ABSTRACT

Algebra and geometry of Maurer-Cartan algebras

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Let R be a commutative ring with 1, A a commutative R-algebra with 1, and let L be an A-module. A *Maurer-Cartan*-algebra structure over L relative to A is a multiplicative R-differential d on $\mathrm{Alt}_A(L, A)$, and the resulting differential graded R-algebra $(\mathrm{Alt}_A(L, A), d)$ is referred to as a *Maurer-Cartan*-algebra over L (relative to A). When L is finitely generated and projective as an A-module, Maurer-Cartan structures over L relative to A and Lie-Rinehart structures on (A, L) are equivalent notions. When A is the algebra of smooth functions on a smooth manifold and L the Lie algebra of vector fields, the corresponding Maurer-Cartan algebra is the ordinary de Rham algebra on the manifold. Maurer-Cartan structures in the graded setting include twilled Lie-Rinehart algebras and quasi-Lie-Rinehart algebras. Under such circumstances, the Maurer-Cartan structure comprises replacements for the ring of functions and the Lie algebra of vector fields, and it organizes the combinatorics needed to handle the higher homotopies which arise in a natural fashion. Examples arise from complex manifolds and from foliations. The corresponding Maurer-Cartan structures have the Hodge-de Rham spectral sequence and the spectral sequence of the foliation under discussion as invariants.