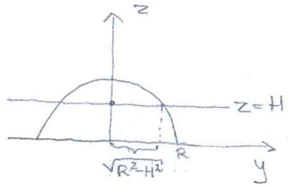
1



$$V=\iiint dx\,dy\,dz \quad , \text{ r-e sfery } x^2+y^2+z^2=R^2 \;\downarrow\; \rho^2+z^2=R^2$$

wsp. cylindryczne: (ρ,φ,z)

$$\boxed{V}=\int_0^{2\pi}d\varphi\int_0^{\sqrt{R^2-H^2}}d\rho\,\rho\int_H^{\sqrt{R^2-\rho^2}}dz=2\pi\int_0^{\sqrt{R^2-H^2}}d\rho\,\rho\left(\sqrt{R^2-\rho^2}-H\right)$$

$$=\left\{\begin{array}{l}t=R^2-\rho^2\\ dt=-2\rho\,d\rho\end{array}\right\}=-\frac{2\pi}{2}\int_{R^2}^{H^2}dt\sqrt{t}-2\pi H\frac{1}{2}\rho^2\Big|_0^{\sqrt{R^2-H^2}}$$

$$=\pi\frac{2}{3}t^{3/2}\Big|_{H^2}^{R^2}-\pi H(R^2-H^2)=\boxed{\frac{1}{3}\pi(R^3+H^3-HR^2)}$$

2

$$\int_C xy\,ds \quad , \quad C=\left\{(x,y)\in\mathbb{R}^2:\ \frac{x^2}{4}+\frac{y^2}{9}=1,\ x\geq 0,\ y\geq 0\right\}$$

$$ds=\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx=\sqrt{1+\frac{9x^2}{16\left(1-\frac{x^2}{4}\right)}}=\frac{1}{4}\cdot\frac{\sqrt{16-4x^2+9x^2}}{\sqrt{1-\frac{x^2}{4}}}$$

$$y=3\sqrt{1-\frac{x^2}{4}}\ \to\ \frac{dy}{dx}=\frac{3\left(-\frac{2x}{4}\right)}{2\sqrt{1-\frac{x^2}{4}}}=-\frac{3x}{4\sqrt{1-\frac{x^2}{4}}}$$

$$\underline{\int_C xy\,ds}=\int_0^2 dx\,x\cdot 3\sqrt{1-\frac{x^2}{4}}\cdot\frac{1}{4}\frac{\sqrt{16+5x^2}}{\sqrt{1-\frac{x^2}{4}}}=\frac{3}{4}\int_0^2 dx\,x\sqrt{16+5x^2}=\left\{\begin{array}{l}t=16+5x^2\\ dt=10x\,dx\end{array}\right\}=\frac{3}{4}\cdot\frac{1}{10}\int_{16}^{36}dt\sqrt{t}=\frac{3}{4}\cdot\frac{1}{10}\cdot\frac{2}{3}t^{3/2}\Big|_{16}^{36}=$$

$$=\frac{1}{20}\left(6^3-4^3\right)=\boxed{\frac{38}{5}}$$

3

$$\operatorname{div}\vec{A},\quad \vec{A}=\frac{1}{r^3}\left(\vec{e}\times\vec{r}\right),\quad r=\sqrt{x^2+y^2+z^2},\quad \vec{e}=(1,1,1)$$

$$\vec{A}=\frac{1}{r^3}\begin{pmatrix}1\\1\\1\end{pmatrix}\times\begin{pmatrix}x\\y\\z\end{pmatrix}=\frac{1}{r^3}\begin{pmatrix}z-y\\x-z\\y-x\end{pmatrix}\quad,\quad \boxed{\operatorname{div}\vec{A}}=\partial_xA_x+\partial_yA_y+\partial_zA_z=(z-y)\partial_x\left[\frac{1}{(x^2+y^2+z^2)^{3/2}}\right]+(x-z)\partial_y\left[\frac{1}{(x^2+y^2+z^2)^{3/2}}\right]+(y-x)\partial_z\left[\frac{1}{(x^2+y^2+z^2)^{3/2}}\right]$$

$$=\frac{-3}{(x^2+y^2+z^2)^{3/2}}\underbrace{\left[(z-y)x+(x-z)y+(y-x)z\right]}_{zx-yx+xy-zy+yz-xz}=\boxed{0}$$

4

$$y'-\frac{y}{x}=\frac{1}{2y}\quad,\quad z=y^2\ \to\ \left\{\begin{array}{l}y=\pm\sqrt{z}\\ y'=\pm\frac{1}{2\sqrt{z}}z'\end{array}\right.\ \to\ \pm\frac{1}{2\sqrt{z}}z'-\frac{\pm\sqrt{z}}{x}=\frac{\pm 1}{2\sqrt{z}}\ \Rightarrow\ z'-\frac{2z}{x}=1$$

r. jednorodne: $z'-\frac{2x}{z}=0\ \Rightarrow\ \int\frac{dz}{z}=2\int\frac{dx}{x}\ \Rightarrow\ \ln z=2\ln x+C_1\ \Rightarrow\ z=Cx^2$

uzmiennienie stałej C: $z(x)=C(x)x^2\ \Rightarrow\ C'x^2+\cancel{2Cx}-\cancel{\frac{2Cx^2}{x}}=1\ \Rightarrow\ C'x^2=1\ \Rightarrow\ C'=\frac{1}{x^2}\ \Rightarrow\ C=-\frac{1}{x}+C_2$

$$z=C_2x^2-x\ \Rightarrow\ y(x)=\pm\sqrt{C_2x^2-x}$$

5

$$yy''-y'^2=6xy^2\ \Rightarrow\ e^u\left(e^u(u')^2+e^uu''\right)-e^{2u}(u')^2=6x\,e^{2u}$$

$y=e^u$

$y'=e^uu'$

$y''=e^u(u')^2+e^uu''$

$$\downarrow$$

$$\cancel{e^{2u}}\left(u'^2+u''\right)-\cancel{e^{2u}}(u')^2=6x\cancel{e^{2u}}\ \Rightarrow\ u'^2+u''-u'^2=6x\ \to\ u''=6x$$

$$u'=6\cdot\frac{1}{2}x^2+C$$

$$u=x^3+Cx+D\ \Leftarrow\ u'=3x^2+C$$

$$\Downarrow$$

$$y(x)=e^{x^3+Cx+D}$$