The world of ancient Greece
Thales’ forecast of the solar eclipse on May 28, 585 B.C.

"After this, war lasted between the Lydians and the Medes for five years... In the sixth year, when they were carrying on the war with nearly equal success, on occasion of an engagement, it happened that in the heat of the battle day was suddenly turned into night. This change of the day Thales the Milesian had foretold to the Ionians, fixing beforehand this year as the very period, in which the change actually took place. The Lydians and Medes seeing night succeeding in the place of day, desisted from fighting, and both showed a great anxiety to make peace..."

Herodotus, *History*, Book I
Ionian philosophers

Thales of Miletus  
Θαλής ὁ Μιλήσιος
(Water)  
(610-545 B.C.)

Anaximander  
Ἀναξιμανδρός
(απειρον)  
(c. 610-545 B.C.)

Anaximenes  
Ἀναξιμένης
(空气)  
(c. 585-525 B.C.)

Heraclitus of Ephesus  
Ἡράκλειτος ὁ Ἑφέσος
(火)  
(c. 540-480 B.C.)
Pythagoras of Samos (ca. 570-497 B.C.)

"Actually, everything that can be known has a Number; for it is impossible to grasp anything with the mind or to recognize it without this"

The universe is κοσμός

The Pythagoreans were the first to have attempted to give a quantitative, mathematical foundation to the knowledge of nature
"...the so-called Pythagoreans, who were the first to take up mathematics, not only advanced this study, but also having been brought up in it they thought its principles were the principles of all things. Since of these principles numbers are by nature the first, and in numbers they seemed to see many resemblances to the things that exist and come into being – more than in fire and earth and water (such and such a modification of numbers being justice, another being soul and reason, another being opportunity – and similarly almost all other things being numerically expressible); since, again, they saw that the modifications and the ratios of the musical scales were expressible in numbers; since, then, all other things seemed in their whole nature to be modelled on numbers, and numbers seemed to be the first things in the whole of nature, they supposed the elements of numbers to be the elements of all things, and the whole heaven to be a musical scale and a number."

Aristotle, *Metaphysics*, Book I, Ch. 5
"And all the properties of numbers and scales which they could show to agree with the attributes and parts and the whole arrangement of the heavens, they collected and fitted into their scheme; and if there was a gap anywhere, they readily made additions so as to make their whole theory coherent. For example, as the number 10 is thought to be perfect and to comprise the whole nature of numbers, they say that the bodies which move through the heavens are ten, but as the visible bodies are only nine, to meet this they invent a tenth - the ‘counter-earth’.

Aristotle, *Metaphysics*, Book I, Ch. 5
"It remains to speak of the earth, where it is, whether it should be classed among things at rest or things in motion, and of its shape. Concerning its position there is some divergence of opinion. Most of those who hold that the whole Universe is finite say that it lies at the centre, but this is contradicted by the Italian school called Pythagoreans. These affirm that the centre is occupied by fire, and that the earth is one of the stars, and creates night and day as it travels in a circle about the centre. In addition they invent another earth, lying opposite our own, which they call counter-earth, not seeking accounts and explanations in conformity with appearances, but trying by violence to bring the appearances into line with accounts and opinions of their own."

Aristotle, *On the Heavens*, Book II, Ch. 13
The Pythagoreans divided numbers into triangular, square, rectangular etc.

triangular numbers

\[ 1 + 2 + 3 + 4 = 10 \text{ (τετρακτύς)} \]

square numbers

1 : 2 octave  \hspace{1cm} 2 : 3 fifth  \hspace{1cm} 3 : 4 fourth
The Pythagoreans divided mathematics into four parts: arithmetic, geometry, music, and astronomy

(music was treated as applied arithmetic, and astronomy - as applied geometry)

-> *quadrivium* in medieval universities
The atoms are indivisible, hard, of different shapes but without colour, taste or smell; they move spontaneously and ceaselessly in the vacuum; they are invisible because of their smallness.
Empedocles of Acragas
(ca. 483-423 B.C.)

Theory of four elements (ῥιξώματα)

Two opposing principles: φιλία - love and νείκος - strife; the first mingle the elements, the second separates them
Acragas was one of the largest Greek cities in the 5th century B.C. The number of its inhabitants was four times larger than it is now.
Anaxagoras of Clazomenae  
(ca. 500-428 B.C.)

He maintained, among other things, that the sun is a fiery stone larger than Peloponnese. Accused of blasphemy, brought to trial, and exiled from Athens.

The first known case of persecution for scientific ideas.
Plato (428-347 B.C.)

Physics problems discussed mainly in *Timaeus* and *Critias*

Founder of [Plato’s] Academy (387 B.C. - 529)
Platonic solids

- **tetrahedron**   **fire**
- cube (hexahedron)   **earth**
- **octahedron**   **air**
- icosahedron   **water**
- dodecahedron   **ether**
"Now all these bodies we must conceive as being so small that each single body in the several kinds cannot for its smallness be seen by us at all; but when many are heaped together, their united mass is seen."

Timaeus Ch. IX
Plato’s Academy and Aristotle’s Lyceum
Eudoxos of Cnidos  
(ca. 408-355 B.C.)

Number of homocentric spheres

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<th>Eudoxos</th>
<th>Calippos</th>
<th>Aristotle</th>
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"But it is necessary, if all the spheres combined are to explain the observed facts, that for each of the planets there should be other spheres (one fewer than those hitherto assigned) which counteract those already mentioned and bring back to the same position the outermost sphere of the star which in each case is situated below the star in question; for only thus can all the forces at work produce the observed motion of the planets. Since, then, the spheres involved in the movement of the planets themselves are - eight for Saturn and Jupiter and twenty five for the others, and of these only those involved in the movement of the lowest-situated planet need not be counteracted, the spheres which counteract those of the outermost two planets will be six in number, and the spheres which counteract those of the next four planets will be sixteen; therefore the number of all the spheres - both those which move the planets and those which counteract these - will be fifty five."

Aristotle, *Metaphysics*, Book XII, Ch. 8
"Behold! human beings living in an underground den, which has a mouth open toward the light and reaching all along the den; here they have been from their childhood, and have their legs and necks chained so that they cannot move, and can only see before them, being prevented by the chains from turning round their heads. Above and behind them a fire is blazing at a distance, and between the fire and the prisoners there is a raised way; and you will see, if you look, a low wall built along the way, like the screen which marionette-players have in front of them, over which they show the puppets... you see men passing along the wall carrying all sorts of vessels, and statues and figures of animals made of wood and stone and various materials, which appear over the wall? Some of them are talking, others silent... They are strange prisoners... and they see only their own shadows, or the shadows of one another, which the fire throws on the opposite wall of the cave..."

Plato, *The Republic*, Book 7
Aristotle of Stagira (384-322 B.C.)

Founded a consistent system of knowledge which comprised all aspects of the world.

Several dozen works on natural philosophy, logic, metaphysics, ethics, politics, art, rhetoric, psychology and biology

[Lyceum (school of peripatetics) founded in 335 B.C.]

(περιπατητικοί - those who walk)
„He was the most eminent of all the pupils of Plato; he had a lisping voice, as is asserted by Timotheus the Athenian, in his work on Lives. He had also very thin legs, they say, and small eyes; but he used to indulge in very conspicuous dress, and rings, and used to dress his hair carefully.”

Diogenes Laertios
Physical ideas of Aristotle are discussed mainly in the following works:

\[ Άριστοτέλους \, \text{φυσικὴ} \, \text{ἀκρόασις} \\
\text{Περὶ} \, \text{Οὐρανοῦ} \\
\text{Μετεωρολογικὰ} \\
\text{Περὶ} \, \text{γενέσεως} \, \text{kai} \, \text{θορᾶς} \\
\text{Τὰ} \, \text{μετὰ} \, \text{τὰ} \, \text{φυσικὰ} \\
\text{Περὶ} \, \text{ψυχῆς} \\
\text{Μηχανικὰ} \, \text{προβλήματα} \\
\text{Περὶ} \, \text{χρωμάτων} \]

\text{Physics} \\
\text{On the Heavens} \\
\text{Meteorology} \\
\text{On generation and corruption} \\
\text{Metaphysics} \\
\text{On the soul} \\
\text{Mechanics*} \\
\text{On colours} \\

* attributed to Aristotle
Summary of Aristotle’s physics

1. World is finite, divided into two separate parts with different laws of physics and different matter:
   - **sublunar sphere** - four elements
   - **supralunar sphere** - ether

2. Matter and form

3. Motion: realisation of a potentiality, requires a cause

4. Causes of four kinds (**material, efficient, formal, final**)

5. Notion of a **natural place**

6. Translatory motion: natural or forced

7. Principles of Aristotle’s dynamics in the sublunar sphere:
   1. A body not acted upon remains at rest
   2. Speed of a body put into motion by an external cause is proportional to acting force and inversely proportional to the resistance of a medium

8. Vacuum can not exist
"Everything comes to be from both **matter** and **form**”

*Physics*, Book I, Ch. 7

Four causes according to Aristotle:

**Material** cause: what is something made of?
**Efficient** cause: what brings something about?
**Formal** cause: what characteristics does an object have?
**Final** cause: what is the reason for something’s existence?

*Physics*, Book II, Ch. 3
Aristotle’s universe was finite

"For the infinite to move at all is thus absolutely impossible; since the very smallest movement conceivable must take an infinity of time. Moreover the heavens certainly revolve, and they complete their circular orbit in a finite time."

*On the Heavens*, Book I, Ch. 9
Aristotle’s system of the world

”It is therefore evident that there is also no place or void or time outside the heaven.”

_On the Heavens_, Book I, Ch. 9
“It is clear, then, that time is number of movement in respect of the before and after, and is continuous since it is an attribute of what is continuous.”

*Physics*, Book IV, 219b
"We must therefore see that we understand the meaning of 'motion'; for if it were unknown, the meaning of 'nature' too would be unknown."

Aristotle, *Physics*, Book III, Ch. 1
”Again, there is no such thing as motion over and above the things. It is always with respect to substance or to quantity or to quality or to place that what changes changes.”

Aristotle, *Physics*, Book III, Ch. 1
Aristotle’s four types of motion:

”The fulfilment of what exists potentially, in so far as it exists potentially, is motion - namely,
- of what is alterable, alteration:
- of what can be increased and its opposite what can be decreased (there is no common name), increase and decrease:
- of what can come to be and can pass away, coming to be and passing away:
- of what can be carried along, locomotion.”

Aristotle, *Physics*, Book III, Ch.1

(Only the last type remained the subject of mechanics!)
“Everything that is in motion must be moved by something. For if it has not the source of its motion in itself it is evident that it is moved by something other than itself, for there must be something else that moves it.”

Aristotle, *Physics*, Book VII, Ch. 1
"Further, in point of fact things that are thrown move though that which gave them their impulse is not touching them, either by reason of mutual replacement, as some maintain, or because the air that has been pushed pushes them with a movement quicker than the natural locomotion of the projectile wherewith it moves to its proper place. But in a void none of these things can take place, nor can anything be moved save as that which is carried is moved. Further, no one could say why a thing once set in motion should stop anywhere; for why should it stop here rather than there? So that a thing will either be at rest or must be moved *ad infinitum*, unless something more powerful gets in its way."

Aristotle, *Physics*, Book IV, Ch. 8
The ancient Greek mathematicians were convinced that one may consider only proportions of homogenous quantities. Therefore they did not define speed as we do now, as the ratio of distance to time, $s/t$.

The speeds of two motions were compared either
1. by comparing the times of covering the same distance
   or
2. by comparing the distances covered during the same time.

(We shall see later that this tradition held until the middle of the XVIII\textsuperscript{th} century, so that even Galileo and Newton did not consider speed as the ratio of distance to time)
”We see the same weight or body moving faster than another for two reasons, either because there is a difference in what it moves through, as between water, air, and earth, or because, other things being equal, the moving body differs from the other owing to excess of weight or of lightness....”

Aristotle, *Physics*, Book IV, Ch. 8
"...A, then, will move through B in time G, and through D, which is thinner, in time E (if the length of B is equal to D), in proportion to the density of the hindering body. For let B be water and D air; then by so much as air is thinner and more incorporeal than water, A will move through D faster than through B. Let the speed have the same ratio to the speed, then, that air has to water. Then if air is twice as thin, the body will traverse B in twice the time that it does D, and the time G will be twice the time E. And always, by so much as the medium is more incorporeal and less resistant and more easily divided, the faster will be the movement."

Aristotle, *Physics*, Book IV, Ch. 8
”Now there is no ratio in which the void is exceeded by body, as there is no ratio of 0 to a number. For if 4 exceeds 3 by 1, and 2 by more than 1, and 1 by still more than it exceeds 2, still there is no ratio by which it exceeds 0; for that which exceeds must be divisible into the excess plus that which is exceeded, so that will be what it exceeds 0 and 0. For this reason, too, a line does not exceed a point unless it is composed of points! Similarly the void can bear no ratio to the full, and therefore neither can movement through the one to movement through the other, but if a thing moves through the thickest medium such and such a distance in such and such a time, it moves through the void with a speed beyond any ratio.”

Aristotle, *Physics*, Book IV, Ch. 8
"For let Z be void, equal in magnitude to B and to D. Then if A is to traverse and move through it in a certain time, H, a time less than E, however, the void will bear this ratio to the full. But in a time equal to H, A will traverse the part O of A. And it will surely also traverse in that time any substance Z which exceeds air in thickness in the ratio which the time E bears to the time H. For if the body Z be as much thinner than D as E exceeds H, A, if it moves through Z, will traverse it in a time inverse to the speed of the movement, i.e. in a time equal to H. If, then, there is no body in Z, A will traverse Z still more quickly. But we supposed that its traverse of Z when Z was void occupied the time H. So that it will traverse Z in an equal time whether Z be full or void. But this is impossible. It is plain, then, that if there is a time in which it will move through any part of the void, this impossible result will follow: it will be found to traverse a certain distance, whether this be full or void, in an equal time; for there will be some body which is in the same ratio to the other body as the time is to the time."

Aristotle, *Physics*, Book IV, Ch. 8
”A given weight moves a given distance in a given time; a weight which is as great and more moves the same distance in a less time, the times being in inverse proportion to the weights. For instance, if one weight is twice another, it will take half as long over a given movement.”
Aristotle, *On the Heavens*, Book I, Ch. 6

”For any two portions of fire, small or great, will exhibit the same ratio of solid to void, but the upward movement of the greater is quicker than that of the less, just as the downward movement of a mass of gold or lead, or of any other body endowed with weight, is quicker in proportion to its size.”
Aristotle, *On the Heavens*, Book IV, Ch. 2
”Fire, air, water, earth, we assert, originate from one another, and each of them exists potentially in each, as all things do that can be resolved into a common and ultimate substrate.”

Aristotle, *Meteorology* Book I, Ch. 3
Museum in Alexandria

Mouseion - a temple dedicated to the muses

Research and educational institution
(founded by Ptolemy I Soter, ca. 300 B.C.)
Great library (Brucheion and Serapeion)
Botanical garden
Zoological garden
Anatomical laboratory
Astronomical observatory
Great Alexandrian library

about 700,000 rolls in the time of Caesar

48/47 B.C.  Brucheion was destroyed by fire during the fighting of Caesar’s forces with the Egyptians in Alexandria

(389) 391  Sarapeion destroyed by the order of Theophilus, the bishop of Alexandria

642  final destruction by the army of caliph Omar
Lecture halls of ancient Alexandria uncovered recently by Polish archeologists from the Centre of Mediterranean Archeology (University of Warsaw) (phot. Małgorzata Krawczyk, 2003)
Euclid (ca. 365-300 B.C.)

Elements in thirteen books
1-6 Plane geometry,
7-10 Arithmetic, number theory
11-13 Solid geometry

Optics
(contains the law of reflection of light)

Catoptrics
Elements in 13 books

"Many propositions in the *Elements* can be ascribed to earlier geometers; we may assume that those which cannot be ascribed to others were discovered by Euclid himself, and their number is considerable. As to the arrangement, it is safe to assume that it is to a large extent Euclid’s own. He created a monument that is as marvelous in its symmetry, inner beauty, and clearness as the Parthenon, but incomparably more complex and more durable."

(George Sarton)
Reflection of light

The angle of incidence equals the angle of reflection

The earliest known law of physics!
Propagation of light along straight lines has been known since very long time and exploited in architecture. But Euclid first adopted it as the principle and basis of geometrical optics which he founded.
Eratosthenes (ca. 276-194 B.C.)

50 x 5,000 stadia = 250,000 stadia

[ = 39,370 km, if 1 stadium = 157.5 m]
”Also, those mathematicians who try to calculate the size of the earth’s circumference arrive at the figure 400,000 stadia. This indicates not only that the earth’s mass is spherical in shape, but also that as compared with the stars it is not of great size.”
Aristotle, *On the Heavens*, Book II, Ch. 14

”…if astronomical demonstrations are correct and the size of the sun is greater than that of the earth and the distance of the stars from the earth many times greater than that of the sun (just as the sun is further from the earth than the moon), then the cone made by the rays of the sun would terminate at no great distance from the earth, and the shadow of the earth (what we call night) would not reach the stars.”
Aristotle, *Meteorology*, Book 1, Ch. 8
Poseidonios (ca. 135-50 B.C.)

(Observations of Canopus)

Horizon in Rhodes

Horizon in Alexandria
Aristarchus of Samos (ca. 310-240 B.C.)

The postulates

1. The moon gets its light from the sun.
2. The earth is a point in the centre of moon’s sphere.
3. When we look at the half moon, the great circle dividing the dark and the bright part points directly at our eyes.
4. The half moon has an angular distance from the sun equal to one quadrant minus 1/30 of a quadrant.
5. The width of the earth’s shadow is twice the diameter of the moon.
6. The moon comprises 1/15 of a zodiacal sign.
Aristarchus of Samos

The results

1. The ratio of distances earth-sun and earth-moon = 19
2. Sun’s diameter = 19 moon’s diameters
3. The radius of moon’s orbit = 9^{1/2} earth’s diameters
4. Sun’s diameter = 6^{3/4} earth’s diameters
5. Earth’s diameter = \frac{57}{20} (= 2.85) moon’s diameters

Present values

(-400) (-400) (-30) (~109) (~3.7)
Aristarchus of Samos

\[ \angle EBH = \angle BAC = 3^\circ \]

\[ \angle FBE = 45^\circ, \angle GBE = 22.5^\circ \]

Theorem. The ratio of large and small segments of a tangent to a circle is larger than the ratio of angles and arcs which subtend them.

\[ \frac{GE}{HE} > \frac{[90^\circ/4]}{[90^\circ/30]} = \frac{15}{2} \]

\[ \frac{FG}{GE} = \frac{FB}{BE} = \sqrt{2} > \frac{7}{5} ; \frac{FE}{GE} > \frac{12}{5} \]

Hence \[ \frac{FE}{HE} = \left( \frac{FE}{GE} \right) \left( \frac{GE}{HE} \right) > \left( \frac{15}{2} \right) \left( \frac{12}{5} \right) = 18 \]

\[ \frac{BH}{HE} = \frac{AB}{BC} > \frac{BE}{HE} = \frac{FE}{HE} \]

and \[ \frac{AB}{BC} > 18 \]
Aristarchus of Samos

\[ \angle EBH = \angle BAC = 3^\circ \]
\[ \angle FBE = 45^\circ, \angle GBE = 22.5^\circ \]

Theorem. The ratio of large and small segments of a chord is smaller than the ratio of arcs which subtend them.

DE subtends an angle of 6° on the semicircle BDE, Side BE/2 of a regular hexagon subtends an angle of 60°, \([BE/2] / DE < 10\), hence \(BE / DE = AB / BC < 20\)

Finally \(18 < AB/BC < 20\)
Thus the sun is about 19 times larger than moon, because they are of similar apparent size
Distance EM = \( \frac{1}{20} \times \frac{17}{19} = \frac{17}{20 \times 19} \)

Distance ES = \( \frac{19}{20} \times \frac{17}{19} = \frac{17}{20} = 19 \) EM

Distance from E to the end of the shadow = \( \frac{17}{19} \times \frac{1}{20} + \frac{2}{19} \)

The ratio of the diameters of E and M is equal to

\[
\frac{2/19 + (17/19)(1/20)}{2/19} = \frac{57}{20}
\]
"Now you are aware that 'universe' is the name given by most astronomers to the sphere whose centre is the centre of the earth and whose radius is equal to the straight line between the centre of the sun and the centre of the earth... But Aristarchus of Samos brought out a book consisting of some hypotheses, in which the premisses lead to the result that the universe is many times greater than that now so called. His hypotheses are that the fixed stars and the sun remain unmoved, that the earth revolves about the sun in the circumference of a circle, the sun lying in the middle of the orbit, and that the sphere of the fixed stars, situated about the same centre as the sun, is so great that the circle in which he supposes the earth to revolve bears such a proportion to the distance of the fixed stars as the centre of the sphere bears to its surface."

Archimedes, *The Sand-Reckoner*
Archimedes (287-212 B.C.)

Most works on pure mathematics

On the Sphere and Cylinder
Quadrature of the Parabola
On Spirals
On Conoids and Spheroids
Measurement of the Circle
Sand-Reckoner

$3 \frac{10}{71} < \pi < 3 \frac{10}{70}$ from the analysis of regular polygons with 96 sides

On the Equilibrium of Planes
On Floating Bodies
Method of exhaustion
Archimedes – *On the equilibrium of planes*

"I postulate the following:

1. Equal weights at equal distances are in equilibrium, and equal weights at unequal distances are not in equilibrium but incline towards the weight which is at the greater distance.
2. If, when weights at certain distances are in equilibrium, something be added to one of the weights, they are not in equilibrium but incline towards that weight to which the addition was made.
3. Similarly, if anything be taken away from one of the weights, they are not in equilibrium but incline towards the weight from which nothing was taken.

Proposition 1. Weights which balance at equal distances are equal.
Proposition 2. Unequal weights at equal distances will not balance but will incline towards the greater weight.
Proposition 3. Unequal weights will balance at unequal distances, the greater weight being at the lesser...

Proposition 6.
Two magnitudes... balance at distances reciprocally proportional to the magnitudes.”
The principle of the lever was given almost a century earlier in Aristotelian *Mechanics* (but without proof):

"As the weight moved is to the weight moving it, so, inversely, is the length of the arm bearing the weight to the length of the arm nearer to the power."

Application of geometry to describe physical objects was considered impossible by both Aristotle and Plato, but it was achieved by Archimedes.

Plato: mathematical theorems are ideal, eternal, therefore real and true, while the world perceived by our senses lacks reality and veracity.

Aristotle: mathematics deals with abstraction while physical objects are real and should be described in the language of forms and qualities.
Resumé of mathematics until the time of Ptolemy

Egyptian and Babylonian mathematics - no proofs, just recipes

Greek mathematics: use of proofs already in the 5th century B.C. Irrational numbers discovered by the Pythagoreans

3rd century B.C.: foundations of geometry completed, beginnings of the number theory, theory of conic sections, ancient forms of the integral (the exhaustion method) and differential calculus.

First applications to mechanics, music, optics.
Later on: trigonometry of chords, spherical geometry, and trigonometry.

That level of mathematics was sufficient even for the complicated system of Ptolemy
"Arabic" multiplication table

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Roman multiplication table

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Seven symbols: I  V  X  L  C  D  M
# Greek multiplication table

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<th>γ</th>
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<th>ε</th>
<th>ζ</th>
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</tr>
</tbody>
</table>

27 symbols: α β γ δ ε ζ η θ ι κ λ μ ν ξ ο π ρ σ τ υ ϕ χ ψ ω ϕη
chord $AB = \text{crd} \alpha = 2 \sin \frac{1}{2} \alpha$
Almagest
Planetary hypotheses
Geography
Optics
Tetrabiblos
Centiloquium
13 Books:
1-2 Daily motion of the celestial sphere, table of chords;
3-4 Length of the year and month, solar theory, theory of moon’s motion;
5 Construction of astrolabe; 6 Eclipses;
7-8 Precession, Catalogue of 1022 stars
   (15 1m, 45 2m, 208 3m, 474 4m, 217 5m, 49 6m and nebular);
9-13 Theory of planetary motion
## Average distance from the earth (in earth’s radii)

<table>
<thead>
<tr>
<th></th>
<th>Ptolemy</th>
<th>al-Battani</th>
<th>Copernicus</th>
<th>Tycho</th>
<th>Now</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td>1,210</td>
<td>1,108</td>
<td>1,142</td>
<td>1,150</td>
<td>23,455</td>
</tr>
<tr>
<td>Saturn</td>
<td>17,026</td>
<td>15,509</td>
<td>10,477</td>
<td>10,550</td>
<td>224,345</td>
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<tr>
<td>Fixed stars</td>
<td>20,000</td>
<td>19,000</td>
<td>-</td>
<td>14,000</td>
<td>$6.35 \times 10^9$ (Proxima)</td>
</tr>
</tbody>
</table>
The tools of Ptolemaic astronomy

Epicycle
Equant
Eccentric

Ptolemy’s lunar theory
The effect of an ellipse produced by an epicycle
The effect of a loop produced by an epicycle
Peculiarities of the Ptolemaic system:

1. The centres of epicycles of internal planets (Mercury and Venus) - always at the sun-earth line
2. The lines joining the external planets (Mars, Jupiter and Saturn) with the centres of their epicycles - always parallel to the sun-earth line

(Note: this simplified diagram does not contain equants and multiple epicycles)
Ptolemy - *Optics*

Refraction of light entering water from air

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\beta_{\text{true}}$</th>
<th>Difference</th>
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<tbody>
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<td>10°</td>
<td>8°</td>
<td>7°29’</td>
<td>+31’</td>
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<tr>
<td>20°</td>
<td>15°30’</td>
<td>14°52’</td>
<td>+38’</td>
</tr>
<tr>
<td>30°</td>
<td>22°30’</td>
<td>22°01’</td>
<td>+29’</td>
</tr>
<tr>
<td>40°</td>
<td>29°</td>
<td>28°49’</td>
<td>+11’</td>
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<tr>
<td>50°</td>
<td>35°</td>
<td>35°04’</td>
<td>- 4’</td>
</tr>
<tr>
<td>60°</td>
<td>40°30’</td>
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</tr>
<tr>
<td>70°</td>
<td>45°30’</td>
<td>44°48’</td>
<td>+42’</td>
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<tr>
<td>80°</td>
<td>50°</td>
<td>47°36’</td>
<td>+2°24’</td>
</tr>
</tbody>
</table>

Law of refraction of light remained unknown
(approximation used: $\alpha/\beta = \text{const}$ or $\beta = a\alpha + b\alpha^2$)

Scheme of Ptolemy’s apparatus for measurements of refraction of light
Hero(n) of Alexandria (ca.10-75)

Mechanics (simple machines)
Pneumatica (automata, vacuum,...)
Catoptrics (mirrors, principle of the shortest path,...)

"...that void spaces exist may be seen from the following considerations: for, if there were no such spaces, neither light, nor heat nor any other material force could penetrate through water, or air, or any other body whatever. How could the rays of the sun, for example, penetrate through water to the bottom of the vessel?...

It is clear, too, that void spaces exist in water from this, that, when wine is poured into water, it is seen to spread itself through every part of the water, which it would not if there no vacua in the water. Again, one light traverses another, for, when several lamps are lighted, all objects are brilliantly illuminated, the rays passing in every direction through each other..."

Pneumatica, Introduction
Heron's principle: the path taken by light in going from some point A to a point B via a reflecting surface is the shortest one when the angle of incidence is equal to the angle of reflection.
Examples of Greek technology

automatic temple door
animation: P. Hausladen RS Voehringen
Examples of Greek technology

vending machine for water and wine

Heron’s aeolipile "wind ball"

water-clock constructed by Ctesibios
The Antikythera device

Found in the wreck of a ship which sank in 65 B.C.
The Antikythera device

Antikythera Mechanism Research Project

Many gears have been decoded and inscriptions deciphered; a reconstruction of the system of gears has been produced.

The examples of Roman engineering

Roman temple in Baalbek (Lebanon) 2nd cent. B.C.

Roman aqueduct Pont du Gard

Mausoleum of Theodoric (Ravenna)
The examples of Roman engineering

Temple of Jupiter in Nimes

Colosseum in Rome
(50,000 seats)
Hipparchus
Lucretius
Varro
Vitruvius
Seneca
Pliny
Heron
Ptolemy

Capella
Boethius
Cassiodorus
Philoponus
Isidore
Roman scientists

**Vitruvius** (Marcus Vitruvius Pollio) 1st cent. B.C.
- *De Architectura libri X*

**Varro** (Marcus Terentius Varro) (116-27 B.C.)
- *Disciplinarum libri IX*
  (Encyclopaedia of 9 disciplines: grammar, dialectics, rhetoric, arithmetic, geometry, astronomy, music, medicine, architecture)
Roman scientists

Lucretius (Titus Lucretius Caro) (ca. 95-55 B.C.) - De rerum natura

Pliny the Elder (Gaius Plinius Secundus) (23-79)
- Naturalis historia (Natural History in 37 books)
- quoted 327 Greek and 146 Roman authors

Seneca the Younger (Lucius Annaeus Seneca) (ca. 3-65) - Quaestiones naturales
"The work of Newton cannot be understood without a knowledge of antique science. Without the stupendous work of Ptolemy, which completed and closed antique astronomy, Kepler’s *Astronomia nova*, and hence the mechanics of Newton, would have been impossible. Without the conic sections of Apollonius, which Newton knew thoroughly, his development of the law of gravitation is equally unthinkable. And Newton’s integral calculus can be understood only as a continuation of Archimedes’ determination of areas and volumes. The history of mechanics as an exact science begins with the laws of the lever, the laws of hydrostatics and the determination of mass centers by Archimedes. In short, all the developments which converge in the work of Newton, those of mathematics, of mechanics and of astronomy, begin in Greece."

*Van der Waerden - Science awakening*
Additional explanatory slides
Solar eclipse from the ISS
The apparent path of Mars in 1956
The apparent path of Mars in 1958
Horizontal parallax of the moon $\alpha \approx 57'$
Cyperus papyrus

papyrus

volumen

codex