boson pair production

DY and Higgs boson production

S. Catani, MG (2007), MG(2008)

S. Catani, L.Cieri, G.Ferrera, D. de Florian, MG (2009)



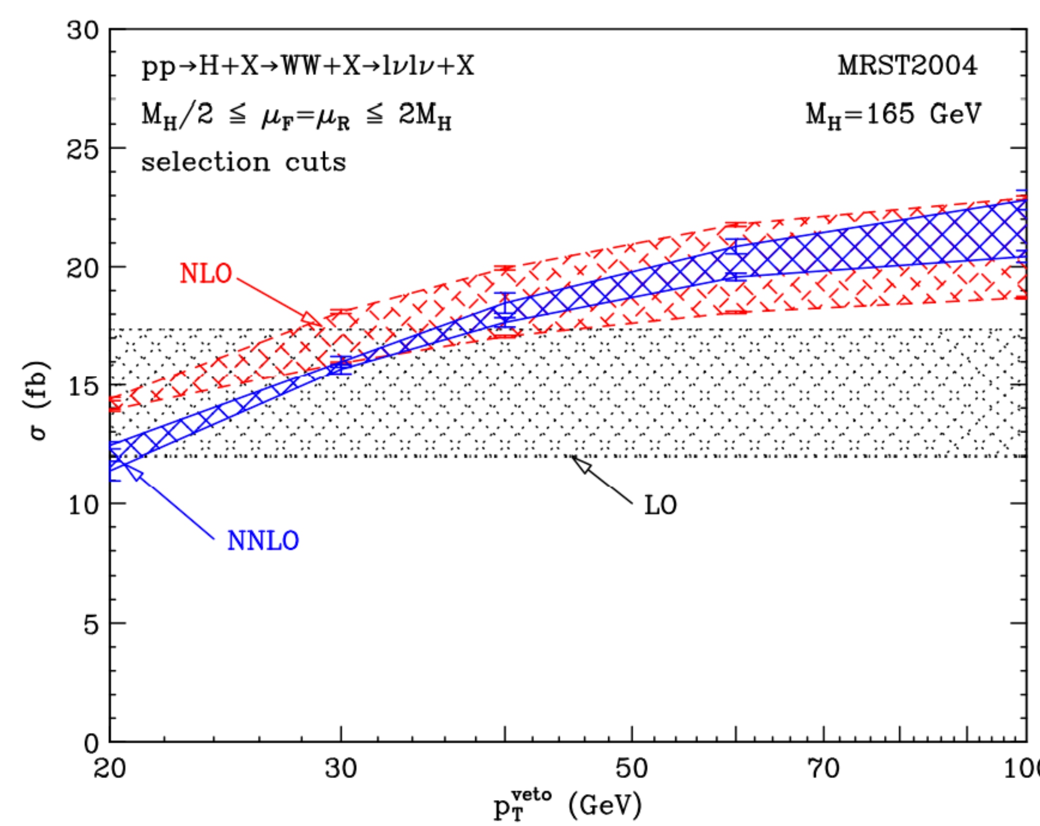
First applications of this method were implemented in two public programs: DYNNLO and HNNLO

DYNNLO: Lepton asymmetry in W decay vs ATLAS data

HNNLO: cross section with a jet veto and study of jet-bin uncertainties

Now also mass effects included

H.Sargsyan, MG(2013)



Status of pp$\rightarrow$VV'+X in QCD

Zγ, Wγ ,WZ, WW, ZZ production known in NLO QCD since quite some time

J.Ohnemus (1993); U.Baur, T.Han, J.Ohnemus (1998)
B.Mele, P.Nason,G.Ridolfi (1991)
S.Frixione, P.Nason,G.Ridolfi (1992); S.Frixione (1993)
L.Dixon,Z.Kunszt,A.Signer (1999)
J.Campbell,K.Ellis (1999); D. de Florian, A.Signer (2000)

Also including leptonic decay

The gluon fusion loop contribution (part of NNLO) to Zγ, ZZ and WW is also known (often assumed to provide the dominant NNLO contribution)

T.Binoth et al. (2005,2008)
M.Duhrssen et al. (2005)
L.Ametller et al. (1985)
J. van der Bij, N.Glover (1988)
K. Adamson, D. de Florian, A.Signer (2000)

$\rightarrow$ **all this implemented in MCFM**

NLO EW corrections have also been studied

W.Hollik, C.Meier (2004)
E.Accomando, A.Denner,C.Meier (2005)
A.Bierweiler, T.Kasprzik,J.Kuhn,S.Uccirati(2012)
M.Billoni,S.Dittmaier,B.Jager,C.Speckner (2013)

Genuine Vγ two-loop amplitude computed

T.Gehrmann, L.Tancredi (2012)

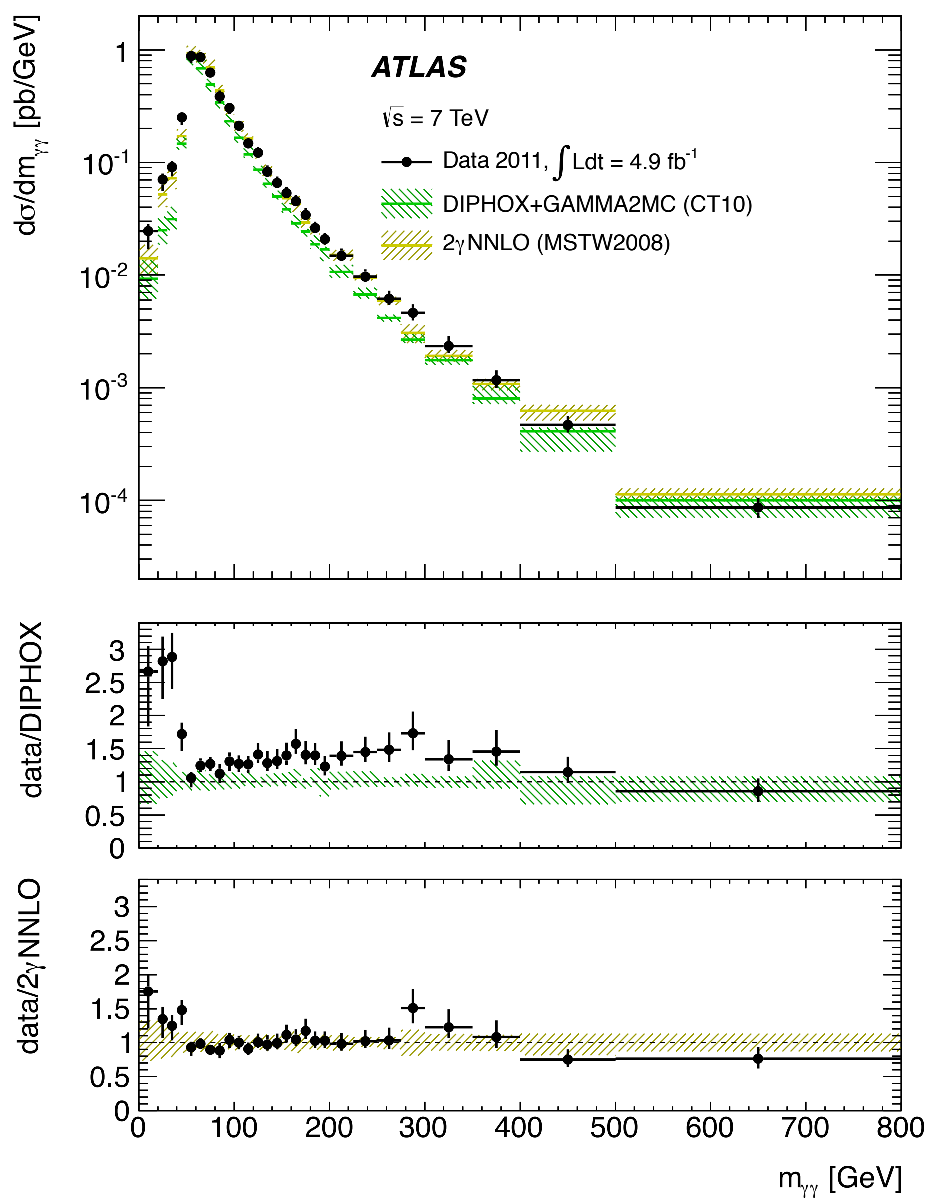
Planar two-loop master integrals for WW,WZ and ZZ production recently evaluated

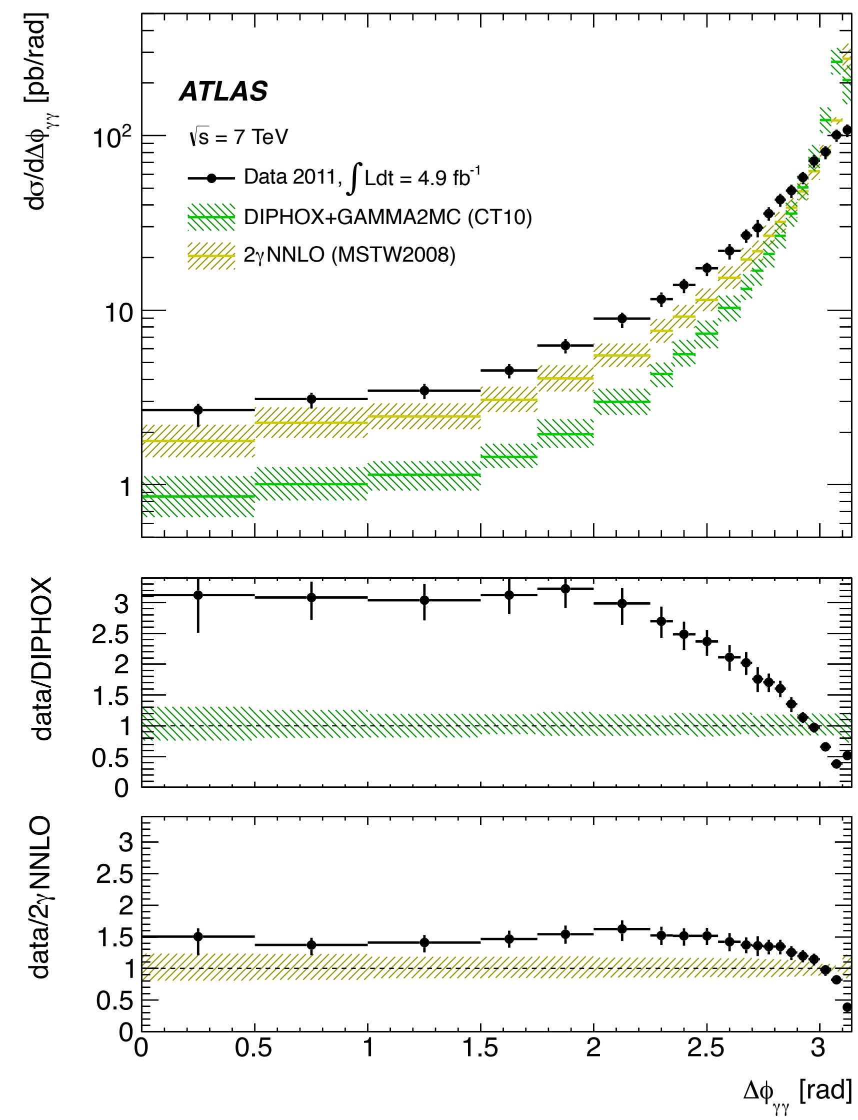
T.Gehrmann, L.Tancredi, E.Weihs (2014)
J.Henn,K.Melnikov, V.Smirnov (2014)

pp→γγ +X at NNLO

Complete NNLO predictions presented for diphoton production

S.Catani, L.Cieri, D.de Florian,,G.Ferrera, MG (2011)





This calculation allowed to resolve discrepancies in the comparison to data

$pp \to V\gamma + X$ at NNLO

Having completed $pp \to \gamma\gamma + X$ the next logical step is $pp \to V\gamma + X$ ($V = Z, W$)

Ingredients for $pp \to V\gamma + X$ at NNLO

- One-loop squared and two-loop amplitudes for $q\bar{q} \to V\gamma$ — W.Van Neerven et al. (1989); T.Gehrmann, L.Tancredi (2012)
- One loop squared $gg \to Z\gamma$ amplitude — L.Amettler et al. (1985); J. van der Bij, N.Glover (1988); K. Adamson, D. de Florian, A.Signer (2000)
- One loop $V\gamma$+1 parton amplitudes — J.Campbell, H.Hartanto, C.Williams (2012)
- Tree-level $V\gamma$+2 parton amplitudes

We obtain the tree-level and one-loop amplitudes with OpenLoops

F.Cascioli, P.Maierhofer, S.Pozzorini (2012)

The OpenLoops generator employs the Denner-Dittmaier algorithm for the numerically stable computation of tensor integrals and allows a fast evaluation of tree-level and one-loop amplitudes within the SM

→ It allows us to keep numerical stability under control in the delicate $q_T \to 0$ region

NEW: Results: $W\gamma$

S.Kallweit, D.Rathlev, A.Torre, MG (to appear)

- ATLAS setup (arXiv:1302.1283)

$p^{\gamma}_T > 15$ GeV $p^{l}_T > 25$ GeV $\Delta R(l/\gamma,jet) > 0.3$

$|\eta^{\gamma}| < 2.37$ $|\eta^{l}| < 2.47$ $\Delta R(l,\gamma) > 0.7$

$p_T^{miss} > 35$ GeV

photon isolation:

$\varepsilon = 0.5$ $R = 0.4$

jets: anti-kt with D=0.4

$p_T^{jet} > 15$ GeV $|\eta^{jet}| < 2.47$

σ (fb)	LO	NLO	NNLO	data
$W^+\gamma$	511.0±0.2	1155.3±0.8 126%	1371 ±5 19%	
$W^-\gamma$	395.3 ± 0.2	909.9±0.4 130%	1085 ±4 19%	
total	906.3 ± 0.3	2065.2 ± 0.9 128%	2456± 6 19%	2770±30(stat)±330(syst)±140(lumi)

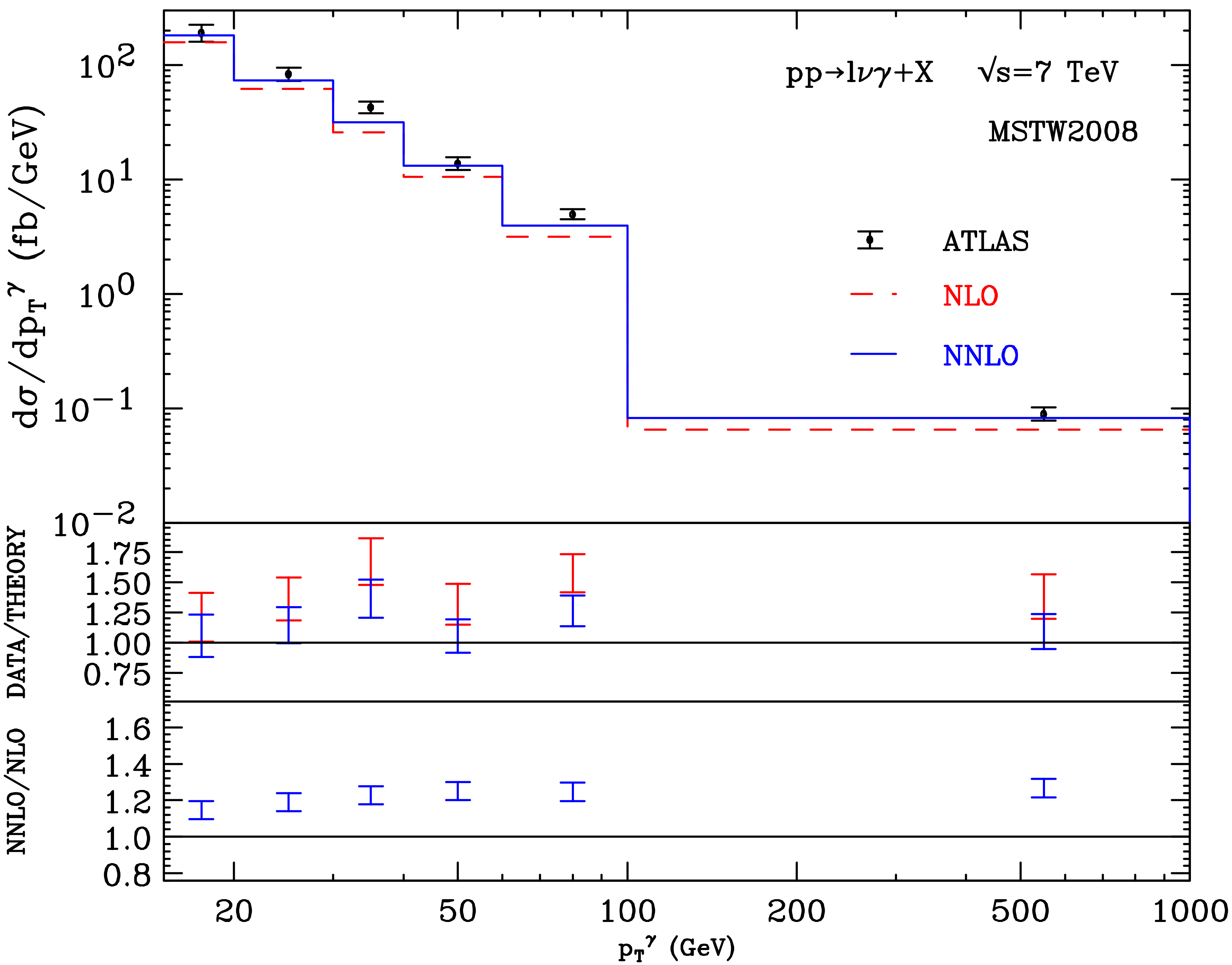
Some tension between data and NLO result

QCD corrections are much larger than for $Z\gamma$

→ NNLO significantly improves Data/Theory agreement

NEW: Results: Wγ

S.Kallweit, D.Rathlev, A.Torre, MG (to appear)

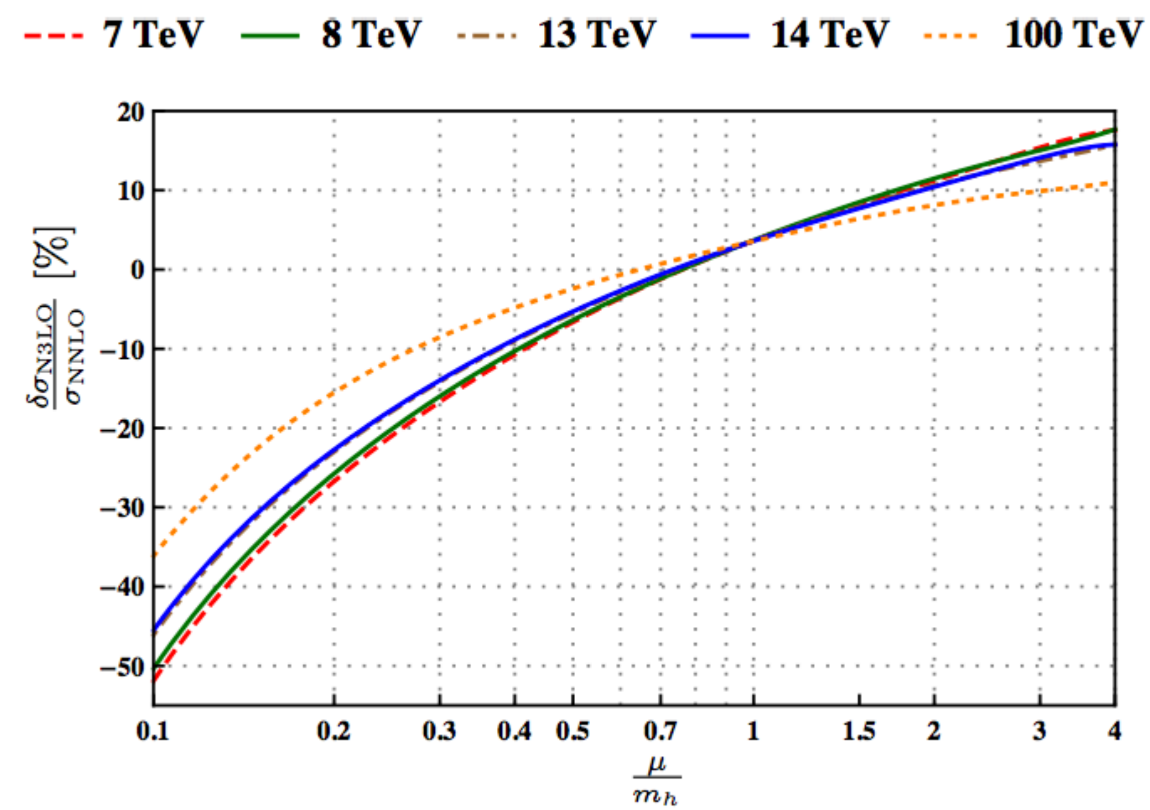


NNLO effect ranges from 15 to 25% as a function of p^{γ}_{T}

Beyond NNLO

C.Anastasiou, C. Duhr, F.Dulat, E.Furlan,
T.Gehrmann, F.Herzog, B.Mistlberger (2014)

In the case of Higgs boson production NNLO corrections are large (+25%) and scale uncertainties still at the O(±10%)



➡ Some brave colleagues are computing the N3LO corrections !

First results, corresponding to the soft-virtual (SV) terms have recently been presented (+4% at $\mu_F=\mu_R=m_H$): further term beyond SV expected soon

It will be interesting to compare with existing NNLL+NNLO calculations that are currently used as reference by ATLAS and CMS

D. de Florian, MG (2009,2011)

Summary

- QCD predictions are essential for a reliable understanding of hard scattering processes at hadron colliders

- NLO calculations should be the standard

  - For many years the bottleneck has been the computation of the relevant one-loop amplitudes: great progress in the last few years

  - The 2->4 barrier has been broken and many new tools have been developed

  The problem is "in principle" solved

- Sophisticated NLO+Parton Shower simulations (not covered in this talk) are now possible

Summary

- I have reviewed the most recent developments in NNLO calculations and discussed some new results

  - ttbar

  - vector boson pair production: $\gamma\gamma$, $W\gamma$ and more to come

- Fully exclusive results are particularly useful

  - they provide a precise estimate of higher order corrections when cuts are applied (no need of large extrapolation factors)

  - the corresponding acceptances can be compared with those obtained with standard MC event generators

- N^3LO barrier broken with first results for Higgs production in the SV approximation