Safe online learning-based control for an aerial robot with manipulator arms

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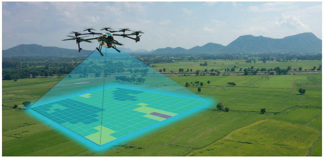
Introduction

Dynamics: can be expressed by rigid bodies' motion.
Control approaches based on feedback linearization and backstepping methods, depend on exact models of the systems and external disturbances to guarantee stability and precise tracking.
An accurate model of typical uncertainties is hard to obtain by using first principles-based techniques. This uncertainty is commonly compound by:

- Impact of air/water flow on aerial/underwater vehicles.
- Interaction with unstructured and a-priori unknown environments.

Why GPs?

Increase of feedback gains to eliminate unknown dynamics → Unfavorable → Saturation of actuators; Large errors in presence of noise

Learning-based oracles: GP

Data-driven modelling tools used in control, machine learning and system identification. The models are highly flexible and can reproduce a large class of different dynamics.

What is a GP?

- A Stochastic process describes systems randomly changing over time. This process can lead to different paths or realizations of the process.
- Each realization defines a position d for every possible timestep t, that is, it corresponds to a function $f(t) = d$. A stochastic process can be seen as random distribution over functions

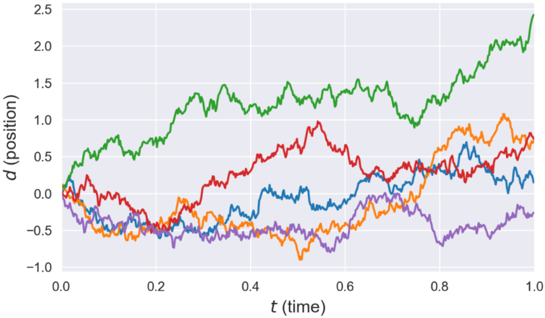


Figure: Different realizations of a Brownian motion

What is a GP?

Gaussian Processes are distributions over functions $f(x)$, where the distribution is defined by a mean function $m(x)$ and the positive definite covariance function $k(x, x^{'})$, with x the function values of the domain $f(x)$ and $(x, x^{'})$ all possible pairs in the domain.

$$f(x) \sim \mathcal{GP}(m(x), k(x, x^{'}))$$

where, for any finite subset $C = \{x_1, \ldots, x_n\}$ of the domain x, the marginal distribution is a multivariate Gaussian distribution

$$f(x) \sim \mathcal{N}(m(X), k(X, X))$$

with mean vector $\mu = m(X)$ and covariance matrix $\Sigma = k(X, X)$.

Why GPs?

GP models

Prediction + measure of the uncertainty of the model

Benefits

- Bias-variance trade-off.
- Strong connection to Bayesian Statistics.

Uses

- Predictive control of models.
- Sliding mode control.
- Tracking of mechanical systems.
- Backtepping control for underactuated vehicles.
- Learn and mitigate unknown dynamics in UAVs models.

The measure of the uncertainty allows us to provide performance and safety guarantees.

Objective

Objective: Employ learning-based approaches based on GP models to learn the uncertainties of our aerial vehicle equipped with manipulator arms (UAM)

- Guarantee the **probabilistic boundedness of the tracking error** to the reconfigured attitude and positions with high probability.
- Simulation tests for the **validation of learning-based controllers**.

Dynamics of an aerial robot with manipulator arms

Consider the **Lie group of rotations in the space $SO(3)$**

$$SO(3) = \{R \in GL(3,\mathbb{R}) | R^\top R = Id, det(R) = 1\}$$

where Id denotes the (3×3)-identity matrix.
Its **Lie algebra** is denoted by $\mathfrak{so}(3)$, which is the set of 3×3 skew-symmetric matrices which have the form

$$\hat{\omega}_i(t) = \begin{pmatrix} 0 & -\omega_i^3(t) & \omega_i^2(t) \\ \omega_i^3(t) & 0 & -\omega_i^1(t) \\ -\omega_i^2(t) & \omega_i^1(t) & 0 \end{pmatrix} \simeq \omega_i$$

where $\omega_i = (\omega_i^1, \omega_i^2, \omega_i^3) \in \mathbb{R}^3$ and where $\hat{\cdot} : \mathbb{R}^3 \to \mathfrak{so}(3)$ denotes the isomorphism between vectors on $\mathbb{R}^3$ with skew-symmetric matrices.

Dynamics of an aerial robot with manipulator arms

We consider a multirotor UAV, consisting on $n+1$ interconnected rigid bodies. The **configuration of the i^{th}-body is denoted by $g_i \in SE(3)$**

$$g_i = \begin{pmatrix} R_i & x_i \\ 0 & 1 \end{pmatrix}$$

where R_i is the orientation in $SO(3)$ and x_i is the position in $\mathbb{R}^3$ of its center of mass. The group operation is the matrix multiplication.
The **inverse of the matrix** is given by

$$g_i^{-1} = \begin{pmatrix} R_i^T & -R_i^T x_i \\ 0 & 1 \end{pmatrix}$$

Dynamics of an aerial robot with manipulator arms

The **Lie algebra of $SE(3)$** is denoted by $\mathfrak{se}(3)$ and any $\xi \in \mathfrak{se}(3)$ is given by

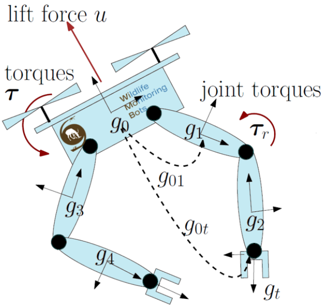
$$\xi = \begin{pmatrix} \hat{\omega} & v \\ 0 & 0 \end{pmatrix}$$

where $\omega_i \in \mathfrak{so}(3) \simeq \mathbb{R}^3$ and $v_i \in \mathbb{R}^3$ are the angular and linear (body) velocities.
The **pose inertia tensor** for each body is given by

$$\mathbb{I}_i = \begin{pmatrix} \mathbb{J}_i & 0 \\ 0 & m_i I_3 \end{pmatrix}$$

where $\mathbb{J}_i$ is the rotational inertia matrix, m_i the mass of i^{th} body, and I_3 the 3×3 identity matrix.
Each body is subject to a potential energy $V : SE(3) \to \mathbb{R}$.
We assume that the **0^{th}-body is subject to forces from the propellers** that result in body-fixed torque $\tau \in \mathbb{R}^3$ and lift force $u > 0$ aligned with the vertical axis $\boldsymbol{e} = (0, 0, 1) \in \mathbb{R}^3$.

Dynamics of an aerial robot with manipulator arms



The system has n joints described by the parameter $\boldsymbol{r} \in M$, $M \subset \mathbb{R}^n$.
The **relative transformation between the base body 0^{th} and the i^{th}** is given by

$$g_i = g_0 g_{0i}(\boldsymbol{r})$$

All torques are controlled using the torque inputs $\tau_r \in \mathbb{R}^m$.

Dynamics of an aerial robot with manipulator arms

The **Lagrangian of the system** is given by

$$L(\mathbf{q}_0, \xi_0) = \frac{1}{2}\xi_0^T M_0(\boldsymbol{r})\xi_0 - \sum_{i=0}^{n} m_i \boldsymbol{a}_g^T x_i$$

where $\boldsymbol{q}_0 = (R_0, x_0, \boldsymbol{r})$ and $\xi_0 = (\omega_0, v_0, \dot{\boldsymbol{r}})$, and where the positions $x_1, \dots, x_n$ are functions of $\boldsymbol{q}_0$, and $\boldsymbol{a}_g$ denotes the gravitational acceleration.
Using the adjoint notation $A_i := Ad_{g_{0i}^{-1}}(\boldsymbol{r})$ and jacobian $J_i = g_{0i}^{-1}(\boldsymbol{r})\partial_r g_{0i}(\boldsymbol{r})$, the **mass matrix M_0** is defined as

$$M_0(r) = \left[\begin{array}{c|c} \mathbb{I}_0 + \sum_{i=1}^{n} A_i^T \mathbb{I}_i A_i & \sum_{i=1}^{n} A_i^T \mathbb{I}_i J_i \\ \hline \sum_{i=1}^{n} J_i^T \mathbb{I}_i A_i & \sum_{i=1}^{n} J_i^T \mathbb{I}_i J_i \end{array}\right]$$

that can be rewritten in terms of $\mathbb{J}, C, M_{\omega\dot{r}}, M_{v\dot{r}}, M_{\dot{r}\dot{r}}$ as

$$M_0(r) = \left[\begin{array}{cc|c} \mathbb{J} & C^T & M_{\omega\dot{r}} \\ C & m\boldsymbol{I}_3 & M_{v\dot{r}} \\ \hline M_{\omega\dot{r}}^T & M_{v\dot{r}}^T & M_{\dot{r}\dot{r}} \end{array}\right], \text{ where } m = \sum_{i=0}^{n} m_i.$$

Dynamics of an aerial robot with manipulator arms

The **equations of motion** in coordinates $\bar{q} = (\boldsymbol{q}, \xi) = (R, x, \boldsymbol{r}, \omega, v, \dot{\boldsymbol{r}})$ are given by

$$\begin{aligned}
\dot{R} =& R\omega, \\
m\ddot{x} =& m\boldsymbol{a}_g + R\boldsymbol{e}u, \\
\begin{bmatrix} \omega \\ \dot{\boldsymbol{r}} \end{bmatrix} =& \bar{M}^{-1}(\boldsymbol{r}) \begin{bmatrix} \mu \\ \nu \end{bmatrix} \\
\begin{bmatrix} \dot{\mu} \\ \dot{\nu} \end{bmatrix} =& \begin{bmatrix} \mu \times \omega \\ \frac{1}{2}\bar{\xi}^T \partial \bar{M}(\boldsymbol{r})\bar{\xi} \end{bmatrix} + \begin{bmatrix} \tau - C^T\boldsymbol{e}u/m \\ \tau_r - M_{v\dot{\mathrm{r}}}^T\boldsymbol{e}u/m \end{bmatrix},
\end{aligned}$$

where $\bar{\xi} = (\omega, \dot{\boldsymbol{r}})$, the mass matrix $\bar{M}(\mathrm{r})$ is given by

$$\bar{M} = \begin{bmatrix} \mathbb{J} - C^T C/m & M_{\omega\dot{r}} - C^T M_{v\dot{r}}/m \\ M_{\omega\dot{r}}^T - M_{v\dot{r}}^T C/m & M_{\dot{r}\dot{r}} - M_{v\dot{r}}^T M_{v\dot{r}}/m \end{bmatrix}$$

Dynamics of an aerial robot with manipulator arms

We consider the system subject to unknown dynamics $\boldsymbol{f}^x_{uk}, \boldsymbol{f}^\omega_{uk}, \boldsymbol{f}^{\dot{r}}_{uk} : SE(3) \times \mathfrak{se}(3) \to \mathfrak{se}(3)^*$.

The **dynamics** of the system are

$$\begin{aligned}
\dot{R} =& R\omega, \\
m\ddot{x} =& m\boldsymbol{a}_g + R\boldsymbol{e}u + \boldsymbol{f}^x_{uk}(\boldsymbol{q}_0, \xi_0), \\
\begin{bmatrix} \omega \\ \dot{\boldsymbol{r}} \end{bmatrix} =& \bar{M}^{-1}(\boldsymbol{r}) \begin{bmatrix} \mu \\ \nu \end{bmatrix} \\
\begin{bmatrix} \dot{\mu} \\ \dot{\nu} \end{bmatrix} =& \begin{bmatrix} \mu \times \omega \\ \frac{1}{2}\bar{\xi}^T \partial \bar{M}(\boldsymbol{r})\bar{\xi} \end{bmatrix} + \begin{bmatrix} \tau - C^T\boldsymbol{e}u/m \\ \tau_r - M^T_{v\dot{\mathrm{r}}}\boldsymbol{e}u/m \end{bmatrix} + \begin{bmatrix} \boldsymbol{f}^\omega_{uk}(\boldsymbol{q}_0, \xi_0) \\ \boldsymbol{f}^{\dot{r}}_{uk}(\boldsymbol{q}_0, \xi_0) \end{bmatrix}.
\end{aligned}$$

Learning for the compensation of unknown dynamics using GPs

We consider an oracle (in our case a GP model) that predicts $\boldsymbol{f}_{uk}^{x}, \boldsymbol{f}_{uk}^{\omega}, \boldsymbol{f}_{uk}^{\dot{r}}$ for a given state $\bar{q} = (R, x, \boldsymbol{r}, \omega, v, \dot{\boldsymbol{r}})$.
We are going to work with a training data set $\mathcal{D} = \{\bar{q}^{\{i\}}, y^{\{i\}}\}_{i=1}^{N}$ such that

$$y = \begin{bmatrix} (m\ddot{x} - m\boldsymbol{a}_g - R\boldsymbol{e}u)^\top \\ (\dot{\mu} - \mu \times \omega + C^\top \boldsymbol{e}u/m - \tau)^\top \\ (\dot{\nu} - \frac{1}{2}\bar{\xi}^\top \partial \bar{M}(\boldsymbol{r})\bar{\xi} + M_{v\dot{r}}^\top \boldsymbol{e}u/m - \tau_r)^\top \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_{uk}^{x} \\ \boldsymbol{f}_{uk}^{\omega} \\ \boldsymbol{f}_{uk}^{\dot{r}} \end{bmatrix}$$

The estimates of the oracle based on the data set $\mathcal{D}$ are denoted by $\hat{\boldsymbol{f}}_{uk}^{x}, \hat{\boldsymbol{f}}_{uk}^{\omega}, \hat{\boldsymbol{f}}_{uk}^{\dot{r}}$

Learning for the compensation of unknown dynamics using GPs

Consider an oracle with the predictions $\hat{\boldsymbol{f}}^{x}_{uk}, \hat{\boldsymbol{f}}^{\omega}_{uk}, \hat{\boldsymbol{f}}^{\dot{r}}_{uk} \in \mathcal{C}^0$ based on the data set $\mathcal{D}$. Let $S_{\mathcal{X}} \subset (SE(3) \times (\mathcal{X} \subset \mathbb{R}^6))$ be a compact set where the derivatives of $\hat{\boldsymbol{f}}^{x}_{uk}, \hat{\boldsymbol{f}}^{\omega}_{uk}, \hat{\boldsymbol{f}}^{\dot{r}}_{uk}$ are bounded on $\mathcal{X}$.
There exists a bounded function $\bar{\rho}\colon S_{\mathcal{X}} \to \mathbb{R}_{\geq 0}$ such that the prediction error is given by

$$\mathrm{P}\left\{ \left\| \begin{bmatrix} \boldsymbol{f}^{x}_{uk}(\bar{q}) - \hat{\boldsymbol{f}}^{x}_{uk}(\bar{q}) \\ \boldsymbol{f}^{\omega}_{uk}(\bar{q}) - \hat{\boldsymbol{f}}^{\omega}_{uk}(\bar{q}) \\ \boldsymbol{f}^{\dot{\mathrm{r}}}_{uk}(\bar{q}) - \hat{\boldsymbol{f}}^{\dot{\mathrm{r}}}_{uk}(\bar{q}) \end{bmatrix} \right\| \leq \bar{\rho}(\bar{q}) \right\} \geq \delta$$

with probability $\delta \in (0, 1]$ for all $\bar{q} = (\boldsymbol{q}, \xi) \in S$.
This ensures that there exists an (probabilistic) upper bound for the error between the prediction $\hat{\boldsymbol{f}}^{x}_{uk}, \hat{\boldsymbol{f}}^{\omega}_{uk}, \hat{\boldsymbol{f}}^{\dot{r}}_{uk}$ and the actual $\boldsymbol{f}^{x}_{uk}, \boldsymbol{f}^{\omega}_{uk}, \boldsymbol{f}^{\dot{r}}_{uk}$ on a compact set.

Learning for the compensation of unknown dynamics using GPs

- **Prediction**: Input matrix $X = [\bar{q}^1, \bar{q}^2, \dots, \bar{q}^N]$. Output matrix $Y^\top = [y^1, y^2, \dots, y^N]$, where y might be corrupted by additive Gaussian noise with $\mathcal{N}(0, \sigma^2 I_6), \sigma \in \mathbb{R}_{\geq 0}$.
  The prediction of the output $y^* \in \mathbb{R}^6$ at a new test point $\bar{q}^* \in S_\mathcal{X}$ is given by

$$\mu_i(y^*|\bar{q}^*, \mathcal{D}) = m_i(\bar{q}^*) + k(\bar{q}^*, X)^\top K^{-1} \left(Y_{:,i} - [m_i(X_{:,1}), \dots, m_i(X_{:,N})]^\top\right)$$

$$\mathrm{var}_i(y^*|\bar{q}^*, \mathcal{D}) = k(\bar{q}^*, \bar{q}^*) - k(\bar{q}^*, X)^\top K^{-1} k(\bar{q}^*, X).$$

Kernel: $k : S_\mathcal{X} \times S_\mathcal{X} \to \mathbb{R}$, correlation of two states $(\bar{q}, \bar{q}^{'})$.
Mean function: $m_i : S_\mathcal{X} \to \mathbb{R}$ to include prior knowledge.
Gram matrix: $K_{j^{'},j} = k(X_{:,j^{'}}, X_{:,j}) + \delta(j, j^{'})\sigma^2$ for all $j, j^{'} \in \{1, \dots, N\}$ where

$$\delta(j, j^{'}) = \begin{cases} 1 & \text{if } j = j^{'} \\ 0 & \text{otherwise} \end{cases}$$

Learning for the compensation of unknown dynamics using GPs

- **Prediction**: Input matrix $X = [\bar{q}^1, \bar{q}^2, \dots, \bar{q}^N]$. Output matrix $Y^\top = [y^1, y^2, \dots, y^N]$, where y might be corrupted by additive Gaussian noise with $\mathcal{N}(0, \sigma^2 I_6), \sigma \in \mathbb{R}_{\geq 0}$.
  The prediction of the output $y^* \in \mathbb{R}^6$ at a new test point $\bar{q}^* \in S_\mathcal{X}$ is given by

$$\begin{aligned} \mu_i(y^*|\bar{q}^*, \mathcal{D}) &= m_i(\bar{q}^*) \\ &\quad + k(\bar{q}^*, X)^\top K^{-1} \left(Y_{:,i} - [m_i(X_{:,1}), \dots, m_i(X_{:,N})]^\top\right) \\ \mathrm{var}_i(y^*|\bar{q}^*, \mathcal{D}) &= k(\bar{q}^*, \bar{q}^*) - k(\bar{q}^*, X)^\top K^{-1} k(\bar{q}^*, X). \end{aligned}$$

  Covariance between $\bar{q}^*$ and input training data X:
  $k : S_\mathcal{X} \times S_\mathcal{X}^N \to \mathbb{R}^N$, such that $k_j = k(\bar{q}^*, X_{:,j})$ for all $j \in \{1, \dots, N\}$

Learning for the compensation of unknown dynamics using GPs

The kernel k is selected such that $\boldsymbol{f}^x_{uk}, \boldsymbol{f}^\omega_{uk}, \boldsymbol{f}^{\dot{r}}_{uk}$ have a bounded reproducing kernel Hilbert space (RKHS) norm on $S_{\mathcal{X}}$.

The model error is bounded by

$$\mathrm{P}\left\{\left\|\mu\left(\begin{bmatrix}\hat{\boldsymbol{f}}^x_{uk}(\bar{q})\\ \hat{\boldsymbol{f}}^\omega_{uk}(\bar{q})\\ \hat{\boldsymbol{f}}^{\dot{r}}_{uk}(\bar{q})\end{bmatrix}\middle|\bar{q},\mathcal{D}\right)-\begin{bmatrix}\boldsymbol{f}^x_{uk}(\bar{q})\\ \boldsymbol{f}^\omega_{uk}(\bar{q})\\ \boldsymbol{f}^{\dot{r}}_{uk}(\bar{q})\end{bmatrix}\right\|\leq\left\|\beta^\top\Sigma^{\frac{1}{2}}\left(\begin{bmatrix}\hat{\boldsymbol{f}}^x_{uk}(\bar{q})\\ \hat{\boldsymbol{f}}^\omega_{uk}(\bar{q})\\ \hat{\boldsymbol{f}}^{\dot{r}}_{uk}(\bar{q})\end{bmatrix}\middle|\bar{q},\mathcal{D}\right)\right\|\right\}\geq\delta$$

T.Beckers, LJ.Colombo, S.Hirche, GJ.Pappas *Online learning-based trajectory tracking for underactuated vehicles with uncertain dynamics*. IEEE Control Systems Letters 6, 2090-2095, 2022.

Design of controllers of a system with partially unknown dynamics

- We want to prove the **stability of the closed-loop** with a control law using a Lyapunov function.
- The data-driven control law with safety guarantees is based on the error terms and modified error terms.

$$
\begin{aligned}
\boldsymbol{e}_x &= \boldsymbol{x} - \boldsymbol{x}_d, \qquad\qquad e_u = u - u_d, \\
\boldsymbol{e}_\omega &= \boldsymbol{\omega} - R^T R_d \boldsymbol{\omega}_d, \ \boldsymbol{e}_R = R_d^T R - I. \\
\tilde{e}_{\dot{u}} &= \dot{e}_u + \frac{1}{k_u} \boldsymbol{e}^T R^T \dot{\boldsymbol{e}}_x \in \mathbb{R} \\
\tilde{\boldsymbol{e}}_\omega &= e_\omega + \frac{1}{k_R} (R_d^T R \boldsymbol{e} u_d) \times R^T \dot{\boldsymbol{e}}_x \in \mathbb{R}^3.
\end{aligned}
$$

M.Kobilarov (2013). *Trajectory control of a class of articulated aerial robots*. IEEE International Conference on Unmanned Aircraft Systems (ICUAS).

Design of controllers of a system with partially unknown dynamics

Let the **Lyapunov function** be

$$V = \frac{k_x}{2}||\boldsymbol{e}_x||^2 + \frac{k_R}{2}||\boldsymbol{e}_R||^2 + \frac{k_r}{2}||\boldsymbol{e}_r||^2 + \frac{k_u}{2}e_u^2 + \frac{m}{2}||\dot{\boldsymbol{e}}_x||^2 + \frac{1}{2}\tilde{e}_{\dot{u}}^2 + \frac{1}{2}\begin{bmatrix}\tilde{\boldsymbol{e}}_\omega \\ \dot{\boldsymbol{e}}_r\end{bmatrix}\bar{M}(\mathbf{r})\begin{bmatrix}\tilde{\boldsymbol{e}}_\omega \\ \dot{\boldsymbol{e}}_r\end{bmatrix}.$$

Denote by $\boldsymbol{\xi} = [\boldsymbol{e}_x^\top, \boldsymbol{e}_R^\top, \boldsymbol{e}_r^\top, e_u^\top, \tilde{e}_{\dot{u}}^\top, \tilde{\boldsymbol{e}_\omega}^\top]^\top$, and observe that

$$\Lambda_1||\boldsymbol{\xi}||^2 \leq V \leq \Lambda_2||\boldsymbol{\xi}||^2,$$

where $\Lambda_1 = \frac{1}{2}\text{min}\{1, k_x, k_R, k_r, k_u, m, \lambda_{min}\}$,
$\Lambda_2 = \frac{1}{2}\text{max}\{1, k_x, k_R, k_r, k_u, m, \lambda_{max}\}$, and λ_1, λ_2 are the minimum and maximum eigenvalues of $\bar{M}(r)$.

Design of controllers of a system with partially unknown dynamics

Considering the control inputs $(u, \boldsymbol{\tau}, \boldsymbol{\tau}_r)$

$$\ddot{u} = -k_u e_u - k_u \tilde{e}_{\dot{u}} + \ddot{u}_d - \frac{1}{k_u}\left(\ddot{\boldsymbol{e}}_u^\top R\boldsymbol{e} + \dot{\boldsymbol{e}}_x^\top R(\boldsymbol{\omega} \times \boldsymbol{e})\right) - \mu\left(\boldsymbol{f}_{uk}^x | \bar{q}, \mathcal{D}\right),$$

$$\begin{bmatrix} \boldsymbol{\tau} \\ \boldsymbol{\tau}_r \end{bmatrix} = \begin{bmatrix} -k_R R^\top R_d \boldsymbol{e}_R - k_\omega \tilde{\boldsymbol{e}}_\omega - \boldsymbol{\mu} \times \boldsymbol{\omega} + C^\top \boldsymbol{e} u/m \\ -k_r \boldsymbol{e}_r - k_{\dot{r}} \dot{\boldsymbol{e}}_r - \frac{1}{2}\bar{\boldsymbol{\xi}}^\top \partial \bar{M}(\boldsymbol{r})\bar{\boldsymbol{\xi}} + M_{v\dot{r}}^\top \boldsymbol{e} u/m \end{bmatrix}$$

$$+ \dot{\boldsymbol{b}} + \frac{1}{2}\dot{\overline{M}}(\boldsymbol{r}) \begin{bmatrix} \tilde{\boldsymbol{e}}_\omega \\ \dot{\boldsymbol{e}}_r \end{bmatrix} - \begin{bmatrix} \mu\left(\boldsymbol{f}_{uk}^\omega | \bar{q}, \mathcal{D}\right) \\ \mu\left(\boldsymbol{f}_{uk}^{\dot{r}} | \bar{q}, \mathcal{D}\right) \end{bmatrix}$$

where

$$\boldsymbol{b} = \overline{M}(\boldsymbol{r}) \begin{bmatrix} R^\top R_d \left(\boldsymbol{\omega}_d - \frac{1}{k_r} B_\vartheta \left(R_d^\top R\right) \boldsymbol{e} u_d \times R^\top \dot{\boldsymbol{e}}_x\right) \\ \dot{\boldsymbol{r}}_d, \end{bmatrix}$$

we have

$$\dot{V} = -k_v ||\dot{\boldsymbol{e}}_x||^2 - k_{\dot{u}} \left(\tilde{e}_{\dot{u}}\right)^2 - \tilde{e}_{\dot{u}} \mu\left(f_{uk}^x | \bar{q}, \mathcal{D}\right) - k_\omega ||\tilde{\boldsymbol{e}}_\omega||^2$$
$$- \tilde{\boldsymbol{e}}_\omega^\top \mu\left(f_{uk}^\omega | \bar{q}, \mathcal{D}\right) - k_{\dot{r}} ||\dot{\boldsymbol{e}}_r||^2 - \dot{\boldsymbol{e}}_r^\top \mu\left(f_{uk}^{\dot{r}} | \bar{q}, \mathcal{D}\right).$$

Design of controllers of a system with partially unknown dynamics

Theorem

Consider the system

$$\begin{aligned}
\dot{R} =& R\omega, \\
m\ddot{x} =& m\boldsymbol{a}_g + R\boldsymbol{e}u + \boldsymbol{f}^{x}_{uk}(\boldsymbol{q}_0, \xi_0), \\
\begin{bmatrix} \omega \\ \dot{\boldsymbol{r}} \end{bmatrix} =& \bar{M}^{-1}(\boldsymbol{r}) \begin{bmatrix} \mu \\ \nu \end{bmatrix} \\
\begin{bmatrix} \dot{\mu} \\ \dot{\nu} \end{bmatrix} =& \begin{bmatrix} \mu \times \omega \\ \frac{1}{2}\bar{\xi}^T \partial \bar{M}(\boldsymbol{r})\bar{\xi} \end{bmatrix} + \begin{bmatrix} \tau - C^T \boldsymbol{e}u/m \\ \tau_r - M^T_{v\dot{\mathrm{r}}} \boldsymbol{e}u/m \end{bmatrix} + \begin{bmatrix} \boldsymbol{f}^{\omega}_{uk}(\boldsymbol{q}_0, \xi_0) \\ \boldsymbol{f}^{\dot{r}}_{uk}(\boldsymbol{q}_0, \xi_0) \end{bmatrix}.
\end{aligned}$$

and a GP model trained with the data set $\mathcal{D} = \{\bar{q}^{\{i\}}, y^{\{i\}}\}_{i=1}^{N}$. *The* ***control law*** *guarantees that the tracking error* $\boldsymbol{\xi}$ *is uniformly ultimately bounded in probability by*

$$P\left\{ ||\boldsymbol{\xi}(t)|| \leq \sqrt{\frac{\Lambda_2}{\Lambda_1}} \max_{q \in S_{\mathcal{X}}} \bar{\rho}(\bar{q}),\ \forall t \geq T \right\} \geq \delta$$

for all $\bar{q} \in S_{\mathcal{X}}$, $\delta \in (0,1)$, *with* $T \in \mathbb{R}_{\geq 0}$, *and renders the tracking error exponentially stable.*

Design of controllers of a system with partially unknown dynamics

Sketch of the theorem

We have seen that

$$\dot{V} = -k_v||\dot{\boldsymbol{e}}_x||^2 - k_{\dot{u}}\left(\tilde{e}_{\dot{u}}\right)^2 - \tilde{e}_{\dot{u}}\mu\left(f^x_{uk}|\bar{q},\mathcal{D}\right) - k_\omega||\tilde{\boldsymbol{e}}_\omega||^2 - \tilde{\boldsymbol{e}}_\omega^\top\mu\left(f^\omega_{uk}|\bar{q},\mathcal{D}\right) - k_{\dot{r}}||\dot{\boldsymbol{e}}_r||^2 - \dot{\boldsymbol{e}}_r^\top\mu\left(f^{\dot{r}}_{uk}|\bar{q},\mathcal{D}\right).$$

Considering that the model error is bounded by

$$\mathrm{P}\left\{\left\|\mu\left(\begin{bmatrix}\hat{\boldsymbol{f}}^x_{uk}(\bar{q})\\ \hat{\boldsymbol{f}}^\omega_{uk}(\bar{q})\\ \hat{\boldsymbol{f}}^{\dot{r}}_{uk}(\bar{q})\end{bmatrix}\middle|\bar{q},\mathcal{D}\right) - \begin{bmatrix}\boldsymbol{f}^x_{uk}(\bar{q})\\ \boldsymbol{f}^\omega_{uk}(\bar{q})\\ \boldsymbol{f}^{\dot{r}}_{uk}(\bar{q})\end{bmatrix}\right\| \le \left\|\beta^\top\Sigma^{\frac{1}{2}}\left(\begin{bmatrix}\hat{\boldsymbol{f}}^x_{uk}(\bar{q})\\ \hat{\boldsymbol{f}}^\omega_{uk}(\bar{q})\\ \hat{\boldsymbol{f}}^{\dot{r}}_{uk}(\bar{q})\end{bmatrix}\middle|\bar{q},\mathcal{D}\right)\right\|\right\} \ge \delta$$

the evolution of the Lyapunov function V can be upper bounded by

$$P\{\dot{V} \le -min\{k_v, k_{\dot{u}}, k_\omega, k_r\}||\xi||^2 - \left(|\tilde{e}_{\dot{u}}| + ||\tilde{\boldsymbol{e}}_\omega|| + ||\dot{\boldsymbol{e}}_r||\right)\bar{\rho}(\bar{q})\} \ge \delta$$

for all $\bar{q}$ on $S_{\mathcal{X}}$, $\delta \in (0,1)$.

Design of controllers of a system with partially unknown dynamics

Sketch of the theorem

$\dot{V}$ is negative with probability δ, for all $\boldsymbol{\xi}$ such that

$$||\boldsymbol{\xi}|| > \max_{\bar{q} \in S_{\mathcal{X}}} \bar{\rho}(\bar{q}) \frac{1}{min\{k_v, k_{\dot{u}}, k_\omega, k_r\}}$$

The Lyapunov function V is lower and upper bounded by

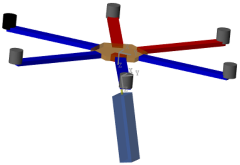
$$\alpha_1(||\boldsymbol{\xi}||) \leq V \leq \alpha_2(||\boldsymbol{\xi}||)$$

where $\alpha_1(r) = \Lambda_1 r^2$ and $\alpha_2(r) = \Lambda_2 r^2$
Finally, we can compute the maximum tracking error $\bar{b} \in \mathbb{R}_{\geq 0}$, such that

$$P\{||\boldsymbol{\xi}|| \leq \bar{b}\} \geq \delta, \text{ by } \bar{b} = \sqrt{\frac{\Lambda_2}{\Lambda_1}} \max_{\bar{q} \in S_{\mathcal{X}}} \bar{\rho}(\bar{q})$$

Simulations

Numerical example of the position tracking of a UAM



Hexarotor with a robotic arm

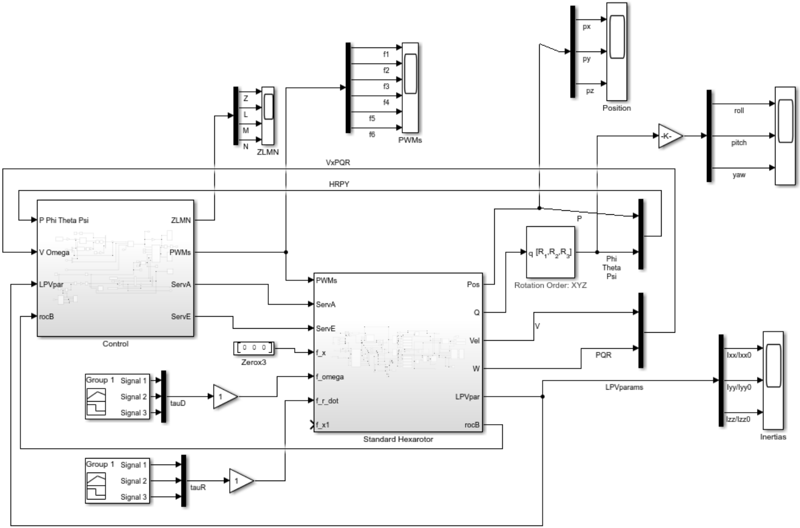
The prescribed trajectory orders the vehicle to execute a controlled takeoff, and the maintenance of a stable hovering position within the xy-axis

Objective: to achieve a positional stability in the lateral plane while effecting a precise vertical ascent of 1 meter along $z-$direction.

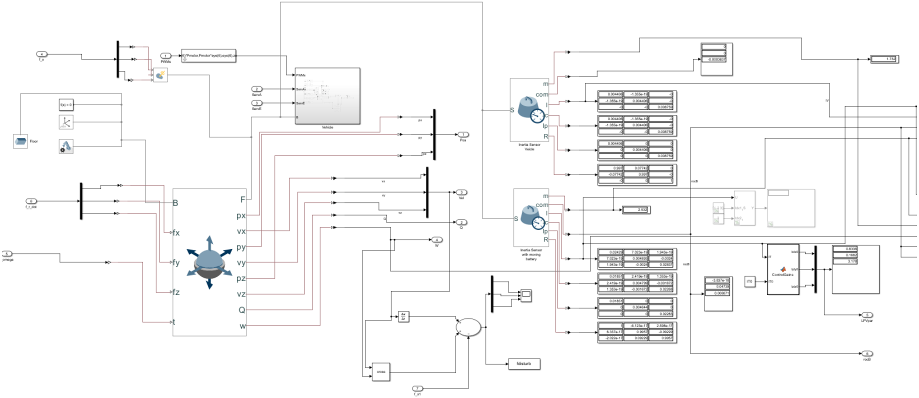
The simulation environment to construct the data set consists on the control allocation for the fault tollerant control of the hexarotor, where the robotic arm is used to stabilize the motion after the failure of a rotor.

C.Pose, J.Giribet and I.Mas. *Adaptive center-of-mass relocation for aerial manipulator fault tolerance.* IEEE Robotics and Automation Letters, 7(2), 5583-5590, 2022.

Simulations



Simulations



Simulations

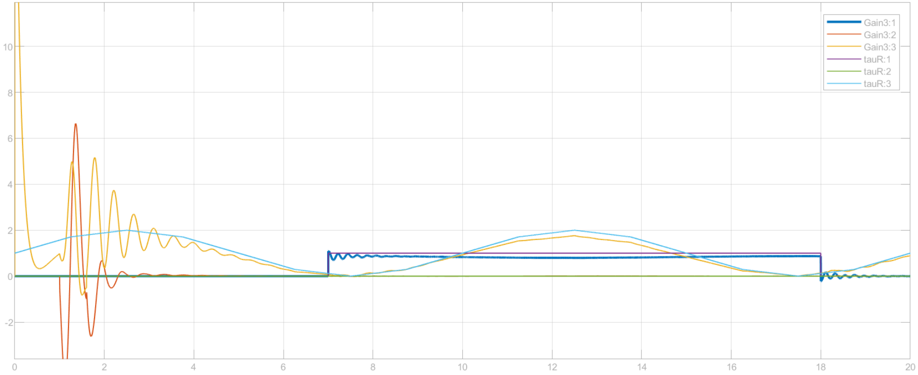


Figure: Disturbances considered in the vehicle

https://youtu.be/bHrvn5kpgns

Simulations

The use of GP process for disturbance estimation

- We use a GP model for the compensation of the unknown dynamics.
- Data set constructed using data related to velocities and positions.
- Squared Exponential kernel. To estimate the parameters of this kernel, the basis function coefficients and the noise standard deviation we have use the fitting model *fitrgp* included in *RegressionGP* model.

To compare the improvement in system performance with the inclusion of the GP for disturbance estimation, two simulations have been done with identical conditions, except for the incorporation of the GP for disturbance estimation.

The disturbances have been introduced into the forces acting on the vehicle.

Simulations

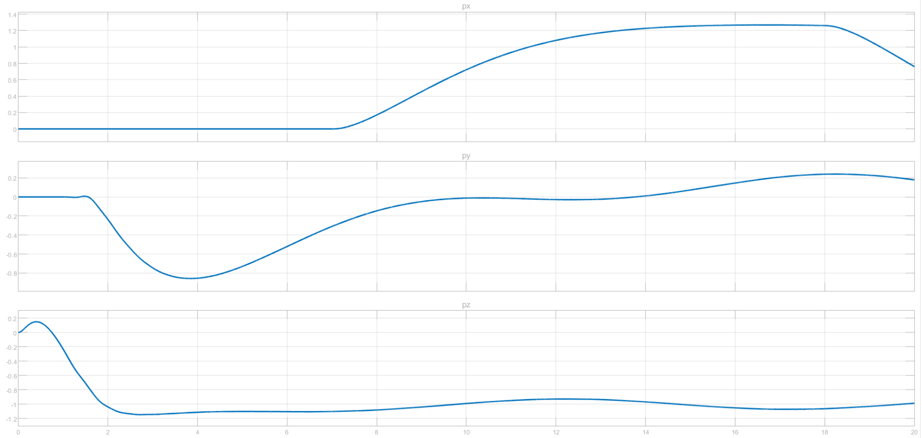


Figure: UAV's position when disturbances affect the vehicle, causing displacement of the center of mass from its desired position in the xy-plane

Simulations

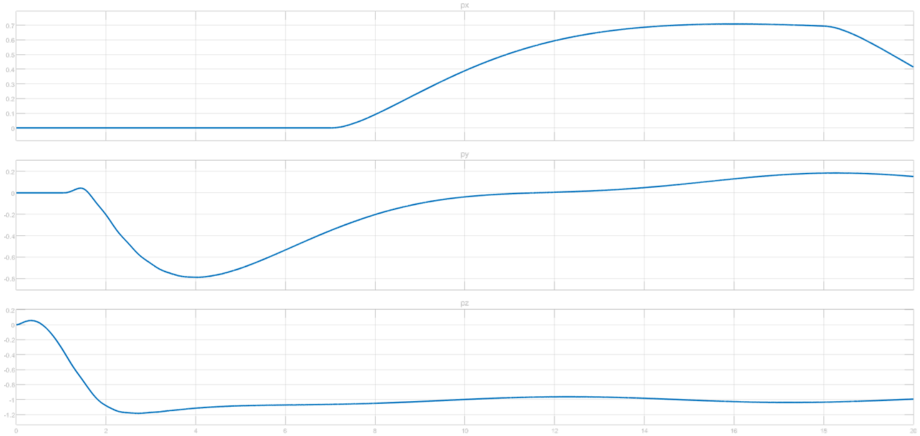


Figure: UAV's position incorporating the estimation provided by the GP into the control action

Conclusions

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- We have used learning-based approaches based on GP to provide a new control law that learns the uncertainties of the UAM.
- We have guaranteed the probabilistic boundedness of the tracking error.
- Using simulations, we have proved that, using GP, the impact of the disturbances on the system can be mitigated.

Thanks for your attention!