ABSTRACT

Algebra and geometry of Maurer-Cartan algebras

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Let $R$ be a commutative ring with 1, $A$ a commutative $R$-algebra with 1, and let $L$ be an $A$-module. A *Maurer-Cartan*-algebra structure over $L$ relative to $A$ is a multiplicative $R$-differential $d$ on $\text{Alt}_A(L, A)$, and the resulting differential graded $R$-algebra $(\text{Alt}_A(L, A), d)$ is referred to as a *Maurer-Cartan*-algebra over $L$ (relative to $A$). When $L$ is finitely generated and projective as an $A$-module, Maurer-Cartan structures over $L$ relative to $A$ and Lie-Rinehart structures on $(A, L)$ are equivalent notions. When $A$ is the algebra of smooth functions on a smooth manifold and $L$ the Lie algebra of vector fields, the corresponding Maurer-Cartan algebra is the ordinary de Rham algebra on the manifold. Maurer-Cartan structures in the graded setting include twilled Lie-Rinehart algebras and quasi-Lie-Rinehart algebras. Under such circumstances, the Maurer-Cartan structure comprises replacements for the ring of functions and the Lie algebra of vector fields, and it organizes the combinatorics needed to handle the higher homotopies which arise in a natural fashion. Examples arise from complex manifolds and from foliations. The corresponding Maurer-Cartan structures have the Hodge-de Rham spectral sequence and the spectral sequence of the foliation under discussion as invariants.