Proton charge radius puzzle

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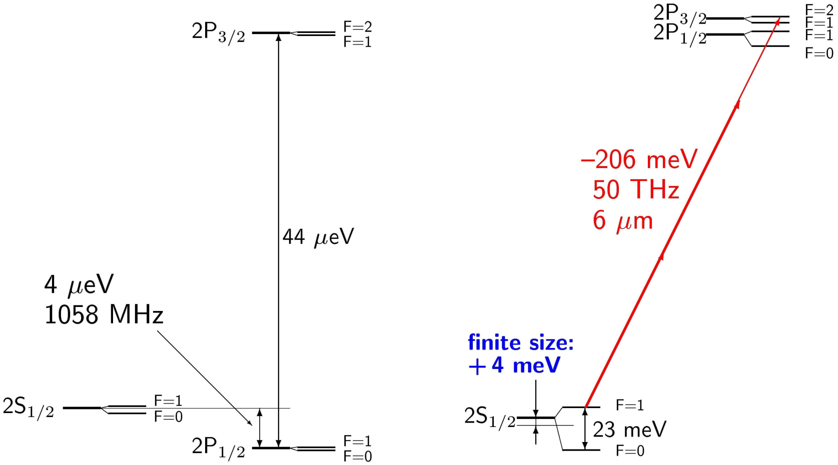
Proton charge radius puzzle

- global fit to H and D spectrum: $r_p = 0.8758(77)$ fm (CODATA 2010)
- $e - p$ scattering: $r_p = 0.8791(79)$ (Bernauer, 2010)
- from muonic hydrogen: $r_p = 0.84089(39)$ fm (PSI, 2010, 2012)

If all these measurements and Lamb shift calculations are correct, this discrepancy does not find explanation within the known description of electroweak and strong interactions.

energy levels of μH in comparison to H



Theory of hydrogen energy levels

- energy according to Dirac equation

$$f(n,j) = \left(1 + \frac{(Z\,\alpha)^2}{[n+\sqrt{(j+1/2)^2-(Z\,\alpha)^2}-j-1/2]^2}\right)^{-1/2}$$

- total energy

$$\begin{aligned} E &= M + m + \mu[f(n,j)-1] - \frac{\mu^2}{2\,M}[f(n,j)-1]^2 \\ &\quad + \frac{(Z\,\alpha)^4\,\mu^3}{2\,n^3\,M}\left[\frac{1}{j+1/2} - \frac{1}{l+1/2}\right](1-\delta_{l0}) + E_L \end{aligned}$$

- $E_L(\alpha) = E^{(5)} + E^{(6)} + E^{(7)} + E^{(8)} + \ldots$ where
  $E^{(n)} \sim \alpha^n\,\mathcal{E}^{(n)}$

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Contributions to the Lamb shift

- one-loop electron self-energy and vacuum polarization
- two-loops
- three-loops
- pure recoil correction
- radiative recoil correction
- finite nuclear size, and polarizability

One-loop contribution

$$\delta E = \frac{\alpha}{\pi} (Z\alpha)^4 \, m \, F(Z\alpha)$$

analytic expansion

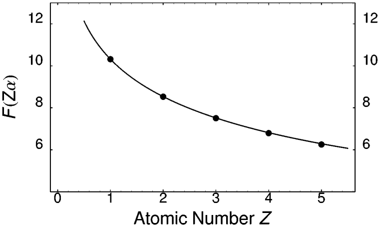
$$\begin{aligned} F(Z\alpha) &= A_{40} + A_{41} \ln(Z\alpha)^{-2} + (Z\alpha) A_{50} \\ &\quad + (Z\alpha)^2 \left[A_{62} \ln^2(Z\alpha)^{-2} + A_{61} \ln(Z\alpha)^{-2} + A_{60} + O(Z\alpha) \right] \end{aligned}$$

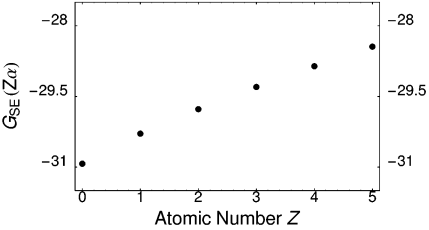
or direct numerical evaluation using
the exact Coulomb-Dirac propagator

Numerical evaluation of the one-loop self-energy

[U. Jentschura, P.J. Mohr, and G. Soff, Phys. Rev. Lett. **82**, 53 (1999)]

$$\delta E = \frac{\alpha}{\pi} (Z\,\alpha)^4 \, m \, F(Z\,\alpha)$$





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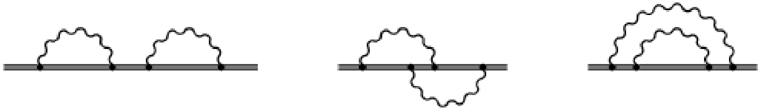
Two-loop electron self-energy correction

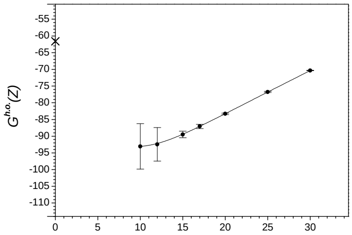


Electron propagators include external Coulomb field, external legs are bound state wave functions.

The expansion of the energy shift in powers of $Z\alpha$

$$\begin{aligned}
\delta^{(2)}E &= m\left(\frac{\alpha}{\pi}\right)^2 F(Z\alpha) \\
F(Z\alpha) &= B_{40} + (Z\alpha)\,B_{50} + (Z\alpha)^2\Big\{[\ln(Z\alpha)^{-2}]^3\,B_{63} \\
&\quad + [\ln(Z\alpha)^{-2}]^2\,B_{62} + \ln(Z\alpha)^{-2}\,B_{61} + G_{60}(Z\alpha)\Big\}
\end{aligned}$$

Direct numerical calculation versus analytical expansion



$G_{60}(1) \approx -86(15)$ (Yerokhin, 2009), uncertainty $\delta E(1S) = \pm 1.5$ kHz

$B_{60} = -61.6(9.2)$ (K.P., U.J., 2003)

uncertainty due to the unknown high energy contribution from the class of about 80 diagrams

discrepancy in the proton charge radius $\rightarrow \delta E(1S) \approx 100$ kHz

Pure recoil corrections

- finite nuclear mass effects, beyond the Dirac equation
- leading $O(\alpha^5)$ terms are known for an arbitrary mass ratio

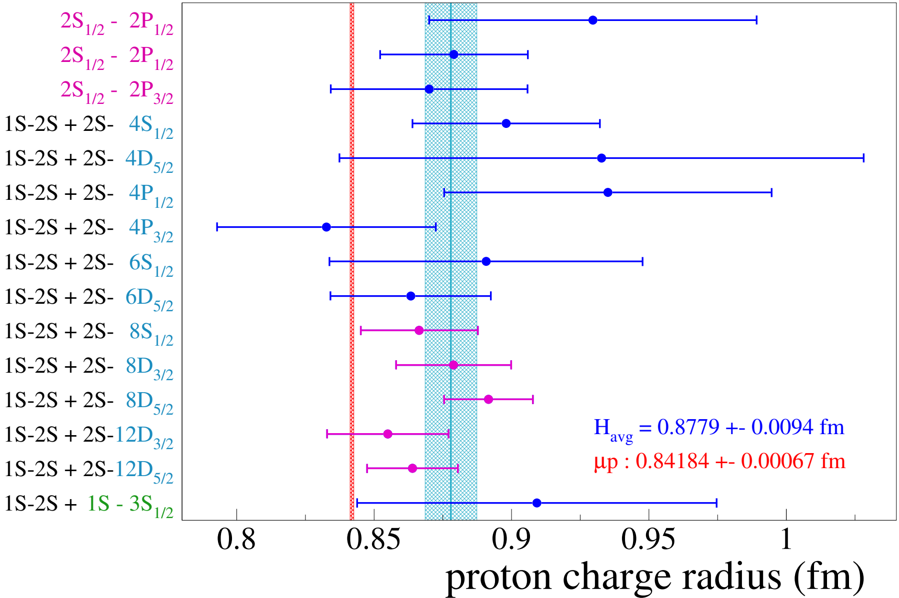
$$
\begin{aligned}
\delta E^{(5)} \quad = \quad & \frac{\mu^3}{m\,M}\,\frac{(Z\,\alpha)^5}{\pi\,n^3}\left\{\frac{1}{3}\,\delta_{l0}\,\ln(Z\,\alpha)^{-2} - \frac{8}{3}\,\ln k_0(n,l) - \frac{1}{9}\,\delta_{l0} - \right. \\
& \left. -\frac{2}{M^2 - m^2}\,\delta_{l0}\left[M^2\,\ln\frac{m}{\mu} - m^2\,\ln\frac{M}{\mu}\right]\right\}
\end{aligned}
$$

where

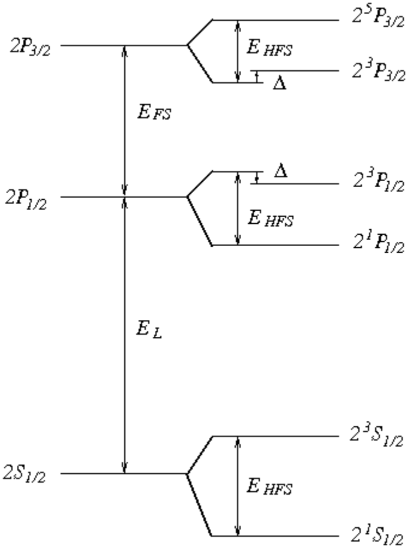
$$
a_n = -2\left[\ln\frac{2}{n} + \left(1 + \frac{1}{2} + \cdots + \frac{1}{n}\right) + 1 - \frac{1}{2\,n}\right]\delta_{l0} + \frac{1 - \delta_{l0}}{l\,(l+1)\,(2\,l+1)}
$$

- $\delta E^{(6)} = \frac{m^2}{M}\left(4\,\ln 2 - \frac{7}{2}\right)$

Experimental results for hydrogen and r_p



energy levels of μH



$$E_L = 202.1 \ meV$$

$$E_{FS} = 8.4 \ meV$$

$$E_{HFS}(2S_{1/2}) = 22.7 \ meV$$

$$E_{HFS}(2P_{1/2}) = 8.0 \ meV$$

$$E_{HFS}(2P_{3/2}) = 3.4 \ meV$$

$$\Delta = 0.1 \ meV$$

Theory of μH energy levels

- μH is essentially a nonrelativistic atomic system
- muon and proton are treated on the same footing
- $m_\mu / m_e = 206.768 \Rightarrow \beta = m_e/(\mu\,\alpha) = 0.737$ the ratio of the Bohr radius to the electron Compton wavelength
- the electron vacuum polarization dominates the Lamb shift

$$E_L = \int d^3r\, V_{vp}(r)\,(\rho_{2P} - \rho_{2S}) = 205.006\,\mathrm{meV}$$

- important corrections: second order, two-loop vacuum polarization, and the muon self-energy
- other corrections are much smaller than the discrepancy of 0.3 meV.

Leading relativistic correction

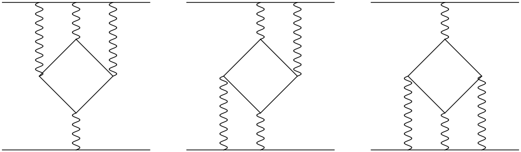
$$\delta H = -\frac{p^4}{8\,m^3} - \frac{p^4}{8\,M^3} + \frac{\alpha}{r^3}\left(\frac{1}{4\,m^2} + \frac{1}{2\,m\,M}\right)\vec{r}\times\vec{p}\cdot\vec{\sigma} + \frac{\pi\,\alpha}{2}\left(\frac{1}{m^2} + \frac{1}{M^2}\right)\delta^3(r) - \frac{\alpha}{2\,m\,M\,r}\left(p^2 + \frac{\vec{r}\,(\vec{r}\vec{p})\,\vec{p}}{r^2}\right)$$

$$\delta E = \langle l, j, m_j|\delta H|l, j, m_j\rangle = \frac{(Z\,\alpha)^4\,\mu^3}{2\,n^3\,m_p^2}\left(\frac{1}{j+\frac{1}{2}} - \frac{1}{l+\frac{1}{2}}\right)(1-\delta_{l0})$$

$$\delta E_L = \frac{\alpha^4\mu^3}{48\,m_p^2} = 0.057\mathrm{meV}$$

valid for an arbitrary mass ratio

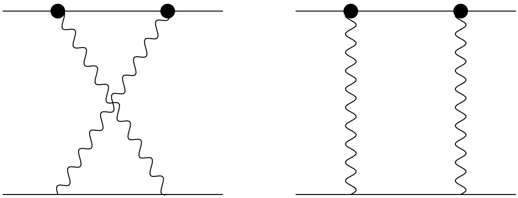
Light by light diagrams



- $\delta E_L = -0.0009$ meV
- significant cancellation between diagrams
- S.G. Karshenboim *et al.*, arXiv:1005.4880

Nuclear structure effects

- when nuclear excitation energy is much larger than the atomic energy, the two-photon exchange scattering amplitude gives the dominating correction
- the total proton structure contribution $\delta E_L = 36.9(2.4)\ \mu\text{eV}$ is much too small to explain the discrepancy



Final results

- $\Delta E = E(2P_{3/2}(F=2)) - E(2S_{1/2}(F=1))$
- experimental result: $\Delta E = 206.2949(32)\ \text{meV}$
- total theoretical result from [U. Jenschura, 2011]

$$\Delta E = \left(209.9974(48) - 5.2262\frac{r_p^2}{\mathrm{fm}^2}\right)\mathrm{meV} \Rightarrow$$

- $r_p = 0.841\,69(66)\ \text{fm}$

Possible sources of the proton radius discrepancy: theory

- mistake in $e - H$ calculations: all corrections calculated independently by at least two groups, uncertainty in the two-loop correction enters at 1 kHz level for 1S state, but this discrepancy corresponds to 100 kHz
- mistake in $\mu - H$: QED theory is quite simple, dominated by nonrelativistic vacuum polarization, everything checked and verified
- missing QED corrections
- significant underestimation of the proton polarizability and of the related subtraction term in dispersion relations (not known from e-p inelastic scattering, (G. Paz and R.J. Hill, J.A. McGovern talk, G.A. Miller talk)
- new interactions between the muon and the proton: a scalar with 1 MeV mass is not completely ruled out, but requires fine tuning (I. Yavin talk)

Possible sources of the proton radius discrepancy: experiment

- μH measurement is not verified by independent experiment
- the determination of r_p from $e - p$ scattering data requires extrapolation to $q^2 = 0$, subject of systematic uncertainties and model dependence, main issue discussed during the conference
- $2S - nS, D$ measurements (mostly from one laboratory, LKB Paris), not confirmed by independent and equally accurate measurements. Highly excited states of H are affected by various systematics, possible hints from E. Hessels talk. As a result the Rydberg constant might be not as accurate as claimed

Ways to go

- determine *Ry* by another accurate measurement in
  - 2S-4P in H (Garching)
  - 2S-nS,D in H (J. Flowers, NPL)
  - 1S-3S (Garching, . . .)
  - transitions between Rydberg states of heavy H-like ions (NIST, N.D. Guise talk)
  - 1S-2S and 1S hfs in $e\,\mu$ (A. Antonini, PSI)
- determine r_p from $2S - 2P$ transition in H: (E. Hessels talk)
- $\mu - p$ elastic scattering (Arrington *et al.*)
- compare charge radii from electronic and muonic spectra of other atomic systems
  - μD data are coming, r_D from very accurate H-D isotope shift (Garching)
  - r_{He} charge radius from 1S-2S (two-photon) transition in He^+, or $2^3S - 2^3P$ in He

New interactions

If discrepancy in r_p is to be explained by a new type of interaction between the proton (neutron) and leptons, than we have two options

- long range $\sim ƛ_e$, not consistent with precise measurements of the Lamb shift in H- and Li-like heavy ions at GSI
- short range $\sim$ 1fm (or shorter), can be seen in μp scatt.

Comparison of nuclear charge radii for H,D,^{3}He and ^{4}He will give hints on the range of new interactions

If it is local, than discrepancy for all these elements can be parametrized by

$$\delta E = (Z\,\delta r_p^2 + (A-Z)\,\delta r_n^2)\,\frac{2\,\delta_{l0}}{3\,n^3}\,Z^3\,\alpha^4\,\mu^3$$

Determination of r_N from muonic atoms spectra requires an accurate calculation of the nuclear polarizability correction (S. Schlesser talk), not necessarily easy task

Proposal to determine α charge radius from the atomic spectroscopy

- $E(2^3S - 2^3P, {}^4\mathrm{He})_{\mathrm{centroid}} = 276\,736\,495\,649.5(2.1)$ kHz, Florence, 2004
- finite size effect: $E_{\mathrm{fs}} = 3\,387$ kHz
- since E_{fs} is proportional to r^2

$$\frac{\Delta r}{r} = \frac{1}{2}\,\frac{\delta E_{\mathrm{fs}}}{E_{\mathrm{fs}}} \approx \frac{1}{2}\,\frac{10}{3\,387} = 1.5 \cdot 10^{-3}$$

- electron scattering gives $r_{\mathrm{He}} = 1.681(4)$ fm, what corresponds to about $2.5 \cdot 10^{-3}$ relative accuracy
- can theoretical predictions be accurate enough ~ 10 kHz ?