Homework problems #2

1. Show that
$$\frac{\partial J^\alpha(x)}{\partial x^\alpha} = 0 \quad \text{for} \quad J^\alpha(x) = \sum_n e_n \delta^3(\bar{x} - \bar{x}_n(t)) \frac{dx_n^\alpha(t)}{dt}\,.$$

2. Derive the Lorentz covariant formula for the electromagnetic force acting on a charged partice:
$$f^\alpha = eF^\alpha{}_\gamma \frac{dx^\gamma}{d\tau}\,.$$

3. Show that modified energy-momentum tensor for a gas of charged particles
$$T^{\alpha\beta} = \sum_n p_n^\alpha \frac{dx_n^\beta}{dt} \delta^3(\bar{x} - \bar{x}_n(t)) + T^{\alpha\beta}_{\text{em}}$$
where $T^{\alpha\beta}_{\text{em}} = F^\alpha{}_\gamma F^{\beta\gamma} - \frac{1}{4}\eta^{\alpha\beta}F_{\gamma\delta}F^{\gamma\delta}$, is conserved, so
$$\frac{\partial T^{\alpha\beta}}{\partial x^\alpha} = 0\,.$$