Moment bezwładności pulsarów i równanie stanu gwiazd neutronowych

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Motivations

- I is the stellar quantity most sensitive to the EOS (pointed out already by *Carter & Quintana 1973*)
- Discovery of PSR J0737-3039A,B
- Discovery of $\sim 2M_{\odot}$ PSR J07051+1807 (in binary with white dwarf)

Plan

- Basic notions and approximations
- Theory: I versus M and other relations for neutron stars and for quark stars
- Constraints on I and EOS from Crab Nebula
- Constraints from timing of PSR J0737-3039A,B
- Constraints from PSR J07051+1807

Pulsars=rotating neutron stars $\Omega = 2\pi/P$
$f = 1/P = \Omega/2\pi$
1983-2005: $f_{\max} = 642$ Hz, since January 2006
$f_{\max} = 716$ Hz

Hydrostatic equilibrium in General Relativity

1-D non-rotating $\mathcal{C}(\rho_c)$

2-D axially symmetric rigidly rotating $\mathcal{C}(\rho_c, \Omega)$

$$I = J/\Omega$$

Slow rotation approximation

Fixed: baryon number A. Condition:

$\Omega \ll \Omega_{\rm ms} \sim \sqrt{GM/R^3} \sim$

$J = I(0)\Omega + \mathcal{O}(\Omega^3)$, $\quad I(\Omega) = I(0) + \mathcal{O}(\Omega^2)$,

$M(\Omega)c^2 = M(0)c^2 + \frac{1}{2}I(0)\Omega^2 + \mathcal{O}(\Omega^4)$ *Hartle 1967*

Crab Pulsar - $\Omega/\Omega_{\rm ms} = P_{\rm ms}/P \sim 1/33$

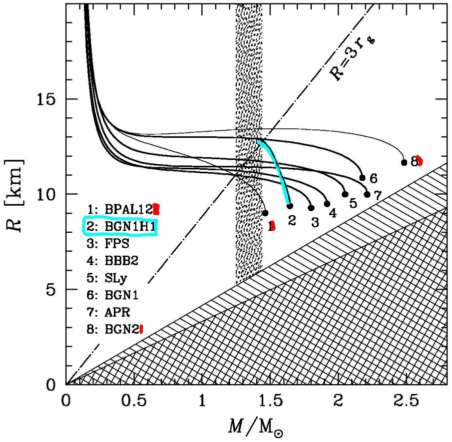
PSR J0737-3039A - $\Omega/\Omega_{\rm ms} = P_{\rm ms}/P \sim 1/22.7$

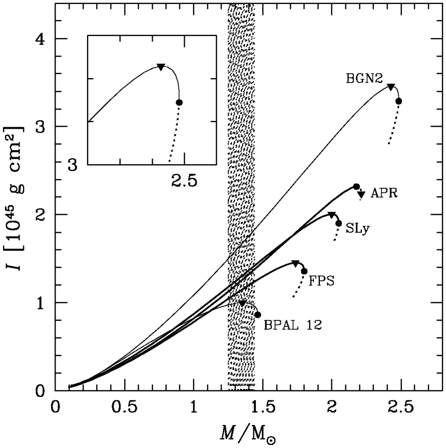
Dragging of local inertial frames

$\overline{\omega}$=local rotation frequency as measured in *local inertial reference frame*

dragging $\Longrightarrow \overline{\omega} < \Omega$

Dependence of $I(M)$ on EOS



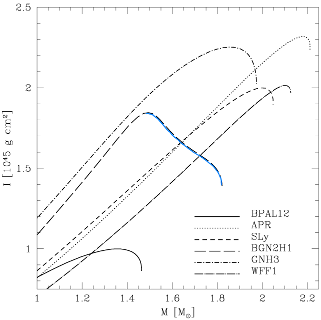


2 - effect of hyperons on $R(M)$

EOS with nucleons only

Softening due to hyperons $\Longrightarrow I_{\max}$ is significantly higher than $I(M_{\max})$

Similar effects due to exotica (meson condensation & quark deconfinement)



Bejger, Bulik & Haensel 2005

Approximate formulae for I

Aim: $I(M,R)$. Measured $I \Longrightarrow$ bounds in $M-R$ plane

compactness parameter $x_{\rm GR} = r_{\rm g}/R$ $r_{\rm g} = 2GM/c^2 = 2.95\ M/M_\odot$ km

Ravenhall & Pethick 1994 $I \simeq 0.21\ MR^2/(1-x_{\rm GR})$

For $x_{\rm GR} = 0.15 - 0.45$ reproduces exact values of I within $\sim 10\%$ for nucleon EOSs; fails for EOSs with high-density softening (hyperons, exotica)

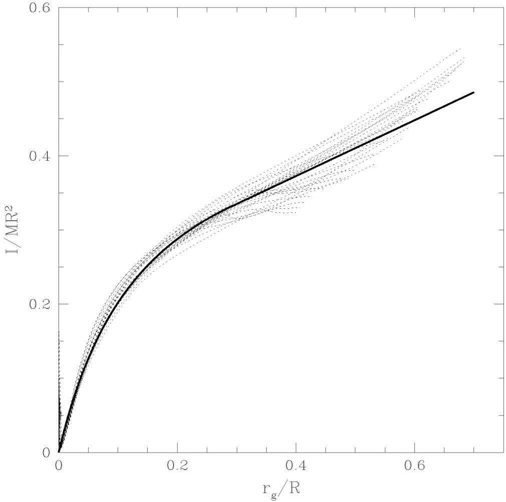
Bejger & Haensel 2002

$$\frac{I}{MR^2} = \begin{cases} x_{\rm GR}/(0.295 + 2\,x_{\rm GR}) & \text{for} \quad x_{\rm GR} \leq 0.3 \\ \frac{2}{9}\,(1 + 1.69\,x_{\rm GR}) & \text{for} \quad x_{\rm GR} > 0.3 \end{cases}$$

$$I_{45}^{\rm max} \simeq (2.414\,x_{\rm GR}^{\rm max} - 0.368)(M_{\rm max}/M_\odot)(R_{M_{\rm max}}/10\ {\rm km})^2\ .$$

More than 30 EOSs. Neutron stars, also with hyperons and exotic cores

Bejger & Haensel 2002⟶



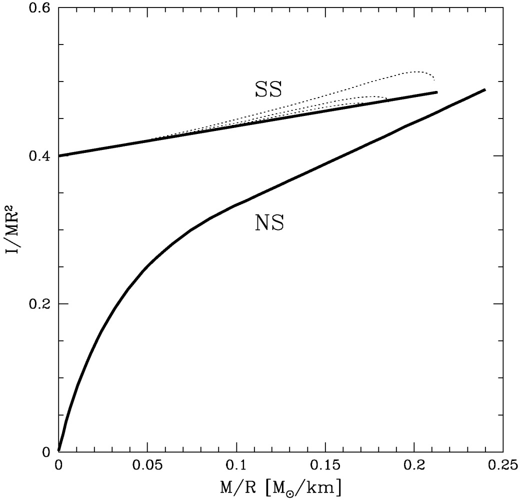
Correlation between I and M, R - quark stars

Quark matter down to the surface.

Flat density profile
for $1.4 M_\odot$
$\rho_c/\rho_{\rm surf} \sim 2$
for $M = M_{\rm max}$
$\rho_c/\rho_{\rm surf} \sim 5.$

$$\frac{I}{MR^2} \simeq \frac{2}{5}\left(1 + 0.34\frac{r_g}{R}\right)$$

Bejger & Haensel 2002



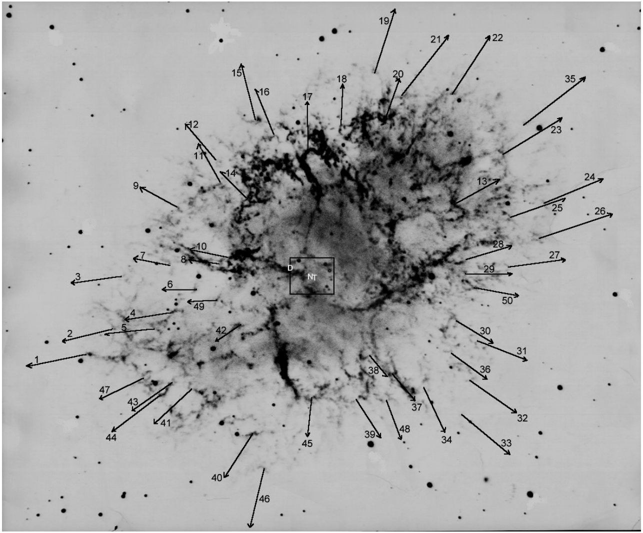
Crab Nebula - remnant of SN1054

Picture taken by 5m Mount Palomar Hale Telescope

Crab Nebula - remnant of SN 1054. Optically shining filaments form a shell whose expansion is **accelerated**. Powered by Crab pulsar (**plerion**) $\dot{E}_{\rm rot} = -I\dot{\Omega}\Omega$.

Measured parameters **now**: $P = 33.41$ ms , $\dot{P} = 4.228 \times 10^{-13}$, breaking index $n \equiv -\Omega\ddot{\Omega}/\dot{\Omega}^2 = 2.509$

Nugent 1998, arrows show motion of 50 filaments in next 50 years at current speed⟶



Braking by radiation losses: $\dot{\Omega}(t) = -K\Omega^n(t)$
Integration yields $\Omega(t) = \Omega_0/\{1 + Kt(n-1)\Omega_0^{n-1}\}^{1/(n-1)}$,
which implies $P_0 = 19.3$ ms (in AD 1054)

At current $v_{\rm p}$ - convergence back in time in AD 1130
$\Longrightarrow$ expansion is accelerated

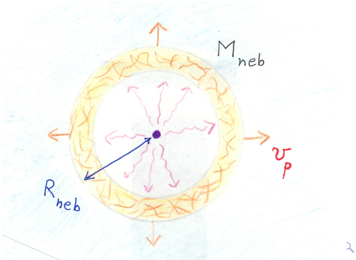
Spin-down energy loss & power bookkeeping

$$\dot{E}_{\rm rot} = \frac{\rm d}{{\rm d}t}\left(\frac{1}{2}I\Omega^2\right) = -I\Omega|\dot{\Omega}| \Longrightarrow I\Omega|\dot{\Omega}| > \dot{E}_{\rm rad} + \dot{E}_{\rm exp}$$

$$\dot{E}_{\rm exp} = \frac{\rm d}{{\rm d}t}\int \frac{1}{2}\rho v^2\,{\rm d}V$$

$$\dot{E}_{\rm exp} = \frac{1}{2}\frac{\rm d}{{\rm d}t}\left(M_{\rm neb}v^2\right) = \dot{E}_{\rm acc} + \dot{E}_{\rm sweep};$$

$$\dot{E}_{\rm acc} = M_{\rm neb}v\dot{v}, \quad \dot{E}_{\rm sweep} = \frac{1}{2}\dot{M}_{\rm neb}v^2$$



Crab Nebula - data.

$M_{\rm neb} = (4.6 \pm 1.8)\ M_{\odot}$ *Fesen et al. 1997*

$R_{\rm neb} = 1.25$ pc *Douvion et al. 2001*

$d_{\rm DF} = 1.83$ kpc *Davidson & Fesen 1985*

$v_0 < v_{\rm p} = 1.5 \times 10^3\ {\rm km\ s^{-1}}$ *Sollerman et al. 2000*

$\dot{E}_{\rm rad}(d) \simeq 1.25\ (d/d_{\rm DF})^2 \times 10^{38}\ \ {\rm erg\ s^{-1}}$ *Petersen 1998*

$$\dot{E}_{\rm acc} = M_{\rm neb} v \dot{v} \gg \dot{E}_{\rm rad},\ \ \dot{E}_{\rm sweep}$$

$$\Delta M_{\rm sweep}(t) = M_{\rm neb}(t) - M_{\rm neb}(0) \ll M_{\rm neb}(t),\ \ \Delta v_{\rm acc} \ll v_{\rm p}$$

Crab Nebula - evolution equations

v_0 - initial speed of expansion, given by SN1054

$E^0_{\rm exp} = \frac{1}{2} M^0_{\rm neb} v_0^2 = 4 \times 10^{49}$ erg $(M_{\rm neb,0}/4M_\odot)(v_0/10^3 \text{ km s}^{-1})^2$

Spherical shell approximation. Crab magnetic field fixed $\Longrightarrow$ braking index etc. constant in time *Bejger & Haensel 2003*

$$\boxed{I_{\rm Crab}\Omega(t)|\dot{\Omega}(t)| \approx M_{\rm neb} v(t) \dot{v}(t)}$$

$$\boxed{R_{\rm neb} = \int_0^{t_{\rm p}} v(t)\ {\rm d}t.}$$

unknown: v_0, $I_{\rm Crab}$. Results:

$$\begin{aligned} M_{\rm neb} &= 4.6\ M_\odot \Longrightarrow I_{\rm Crab,45} = 2.2; \\ M_{\rm neb} &= 6.4\ M_\odot \Longrightarrow I_{\rm Crab,45} = 3.1\ . \end{aligned}$$

Time dependent acceleration is **crucial**; if one uses $\dot{v} = const.$ then $I_{\rm Crab}$ is too large. Notice that what we expect on evolutionary grounds is $M_{\rm neb} > 6 M_\odot$!

I and M of PSR J0737-3039A,B

Close, relativistic binary PSR+PSR, discovery announced January 2005. Two-months of timing analysis give similar precision for masses as a decade for Hulse-Taylor binary.

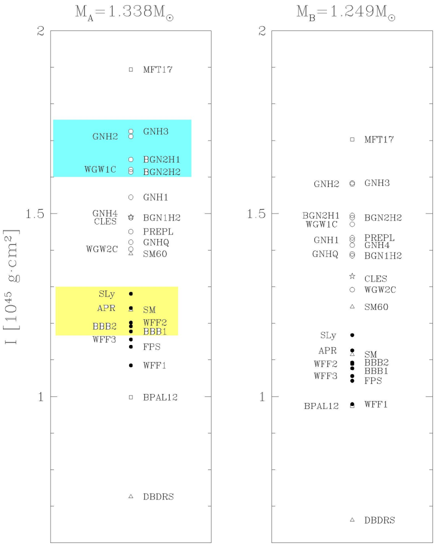
Nearly edge-on. $P_{\rm A} = 22.7{\rm ms}$, $P_{\rm B} = 2.77$ s. Periastron advance $\dot{\omega}$ affected by the spin-orbit coupling (*Damour & Schaeffer 1988*)

$$0 > (\dot{\omega})_{J_{\rm A}} \propto I_{\rm A}/(P_{\rm A} M_{\rm A}^2) \Longrightarrow I_{\rm A}$$

in a few years $I_{\rm A}$ known within 10%
Morrison et al. 2004, Lattimer & Schutz 2005!

Bejger, Bulik & Haensel 2005 ⟶

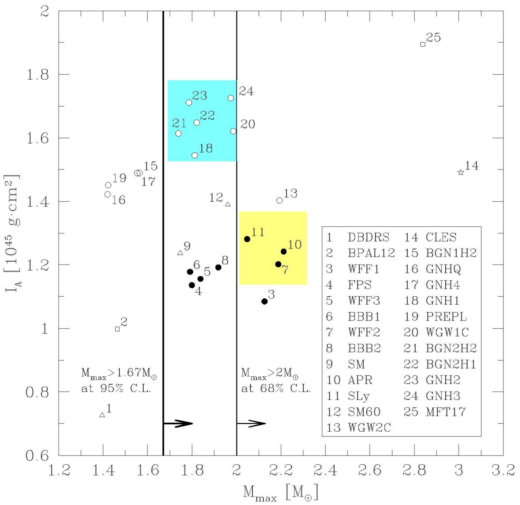
Low-mass NS; sensitive to EOS at $(3-5) \times \rho_0$



I_A and M(PSR J0751+1807)

Several candidates for very massive NS in close, relativistic NS-WD binaries.
At 95% confidence level PSR J07051+1807 has $M = 2.1^{+0.4}_{-0.5}\ M_\odot$.
Different segments of EOS tested:
PSR J07051+1807 - $(6 - 10) \times \rho_0$
PSR J0737-3039A - $(3 - 5) \times \rho_0$
normal nuclear density $\rho_0 = 2.7 \times 10^{14}$ g/cm^3

Bejger, Bulik & Haensel 2005⟶



Conclusion

- I of Crab Pulsar seems to indicate a rather stiff EOS
- Constant acceleration of Crab Nebula expansion and $M_{\rm neb} > 6M_{\odot}$ are inconsistent with "reasonable EOSs"
- Using a simple model of Crab Nebula accelerated expansion one finds that I of Crab Pulsar indicates a stiff EOS
- Precise measurement of the spin-orbit component of the periastron advance in PSR J0737A,B will yield $I_{\rm A}$ for $M_{\rm A} = 1.338M_{\odot}$, testing EOS at $(3-5) \times \rho_0$
- Increase of precision of M for PSR J07051+1807 converging to $2M_{\odot}$ together with $I_{\rm A}$ will hopefully enable us to pick up the correct EOS for $\rho = (3-10) \times \rho_0$