

Homework problems # 1

1. Show that the Lorentz transformations are the only non-singular coordinate transformations $x \rightarrow x'$ that leave $d\tau^2$ invariant (non-singularity means that both $\partial x'^\alpha / \partial x^\beta$ and $\partial x^\beta / \partial x'^\alpha$ exists).

2. Prove that

$$\varepsilon^{\varepsilon\xi\kappa\lambda}\Lambda^\alpha_{\varepsilon}\Lambda^\beta_{\xi}\Lambda^\gamma_{\kappa}\Lambda^\delta_{\lambda} = \varepsilon^{\alpha\beta\gamma\delta},$$

then show that $\varepsilon^{\alpha\beta\gamma\delta}$ is a contravariant Lorentz tensor which transforms into itself.

3. Derive the Lorentz covariant formula for the electromagnetic force acting on a charged particle:

$$f^\alpha = eF^\alpha_{\gamma} \frac{dx^\gamma}{d\tau}.$$