Quantum Field Theory
on
LQC Bianchi Spacetimes

Andrea Dapor, Jerzy Lewandowski, Yaser Tavakoli

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Quantum Field Theory on LQC Bianchi Spacetimes

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Introduction

QFT and Effective Geometry

Next Order
Deformed mass-shell constraint:

\[ p^2 = E^2 \left[ 1 + \xi \frac{E^\alpha}{E_{pl}^\alpha} \right] \]
Lorentz Violation

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Deviation from "conventional" speed of light (\( c = 1 \)): GRB?
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Solution: consider more gravitational dof’s: \textit{Bianchi I}. 

Introduction

Quantum Field Theory on LQC Bianchi Spacetimes

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Quantum Bianchi I Spacetime

\[ g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + \sum_{i=1}^{3} a_i^2(t)(dx^i)^2 \]
Quantum Bianchi I Spacetime

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Ashtekar variables: \( A_i^a = c_i \delta^a_i \) and \( E_a^i = p_i \delta_a^i \).

\[ p_1 = a_2 a_3, \quad p_2 = a_3 a_1, \quad p_3 = a_1 a_2 \]
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\( \Rightarrow \mathcal{H}_{geo} \) spanned by \( \hat{p}_i \)-eigenstates \( |\tilde{\lambda} \rangle := |\lambda_1, \lambda_2, \lambda_3 \rangle \)
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\[-i\hbar \partial_T \Psi_0(T, \tilde{\lambda}) = \hat{H}_0 \Psi_0(T, \tilde{\lambda})\]
Real scalar field $\phi$

$$L_\phi = \frac{1}{2}(g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - m^2 \phi^2)$$
Real Scalar Field

Real scalar field $\phi$

$$\mathcal{L}_\phi = \frac{1}{2} (g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - m^2 \phi^2)$$

Mode decomposition:

$$H_{\phi}(T) = \sum_{\vec{k} \in \mathcal{L}} H_{\vec{k}}(T)$$

where $H_{\vec{k}}$ is Hamiltonian of a h.o.
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Single mode $\vec{k}$:

$$\mathcal{H}_{kin}^{(\vec{k})} = \mathcal{H}_{\vec{k}} \otimes \mathcal{H}_{geo}, \quad \hat{C}_{(\vec{k})} = \hat{H}_{\vec{k}} + \hat{C}_{geo}$$
Single mode $\vec{k}$:

$$\mathcal{H}^{(\vec{k})}_{kin} = \mathcal{H}_{\vec{k}} \otimes \mathcal{H}_{geo}, \quad \mathcal{C}^{(\vec{k})} = \hat{H}_{\vec{k}} + \hat{C}_{geo}$$

$$-i\hbar \partial_T \Psi(T, \vec{\lambda}, q_{\vec{k}}) = \left[ \hat{H}_0 - \hat{H}_0^{-\frac{1}{2}} \hat{H}_{\vec{k}} \hat{H}_0^{-\frac{1}{2}} \right] \Psi(T, \vec{\lambda}, q_{\vec{k}})$$
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\]

We want to extract an equation for matter only.
QFT on QS

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We want to extract an equation for matter only.
$\Rightarrow$ Take the ”classical geometry” limit.
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$$-i\hbar \partial_T \Psi(T, \vec{\lambda}, q_{\vec{k}}) = \left[ \hat{H}_0 - \hat{H}_{\vec{k}}^{-\frac{1}{2}} \hat{H}_{\vec{k}} \hat{H}_0^{-\frac{1}{2}} \right] \Psi(T, \vec{\lambda}, q_{\vec{k}})$$

We want to extract an equation for matter only.
⇒ Take the ”classical geometry” limit.

Test field approximation:

$$\Psi(T, \vec{\lambda}, q_{\vec{k}}) = \Psi_0(T, \vec{\lambda}) \otimes \psi(T, q_{\vec{k}}), \quad -i\hbar \partial_T \Psi_0 = \hat{H}_0 \Psi_0$$
Single mode $\vec{k}$:

$$\mathcal{H}^{(\vec{k})}_{kin} = \mathcal{H}^\vec{k}_{kin} \otimes \mathcal{H}_{geo}, \quad \mathcal{C}^{(\vec{k})} = \mathcal{H}^\vec{k} + \mathcal{C}_{geo}$$

$$-i\hbar \partial_T \Psi(T, \vec{\lambda}, q_{\vec{k}}) = \left[ \hat{H}_0 - \hat{H}_0^{-\frac{1}{2}} \hat{H}^\vec{k} \hat{H}_0^{-\frac{1}{2}} \right] \Psi(T, \vec{\lambda}, q_{\vec{k}})$$

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⇒

$$i\hbar \partial_T \psi = \langle \hat{H}^{-\frac{1}{2}}_0 \hat{H}^\vec{k} \hat{H}_0^{-\frac{1}{2}} \rangle_{\Psi_0(T, \vec{\lambda})} \psi = \langle \hat{H}^{-\frac{1}{2}}_0 \hat{H}^\vec{k}(T) \hat{H}_0^{-\frac{1}{2}} \rangle_0 \psi =$$

$$= \frac{1}{2} \left[ \langle \hat{H}_0^{-1} \rangle_0 \hat{p}_{\vec{k}}^2 + \sum_{i=1}^3 \langle \hat{H}_0^{-\frac{1}{2}} \hat{p}_i \hat{H}_0^{-\frac{1}{2}} \hat{H}_0^{-\frac{1}{2}} \rangle_0 m_i^2 \right] \hat{q}_{\vec{k}}^2 \psi$$
Effective ”classical” spacetime (ECS) of the Bianchi I form:

\[ \bar{g}_{\mu\nu}dx^\mu dx^\nu = -\tilde{N}^2 dT^2 + |\bar{p}_1\bar{p}_2\bar{p}_3| \sum_{i=1}^{3} \frac{(dx^i)^2}{\bar{p}_i^2} \]
Effective ”classical” spacetime (ECS) of the Bianchi I form:

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Single mode $\vec{k}$:

$$\mathcal{H}_{\vec{k}} = L^2(\mathbb{R}, dq_{\vec{k}})$$
QFT on ECS

Effective ”classical” spacetime (ECS) of the Bianchi I form:

\[ \tilde{g}_{\mu\nu} dx^\mu dx^\nu = -\tilde{N}^2 dT^2 + |\tilde{p}_1\tilde{p}_2\tilde{p}_3| \sum_{i=1}^{3} \frac{(dx^i)^2}{\tilde{p}_i^2} \]

Single mode \( \vec{k} \):

\[ \mathcal{H}_{\vec{k}} = L^2(\mathbb{R}, dq_{\vec{k}}) \]

\[ i\hbar \partial_T \psi(T, q_{\vec{k}}) = \frac{\tilde{N}}{2 \sqrt{|\tilde{p}_1\tilde{p}_2\tilde{p}_3|}} \left[ \tilde{p}_{\vec{k}}^2 + \left( \sum_{i=1}^{3} (\tilde{p}_i k_i)^2 + |\tilde{p}_1\tilde{p}_2\tilde{p}_3| m^2 \right) \tilde{q}_{\vec{k}}^2 \right] \psi(T, q_{\vec{k}}) \]
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Comparison

QFT on QS:

\[ i\hbar \partial_T \psi = \frac{1}{2} \left[ \langle \hat{H}_0^{-1} \rangle_0 \hat{p}_k^2 + \left( \sum_{i=1}^{3} \langle \hat{H}_0^{-\frac{1}{2}} \hat{p}_i \hat{H}_0^{-\frac{1}{2}} \rangle_0 \hat{k}_i^2 + \langle \hat{H}_0^{-\frac{1}{2}} [\hat{p}_1 \hat{p}_2 \hat{p}_3] \hat{H}_0^{-\frac{1}{2}} \rangle_0 m^2 \right) \hat{q}_k^2 \right] \psi \]
**Comparison**

**QFT on QS:**

\[
\begin{align*}
\frac{i\hbar}{2} \partial_T \psi &= \frac{1}{2} \left[ \langle \hat{H}_0^{-1} \rangle_0 \hat{p}_k^2 + \left( \sum_{i=1}^{3} \langle \hat{H}_0^{-\frac{1}{2}} \hat{p}_i \hat{H}_0^{-\frac{1}{2}} \rangle_0 k_i^2 + \langle \hat{H}_0^{-\frac{1}{2}} | \hat{p}_1 \hat{p}_2 \hat{p}_3 | \hat{H}_0^{-\frac{1}{2}} \rangle_0 m^2 \right) \hat{q}_k^2 \right] \psi 
\end{align*}
\]

**QFT on ECS:**

\[
\begin{align*}
\frac{i\hbar}{2} \partial_T \psi &= \frac{\tilde{N}}{2 \sqrt{|\tilde{p}_1 \tilde{p}_2 \tilde{p}_3|}} \left[ \hat{p}_k^2 + \left( \sum_{i=1}^{3} (\tilde{p}_i k_i)^2 + |\tilde{p}_1 \tilde{p}_2 \tilde{p}_3| m^2 \right) \hat{q}_k^2 \right] \psi 
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\[ i\hbar \partial_T \psi = \frac{1}{2} \left[ \langle \hat{H}_0^{-1} \rangle_0 \hat{p}_k^2 + \left( \sum_{i=1}^{3} \langle \hat{H}_0^{-\frac{1}{2}} \hat{p}_i \hat{H}_0^{-\frac{1}{2}} \rangle_0 k_i^2 + \langle \hat{H}_0^{-\frac{1}{2}} [\hat{p}_1 \hat{p}_2 \hat{p}_3 | \hat{H}_0^{-\frac{1}{2}} \rangle_0 m^2 \right) \hat{q}_k^2 \right] \psi \]

QFT on ECS:

\[ i\hbar \partial_T \psi = \frac{\tilde{N}}{2 \sqrt{|\tilde{p}_1 \tilde{p}_2 \tilde{p}_3|}} \left[ \hat{p}_k^2 + \left( \sum_{i=1}^{3} (\tilde{p}_i k_i)^2 + |\tilde{p}_1 \tilde{p}_2 \tilde{p}_3| m^2 \right) \hat{q}_k^2 \right] \psi \]

\[ \Rightarrow \text{effective metric:} \]

\[ \tilde{g}_{\mu\nu} dx^\mu dx^\nu = \langle \hat{H}_0^{-1} \rangle_0^{1/2} \left( \prod_{i=1}^{3} \langle \hat{H}_0^{-\frac{1}{2}} \hat{p}_i^2 (T) \hat{H}_0^{-\frac{1}{2}} \rangle_0 \right)^{1/2} \times \]

\[ \times \left[ -dT^2 + \sum_{i=1}^{3} \frac{1}{\langle \hat{H}_0^{-1} \rangle_0 \langle \hat{H}_0^{-\frac{1}{2}} \hat{p}_i^2 (T) \hat{H}_0^{-\frac{1}{2}} \rangle_0} (dx^i)^2 \right] \]
QFT on QS:

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i\hbar \partial_T \psi = \frac{1}{2} \left[ \langle \hat{H}_0^{-1} \rangle_0 \hat{p}_k^2 + \left( \sum_{i=1}^{3} \langle \hat{H}_0^{-\frac{1}{2}} \hat{p}_i \hat{H}_0^{-\frac{1}{2}} \rangle_0^2 k_i^2 + \langle \hat{H}_0^{-\frac{1}{2}} |\hat{p}_1 \hat{p}_2 \hat{p}_3| \hat{H}_0^{-\frac{1}{2}} \rangle_0 m^2 \right) \hat{q}_k^2 \right] \psi
\]

QFT on ECS:

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\Rightarrow \text{effective metric:}

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\bar{g}_{\mu\nu} dx^\mu dx^\nu = \langle \hat{H}_0^{-1} \rangle_0^{1/2} \left( \prod_{i=1}^{3} \langle \hat{H}_0^{-\frac{1}{2}} \hat{p}_i^2 (T) \hat{H}_0^{-\frac{1}{2}} \rangle_0 \right)^{1/2} \times
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\[
\times \left[ -dT^2 + \sum_{i=1}^{3} \frac{1}{\langle \hat{H}_0^{-1} \rangle_0 \langle \hat{H}_0^{-\frac{1}{2}} \hat{p}_i^2 (T) \hat{H}_0^{-\frac{1}{2}} \rangle_0} (dx^i)^2 \right]
\]

Dispersion relation for mode \( \vec{k} \) on the background \( \bar{g}_{\mu\nu} \) gives

\[
\nu = 1
\]
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Atomic B-O

\[-i\hbar \partial_t \Psi = \hat{H} \Psi = [T_n + \hat{H}_e] \Psi\]
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Atomic B-O

\[-i\hbar \partial_t \Psi = \hat{H}\Psi = [\hat{T_n} + \hat{H}_e]\Psi\]

- heavy dof’s: nucleus position, \(n\)
Atomic B-O

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Atomic B-O

\[-i\hbar \partial_t \Psi = \hat{H}\Psi = [\hat{T}_n + \hat{H}_e]\Psi\]

- heavy dof’s: nucleus position, \(n\)
- light dof’s: electron position, \(e\)

On the (Coulomb) background, solve the eigenequation for \(\hat{H}_e\):

\[\hat{H}_e \chi_i(e) = \epsilon_i(n) \chi_i(e)\]
Atomic B-O

\[-i\hbar \partial_t \Psi = \hat{H} \Psi = [\hat{T}_n + \hat{H}_e] \Psi\]

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On the (Coulomb) background, solve the eigenequation for \( \hat{H}_e \):

\[\hat{H}_e \chi_i(e) = \epsilon_i(n) \chi_i(e)\]

Substitute back, and solve the eigenequation for \( \hat{H} \):

\[\Phi_\alpha = \sum_i \varphi_{i,\alpha}(n) \chi_i(e), \quad [\hat{T}_n + \epsilon_i(n)] \varphi_{i,\alpha}(n) = E_\alpha \varphi_{i,\alpha}(n)\]
Atomic B-O

\[ -i\hbar \partial_t \Psi = \hat{H}\Psi = [\hat{T}_n + \hat{H}_e]\Psi \]

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The ”corrected” state of the system:

\[ \Psi_0 = \sum_\alpha c_\alpha \Phi_\alpha^0 \quad \rightarrow \quad \Psi = \sum_\alpha c_\alpha \Phi_\alpha = \sum_{i,\alpha} c_\alpha \varphi_{i,\alpha} \chi_i \]
Cosmological B-O

\[-i\hbar \partial_T \Psi = \left[ \frac{1}{2} \hat{H}_0^2 - \hat{H}_k \right] \Psi\]
Cosmological B-O

\[-i\hbar \partial_{\vec{T}} \Psi = \left[ \frac{1}{2} \hat{H}_0^2 - \hat{H}_k \right] \Psi\]

- heavy dof’s: geometry, \( \lambda \)
Cosmological B-O

\[-i\hbar \partial T \Psi = \left[ \frac{1}{2} \hat{H}_0^2 - \hat{H}_k \right] \Psi\]

- heavy dof’s: geometry, \(\lambda\)
- light dof’s: matter, \(q_k\)
Cosmological B-O

\[-i\hbar \partial_{\overline{T}} \Psi = \left[ \frac{1}{2} \hat{H}_0^2 - \hat{H}_k \right] \Psi\]

- heavy dof’s: geometry, \( \lambda \)
- light dof’s: matter, \( q_{\vec{k}} \)

On the background \( \Psi_0 \), solve the eigenequation for \( \hat{H}_k \):

\[\hat{H}_k \chi_i(q_{\vec{k}}) = \epsilon_i(p) \chi_i(q_{\vec{k}})\]
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Cosmological B-O

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- heavy dof’s: geometry, \(\lambda\)
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On the background \(\Psi_0\), solve the eigenequation for \(\hat{H}_k\):

\[\hat{H}_k \chi_i(q_{k}) = \epsilon_i(p) \chi_i(q_{k})\]

Substitute back:

\[\Phi_\alpha = \sum_i \varphi_{i,\alpha}(\lambda) \chi_i(q_{k}), \quad \left[ \frac{1}{2} \hat{H}_0^2 + \epsilon_i(\hat{p}) \right] \varphi_{i,\alpha}(\lambda) = E_\alpha \varphi_{i,\alpha}(\lambda)\]
Cosmological B-O

\[-i\hbar \partial_{\bar{T}} \Psi = \left[ \frac{1}{2} \widehat{H}_0^2 - \widehat{H}_k \right] \Psi\]

- heavy dof’s: geometry, \(\lambda\)
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Substitute back:

\[\Phi_{\alpha} = \sum_i \varphi_{i,\alpha}(\lambda) \chi_i(q_{\vec{k}}), \quad \left[ \frac{1}{2} \widehat{H}_0^2 + \epsilon_i(\hat{p}) \right] \varphi_{i,\alpha}(\lambda) = E_{\alpha} \varphi_{i,\alpha}(\lambda)\]

The ”corrected” state of the system:

\[\Psi_0 = \sum_{\alpha} c_{\alpha} \Phi_{\alpha}^0 \rightarrow \Psi = \sum_{\alpha} c_{\alpha} \Phi_{\alpha} = \sum_{i,\alpha} c_{\alpha} \varphi_{i,\alpha} \chi_i\]
Lorentz violation

\[ \Psi = \Psi_0 \otimes \psi + \delta \Psi \]

where

\[ \delta \Psi = \sum_{i, \alpha, \beta \neq \alpha} c_\alpha \frac{\langle \Phi^0_\beta | \epsilon_i (\hat{p}) | \Phi^0_\alpha \rangle}{E^0_\beta - E^0_\alpha} \Phi^0_\beta \chi_i \]
Lorentz violation

\[ \Psi = \Psi_0 \otimes \psi + \delta \Psi \]

where

\[ \delta \Psi = \sum_{i,\alpha,\beta \neq \alpha} c_\alpha \frac{\langle \Phi_0^0 | \epsilon_i(\hat{p}) | \Phi_0^0 \rangle}{E_\beta^0 - E_\alpha^0} \Phi_\beta^0 \chi_i \propto k \]
Lorentz violation

\[ \Psi = \Psi_0 \otimes \psi + \delta \Psi \]

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\[ \delta \Psi = \sum_{i, \alpha, \beta \neq \alpha} c_{\alpha} \frac{\langle \Phi_0^0 | \epsilon_i(\hat{p}) | \Phi_0^0 \rangle}{E_0^0 - E_0^0} \Phi_0^0 \chi_i \propto k \]

\[ \Rightarrow \text{"corrected" effective metric:} \]

\[ \bar{g}_{\mu\nu} dx^\mu dx^\nu = -(1 + \xi k \ell_P^2) \langle \hat{p}^2 \rangle^{3/2} \bar{T}^2 + \langle \hat{p}^2 \rangle^{1/2} \sum_i (dx^i)^2 \]
Lorentz violation

\[ \Psi = \Psi_0 \otimes \psi + \delta \Psi \]

where

\[ \delta \Psi = \sum_{i, \alpha, \beta \neq \alpha} c_\alpha \frac{\langle \Phi^0_\beta | \epsilon_i(\hat{p}) | \Phi^0_\alpha \rangle}{E^0_\beta - E^0_\alpha} \Phi^0_\beta \chi_i \propto k \]

⇒ ”corrected” effective metric:

\[ \bar{g}_{\mu \nu} dx^\mu dx^\nu = -(1 + \xi k \ell_{Pl})^2 \langle \hat{p}^2 \rangle^{3/2} dT^2 + \langle \hat{p}^2 \rangle^{1/2} \sum_i (dx^i)^2 \]

Dispersion relation for mode \( \tilde{k} \) on the background \( \bar{g}_{\mu \nu} \) gives

\[ \nu = 1 + \frac{\xi}{2} k \ell_{Pl} \]
Lorentz violation

\[ \Psi = \Psi_0 \otimes \psi + \delta \Psi \]

where

\[ \delta \Psi = \sum_{i, \alpha, \beta \neq \alpha} c_\alpha \frac{\langle \Phi_0^0 | \epsilon_i(\hat{p}) | \Phi_0^0 \rangle}{E_\beta^0 - E_\alpha^0} \Phi_0^0 \chi_i \propto k \]

⇒ ”corrected” effective metric:

\[ \bar{g}_{\mu\nu} dx^\mu dx^\nu = -(1 + \xi k \ell_{Pl})^2 \langle \hat{p}^2 \rangle^{3/2} d\tilde{T}^2 + \langle \hat{p}^2 \rangle^{1/2} \sum_i (dx^i)^2 \]

Dispersion relation for mode \( \vec{k} \) on the background \( \bar{g}_{\mu\nu} \) gives

\[ v = 1 + \frac{\xi}{2} k \ell_{Pl} \]

Lorentz violation around \( E \sim E_{Pl} \) (GRB bound: \( \sim 10^{-2} E_{Pl} \)).
Conclusions

What we saw:

• first steps toward QFT on Bianchi LQC spacetimes
• concept of “effective geometry” felt by quanta of matter
• no L-violation in Bianchi I case at 0th order (test field approx.)
• possible L-violation when backreaction is taken into account

What next:

• try to include massive fields
• refine the QFT part: consider an infinite number of modes
• check the validity of B-O scheme, and explicitly compute ξ
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