Homework problems #4

1. Show that for a general coordinate transformation $x^{'} \to x$ the Levi-Civita symbol $\varepsilon^{\mu\nu\lambda\kappa}$ satisfies the following relation

$$\frac{\partial x^{'\rho}}{\partial x^{\mu}} \frac{\partial x^{'\sigma}}{\partial x^{\nu}} \frac{\partial x^{'\eta}}{\partial x^{\lambda}} \frac{\partial x^{'\xi}}{\partial x^{\kappa}} \varepsilon^{\mu\nu\lambda\kappa} = \left| \frac{\partial x^{'}}{\partial x} \right| \varepsilon^{\rho\sigma\eta\xi}$$

2. Show that $\varepsilon^{\rho\sigma\eta\xi}$ and $\varepsilon_{\rho\sigma\eta\xi}$ are related by

$$\varepsilon_{\rho\sigma\eta\xi} = -g \, \varepsilon^{\rho\sigma\eta\xi} \, ,$$

where $g \equiv -\mathrm{Det}(g_{\mu\nu})$.