Analiza I - 2013/14

Zadania domowe - seria 13

Zadanie 1. Znaleźć funkcje pierwotne:

a) $\displaystyle\int \frac{1}{1+\operatorname{ctg} x}\,dx\,;$

b) $\displaystyle\int \frac{\sin x \sin 2x}{\cos 2x}\,dx\,;$

c) $\displaystyle\int \frac{1}{2+\sin x+\cos x}\,dx\,.$

Zadanie 2. Obliczyć funkcje pierwotne :

a) $\displaystyle\int \frac{x^2+1}{\sqrt{x^2-3x+2}}\,dx\,;$

b) $\displaystyle\int \frac{\sqrt{x^2+2x+2}}{x}\,dx\,;$

c) $\displaystyle\int \frac{x}{(1+x)\sqrt{1-x-x^2}}\,dx\,.$

Zadanie 3. Obliczyc:

a) $\displaystyle\int_0^{\pi} x^3\cos x\,dx;$

b) $\displaystyle\int_0^{\frac{1}{2}} \sqrt{e^{2x}-1}\,dx;$

c) $\displaystyle\int_0^{e} \log(1+x)\,dx;$

d) $\displaystyle\int_0^{a} x^2\sqrt{a^2-x^2}\,dx \quad (a>0);$

e) $\displaystyle\int_0^{1} \frac{1}{x+\sqrt{x^2-x+1}}\,dx$

f) $\displaystyle F(\alpha) := \int_0^{+\infty} \frac{1}{(x^2+1)(x^\alpha+1)}\,dx \quad (\alpha>0);$

g) $\displaystyle G(a,b) := \int_0^{2\pi} \frac{\sin^2 x}{a-b\cos x}\,dx \quad (a>b>0).$

Zadanie 4. Zbadać zbieżność szeregu:

a) $\sqrt{2}+\sqrt{2-\sqrt{2}}+\sqrt{2-\sqrt{2+\sqrt{2}}}+\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2}}}}+\ldots$

Wskazówka: $\sqrt{2}=2\cos\frac{\pi}{4}$.

b) $\displaystyle\sum_{n=1}^{+\infty}(\sqrt{2}-\sqrt[3]{2})(\sqrt{2}-\sqrt[5]{2})\cdot\ldots\cdot(\sqrt{2}-\sqrt[2n+1]{2})$

c) $\displaystyle\sum_{n=1}^{+\infty}\frac{1}{\sqrt{n(n+1)}}\,;$

d) $\displaystyle\sum_{n=1}^{+\infty}\frac{2+(-1)^n}{2^n}\,;$

e) $\displaystyle\sum_{n=2}^{+\infty} \frac{\sqrt{n+2}-\sqrt{n-2}}{n^\alpha}$ w zależności od $\alpha \in \mathbb{R}$

f) $\displaystyle\sum_{n=2}^{+\infty} \frac{1}{\log(n!)}$;

g) $\displaystyle\sum_{n=1}^{+\infty} (-1)^n \frac{\sqrt{n}}{n+4}$;

h) $\displaystyle\sum_{n=1}^{+\infty} \frac{(-1)^n}{\sqrt[n]{n}}$;

i) $\displaystyle\sum_{n=1}^{+\infty} (-1)^n \frac{\sin^2 n}{n}$.