Homework problems #4

1. Derive equation of motion for a massive photon, so use the following Lagrangian density

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2A_{\mu}A^{\mu}$$

and show the for $m \neq 0$ the equation implies $A_{\mu}{}^{,\mu} = 0$.

2. For polarization tensors $\varepsilon_{\mu\nu}^{(a)}(k)$ of a massive graviton, write down the most general form for $\sum_{a} \varepsilon_{\mu\nu}^{(a)}(k) \varepsilon_{\lambda\sigma}^{(a)}(k)$ using symmetry repeatedly. For example, it must be invariant under the exchange $\{\mu\nu \leftrightarrow \lambda\sigma\}$. You might end up with something like

$$AG_{\mu\nu}G_{\lambda\sigma} + B(G_{\mu\lambda}G_{\nu\sigma} + G_{\mu\sigma}G_{\nu\lambda}) + C(G_{\mu\nu}k_{\lambda}k_{\sigma} + k_{\mu}k_{\nu}G_{\lambda\sigma}) + D(k_{\mu}k_{\lambda}G_{\nu\sigma} + k_{\mu}k_{\sigma}G_{\lambda\nu} + k_{\nu}k_{\sigma}G_{\mu\lambda} + k_{\nu}k_{\lambda}G_{\mu\sigma}) + Ek_{\mu}k_{\nu}k_{\lambda}k_{\sigma}$$

where

$$G_{\mu\nu} = \eta_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{m^2}$$

with various unknown A,...,E. Apply $k^{\mu} \sum_{a} \varepsilon_{\mu\nu}^{(a)}(k) \varepsilon_{\lambda\sigma}^{(a)}(k) = 0$ and find out what that implies for the constants. Proceeding this way, derive

$$\sum_{a} \varepsilon_{\mu\nu}^{(a)}(k) \varepsilon_{\lambda\sigma}^{(a)}(k) = (G_{\mu\lambda}G_{\nu\sigma} + G_{\mu\sigma}G_{\nu\lambda}) - \frac{2}{3}G_{\mu\nu}G_{\lambda\sigma}$$