F8

Faraday effect in CdTe and Cd$_{1-x}$Mn$_x$Te.

The objective of this experiment is to measure the absorption spectrum and the Faraday effect in crystals of Cd$_{1-x}$Mn$_x$Te with different concentrations of Mn. The outcome of such measurements can, then, be employed to obtain the Vendret constant of the materials considered.

1. **What should you know before attempting to perform this experiment?**
   a. Basic knowledge of semiconducting physics: Crystal structure, electronic band structure E(k) and band models,
   b. Optical properties of semiconductors [1,2,3]:
      i. Optical transitions near the absorption edge, multiband and excitonic absorption,
      ii. Refractive index,
   c. Correlation between optical constants (α and *n*), and the relation of Kramers-Kronig [4,5],
   d. Faraday effect [1,5,6]:
      i. Phenomenological explanation,
      ii. Correlation between Faraday effect and absorption lines and absorption edges,
   e. Parameters of the Cd$_{1-x}$Mn$_x$Te crystal:
      i. Crystal structure and band structure near k = 0,
      ii. Energy gap and its relation to the Mn concentration x,
      iii. Faraday effect in Cd$_{1-x}$Mn$_x$Te,
   f. Basic knowledge about optical instrumentation:
      i. Construction of a monochromator,
      ii. Inner workings of a CCD.

You should be familiar with the text below prior to the experiment, getting acquainted with words and expressions marked in **bold** letters.

In a crystalline solid, electrons are subjected to a periodic potential. Their wavefunction, in this situation, can be described in terms of **Bloch functions**. Free carriers in this material, thus, do not behave as non-interacting electrons in vacuum. Instead, they have different values of **mass** and can possess positive charge. The latter are conventionally termed as **holes**, and usually inhabit the **valence band** of a material. Meanwhile, carriers with negative mass, called electrons, exist in the **conduction band**. In insulators or semiconductors, the conduction and valence bands are separated by an **energy gap**.

Besides the well-known monoatomic semiconductors of the group IV of the periodic table, there exist a family of semiconductors composed by alloying elements of the family III and V of the periodic table, as well as alloying those of the family II and VI. These are named, rather dully, as II-V and II-VI semiconductors. A diagram representing different materials of these groups are presented in Fig. 1, together with their respective energy gaps.

Semiconductors of the II-VI family are compounds in which cations are metals of the group II of the periodic table (Zn, Mg, Cd, Hg, Mg), whereas anions are elements of group VI of the periodic table (O, S, Se, Te). The width of the Energy gap in this family ranges from values close to 0 eV (CdHgTe) to those above 3.5 eV (ZnS). The possibility to achieve mixtures of alloys with fractional stoichiometries (e.g. Cd$_{0.3}$Mn$_{0.7}$Te as an alloy composed by 30% CdTe and 70% MnTe) allows the precise engineering of the energy gap of the semiconductors of this family

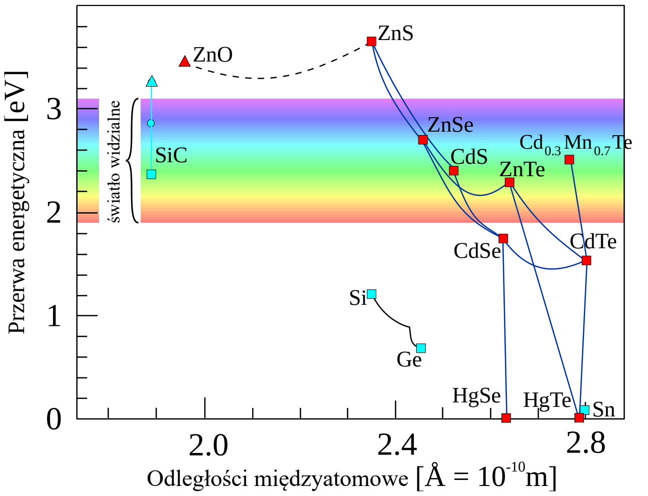

Figure 1 - Gap vs. lattice parameter of alloys pertaining to the II-IV family

Alloys containing manganese (Mn) usually present strong magnetic properties. Among those, are materials known as “magnetic semiconductors”. These materials present a property dubbed „giant Faraday effect”, which allows their application in, for example, spintronic devices.

In the case of the admixed alloy Cd$_{(1-x)}$Mn$_x$Te, the gap width will depend on the Mn concentration, according to

$$E_g = E_{CdTe} + x \times 1.39\ eV \tag{1}$$

(Please refer to N. T. Khoi, J.A. Gaj, Phys. Stat. Solidi (b) **83**, K133 (1977) and M. P. Vecchi, W. Giriat, L Videla, Appl. Phys. Lett. 38 p. 99 (1981))

Light is an electromagnetic wave. As such, it must obey the Maxwell equations

$$\nabla \times \boldsymbol{E} = \frac{\partial \boldsymbol{B}}{\partial t} \tag{1.2A}$$

$$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} + \boldsymbol{J} \tag{1.2B}$$

Equation 1.2A states that the electric vector $\boldsymbol{E}$ and its associated magnetic field $\boldsymbol{B}$ are orthogonal to each other and to the wave propagation direction, usually denoted by $\boldsymbol{k}$. Such a result means that light is a transversal wave, from which we may define its polarization. As a convention, the direction of the vector $\boldsymbol{E}$ is chosen as the **polarization axis** of the light. Whenever $\boldsymbol{E}$ has a well-defined direction (e.g. along the x- or y-axis), the light is termed as “linearly polarized”. However, if the light is composed by the sum of two waves polarized along different directions (e.g. x and y- axis), with one of the components out-of-phase by a factor of $\pi/2$ (a quarter of a wavelength), the vector $\boldsymbol{E}$ no longer will be confined to a single axis. Instead, the electric field will rotate on the plane perpendicular to $\boldsymbol{k}$, with an angular frequency ω – identical to the frequency of the propagating wave. In this situation, the light is termed as **circularly polarized**. Depending on the direction of rotation of $\boldsymbol{E}$, the light will be dubbed as right – or left hand- polarized (also denoted by the symbols + and -).

The propagation speed of an electromagnetic wave in a medium is defined as c*. This value is invariably smaller than the speed of light in the vacuum c by a factor n, according to $c^* = c/n$. The number n recieves the name of the medium’s **refractive index**.

Because both the refractive index n and the **absorption coefficient** α of a medium depend on the material dielectric constant ε, it is possible to reconstruct $\alpha(\omega)$ by knowing the behavior of $n(\omega)$ over a wide spectrum. The connection between α and n if provided by the **Kramers-Kroniger** equation.

In **non-isotropic** media, the speed of light c^* might depend on the polarization of the propagating wave. If the propagation of circularly-polarized light is affected by magnetic fields, the light speed and refractive index of photons travelling with each polarization will differ ($n_+ \neq n_-$). For a linearly polarized light travelling in such a media, the discrepancy between n_+ and n_- results in a rotation of the initial (incident) polarization by an angle θ

$$\theta = \frac{\omega(n_+ - n_-)l}{2c} \equiv VlB, \tag{1.3}$$

where l is the distance travelled in the medium, and B is the induction field strength along the direction of $\boldsymbol{k}$ in the material. The parameter V is valled the **Vendret constant**.

The outcome of the Faraday effect on the light spectrum of a material depends on the kind of optical transitions involved. For example, if two damped harmonic oscillators with energies E_+ and E_- and bandwidth Γ (e.g. **excitons** with T>0) are **allowed** in two different polarizations, the spectrum of the system studied will be described by the equation

$$V(E) = A\left[\frac{E^2 - E_+^2}{(E^2 - E_+^2)^2 + \Gamma^2 E^2} - \frac{E^2 - E_-^2}{(E^2 - E_-^2)^2 + \Gamma^2 E^2}\right], \tag{1.4}$$

with $E_+ = E_g + \Delta$, $E_- = E_g - \Delta$. For $\Delta \ll E_g$, eq. 1.4 becomes

$$V(E) = \frac{\beta\left(E_g - E\right)}{\left(E^2 - E_g^2\right) + \Gamma^2 E^2}, \tag{1.5}$$

where β is a phenomenological constant and E_g can be interpreted as the average excitonic energy gap.

In equation 1.4, the absorption near the gap is smeared by a parameter Γ. As a matter of fact, such a broadening might have different origins (e.g. attenuation, variation in the density of states, disorder, etc…). Conventionally, absorption plateaus around 10^{-4} cm^{-1}. This means that light is allowed to propagate a few micrometers inside the material, which is a distance much smaller than the sample width. The widening of the absorption lines means that photons with energies of the order Γ below E_g also will also be subjected to absorption values of the order $\alpha \approx 10^{-4}$ cm^{-1}, and therefore will not be allowed to propagate in the material. Under these circumstances, the cutoff frequency for light propagation in the materials considered does not correspond to the energy gap, but rather to a lower energy, labelled E_a.

2. Experimental procedure

A. Calibrate the electromagnet. The intensity of the magnetic field H is proportional to the current I_M flowing through its coils. The magnetic induction B can be measured with a gaussmeter.

B. Prepare the optical setup for measurements. Before any measurements, the setup should be aligned, and a full spectra of the excitation lamp should be collected. It is also necessary to measure the optical response f the polarizers employed. Prior to measurements, the integration time of the CCD should be adjusted as to avoid unwanted saturations in the entire measured spectrum.

C. Obtain the figure of merit of the polarizers employed.

$$d(\omega) = \frac{\left(I_{\parallel}(\omega) - I_{\perp}(\omega)\right)}{I_{\parallel}(\omega) - I_{\perp}(\omega)}, \tag{2.1}$$

where $I_{\parallel}$ and $I_{\perp}$ are the light intensity transmitted in the co- and cross-polarized configurations, respectively.

D. In the absence of magnetic fields, perform a measurement of the absorption edge of un-doped CdTe (E_{CdTe}) and its Mn-doped counterparts.

E. Perform measurements of the Faraday effect as a function of magnetic field in crystals of $Cd_{(1-x)}Mn_xTe$ with different x. For that, a series of spectra should be registered at a fixed magnetic field, for different relative orientations between the polarizers employed. This procedure should be repeated for different magnetic fields.

F. Analyze:
    a. The spectral dependence of the Vendret constant
    b. Interpret the spectrum of the Vendret constant in terms of a simple absorption model.

3. Report preparation

The report should contain the following sections:

a. Abstract,
b. Theoretical introduction,
c. Description of the samples and experimental setup,
d. Results and its analysis,
e. Summary and conclusions,
f. Bibliography.

Where:

a. Abstract should contain a short (few sentences) summary of the tasks performed and the results obtained
b. All equations used must be presented and justified.
c. In the description of the experimental setup, the characterization of the polarizers, lamp and magneti should be resented (calibration curves, figure of merit, etc…)
d. In the results section, the spectra of doped and un-doped CdTe should be presented, together with measurements of the Faraday effect at different magnetic fields and cross-polarization angles.

The absorption spectra can be employed to define the absorption edge of the different samples. Such a value allows the extraction of the Mn content x, following eq. 1.1. However, note that the absorption edge does not provide E_{CdTe}, but rather $E_a \equiv E_{CdTe} + \Delta E$ (see sec. 2). To determine x using eq. 1.1, it is convenient to assume that ΔE does not change with x (is it true?).

Analyze all measured spectra to obtain the variation of $I(\omega)$ as a function of angle ϕ between polarizers. Curves showcasing $I(\phi)$ for constant ω should be plotted for few values of B. The obtained curves can be well-described by

$$I(\phi) = I_0 \times \cos^2(\phi - \theta) + I_L, \tag{3.1}$$

Where I_o, θ, and I_L are constants that depend on B and ω.

The parameter θ represents the rotation angle of the incident linear polarization, and should be plotted as a function of B, thus allowing the extraction of the Vendret constant for a fixed ω through equation 1.3. This process should be repeated for different photon energies, in order to obtain the spectral response $V(\omega)$. The latter should, then, be described through the theoretical model presented in eq. 1.5.

A possible obstacle in using eq. 1.5 regards the lack of an accurate value of E_g. This can be remedied by utilizing literature values of E_g for un-doped CdAs, in order to determine ΔE from the experimental data. Such a value can then be employed for samples with different x, thus allowing a more accurate estimation of E_g for all samples, which is the parameter that should be present in eq. 1.5.

e. When preparing the presentation, please remember that figures must be correctly labelled (e.g. Fig. 1, 2, 3, etc…), and should be referred by their labels (NOT by “see the figure below”). Figures should be accompanied by a description below the figure. Tables (if any) should also be numbered. However, differently from figures, tables are titled (i.e. the description is atop the table, not below it). Whenever a reference is used, or results from other authors are cited, they should follow with the appropriate reference between brackets (e.g. [1]). The referencing style is up to the author of the report, but should be consistent.

4. Further reading

1. J. Ginter - „Wstęp do fizyki atomu, cząsteczki i ciała stałego.”
2. K. W. Szalimowa - Fizyka półprzewodników.
3. J. Pankove - Zjawiska optyczne w półprzewodnikach
4. F.W. Byron, R.W. Fuller - Matematyka w fizyce klasycznej i kwantowej
5. J. Houghton, S. D. Smith - Fizyka podczerwieni
6. J. A. Gaj, 'Oddziaływanie jonów paramagnetycznych z elektronami pasmowymi w $Cd_{1-x}Mn_xTe$', Rozprawy Uniwersytetu Warszawskiego **204**, Wydawnictwo Uniwersytetu Warszawskiego, Warszawa 1981
7. J. A. Gaj, R.R. Gałązka, M. Nawrocki, „Giant Exciton Faraday Rotation in $Cd_{1-x}Mn_xTe$ Mixed Cristals.” Solid State Communications **25**, 193 (1978)
8. Internet: Kramers-Kroniga relation, Faraday efect, CCD camera
9. Encyklopedia Fizyki, hasła: relacje Kramersa-Kroniga, efekt Faradaya, monochromator

K.K., R.S., BC 05 IV 2023

Appendix A

How does a CCD work?

- Check wikipedia. A good explanation can also be found on the link below
http://www.optique-ingenieur.org/en/courses/OPI_ang_M05_C06/co/Contenu_04.html
(verified on April 05th, 2023)

Appendix B

The illustration B1 below provides a self-explanatory method to extract the parameter of interest θ from $I(\phi)$ curves measured at different B and constant ω. We leave to the author of the report to implement the numerical process necessary to extract θ of different sets of curves.

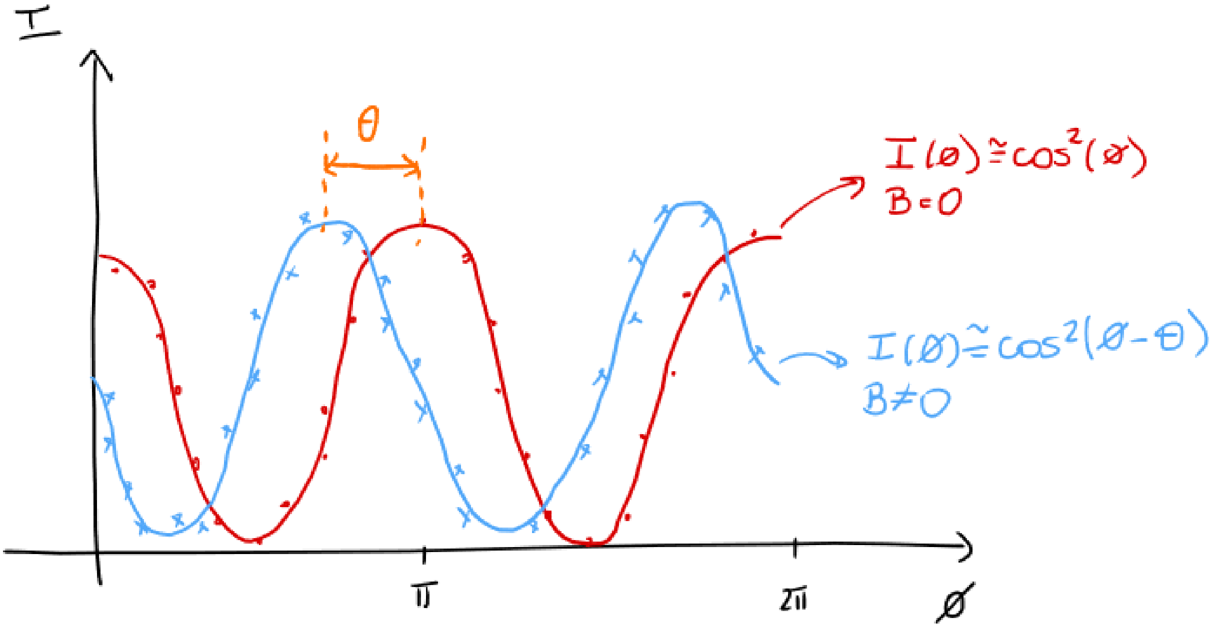

Figure B 1 – Illustration of $I(\phi)$ measured at the same value of ω with and without applied magnetic field.

Also refer to the instructions of the experiment in Polish, where an analytical procedure is provided.
https://www.fuw.edu.pl/IIPRACOWNIA/home/Opisy-cwiczen/F8__2022.pdf
(verified on April 05th, 2023)