EXTERNAL EXAMINER REPORT ON THE PHD THESIS OF GABRIEL PEREIRA ALVES, UNIVERSITY OF WARSAW

MÁTÉ FARKAS

This is an external examiner report on the PhD thesis of Gabriel Pereira Alves (in what follows, 'the candidate'), who conducted their PhD studies at the Faculty of Physics at the University of Warsaw under the supervision of dr hab. Jędrzej Kaniewski.

1. Summary

The work in the thesis entitled 'Bell non-locality and certification in quantum devices' falls under the broad umbrella of classical certification of quantum devices. This research topic is timely in the theoretical quantum information science community: the emergence of quantum technologies calls for efficient classical means of verifying these devices. Such verification will find applications in quantum cryptography, communication and computation, and potentially in other sub-fields of quantum information science. The thesis therefore addresses an important topic within its broader field.

The thesis comprises three research papers, two of which are published in *Physical Review A*, a reputable international journal in quantum information science. The third paper is currently being considered for publication at an undisclosed venue, but I have no doubts that it is publishable in a journal of similar quality to *Physical Review A*. The candidate is the first author of all three papers, implying a significant contribution to each of them, as also evidenced by the comments in the thesis outlining the candidate's specific contributions to each of these works.

The thesis document itself consists of 17 pages of introductory material to the above three papers, discussing the broader scientific context and some of the mathematical tools used in the papers. This introductory material is then followed by the papers themselves, each of them with a short two-paragraph 'comment' summarising the main findings of the papers and the candidate's contributions.

The candidate's contributions concern a family of certification results in Bell scenarios ('device-independent certification' or 'self-testing', the strongest form of quantum certification), some general results on optimal quantum strategies and quantum certification in prepare-and-measure scenarios ('semi-device-independent certification') and the application of machine learning techniques to characterising Bell scenarios. All of these contributions are highly original and of excellent research quality on the international level.

2. Overall assessment of the introductory material

The introductory material starts with a historical account on Bell non-locality and its foundational relevance. This account is of exceptional quality, very clearly explaining highly complex metaphysical concepts, providing ample references guiding the reader. I would argue that the strongest feat of this thesis (apart from the results themselves) is this highly mature presentation of the broad scientific context. One does not often find this in PhD theses. The presentation even inspired me to look into some of the historical debates about local hidden variables (von Neumann's impossibility theorem). While there might be a few confusing statements about metaphysics (see the specific comments at the end of the report), the overall presentation of the history of Bell non-locality is of outstanding quality.

The thesis then moves on to mathematically formulate the famous Bell's theorem. This is again done in a very logical and intuitive fashion. What I'm missing here perhaps the most is a self-contained section of mathematical preliminaries for quantum theory, describing the Hilbert space formalism, quantum states and measurements. Throughout the introductory material, the text assumes a lot of prior knowledge, which somewhat seems to defeat the purpose of 'addressing the conceptual gaps' (quoting the candidate) that the papers leave. I would argue that it's probably very difficult to read the more technical bits of the introductory material without extensive prior knowledge in the very specific topics that the thesis addresses (Bell non-locality, quantum random access codes, semidefinite programming hierarchies, measurement incompatibility).

Generally speaking, a lot of mathematical concepts are not explained, a lot of details are left out, and—perhaps as a consequence—sometimes it's not even fully clear whether some of the statements are mathematically correct. I provide a non-exhaustive list in the 'specific comments' section, but as an example, above Eq. (2.6) it is written "an isometry which acts locally in the Hilbert space of Alice must be a unitary operator", which appears to be incorrect, or after Eq. (2.7) the matrices A, B_i and C_j are claimed to always be square, which is not necessarily the case. Similar inconsistencies, small typos and left out details riddle the rest of the technical preliminaries. Overall, I feel that the right balance between detail and clarity has not been found: there are quite a few complex mathematical concepts in the introductory material, but they are almost impenetrable if the reader has no extensive prior knowledge (in which case, the introductory material would not be necessary). I have no doubt that the candidate has mastered these mathematical concepts (after all, the publications speak for themselves), but the presentation of them lacks the mathematical rigour that would make it a truly useful material for less experienced readers (and a PhD examiner could very well be less experienced in some of the specific topics than the candidate).

3. Overall assessment of the papers

As mentioned above, all three papers are of very high standard. In my view, the first paper *"Optimality of any pair of incompatible rank-one projective measurements for some nontrivial Bell inequality"* is the strongest out of the three. I myself have used these results and have been inspired by them in some of my recent research. I believe that this work would have been possible to publish in a journal of 'higher impact' than that of *Physical Review Letters*. For example, as an editor of the journal *Quantum*, I can say that I believe it would have had good chances of being published there.

The paper builds on previous work that proved a self-testing statement for a maximally entangled state, also certifying projective measurements that have a uniform overlap. In this work, the candidate (together with the supervisor) generalises this self-test to projective measurements with arbitrary (non-trivial) overlap structure. This work demonstrates a thorough understanding of non-locality and matrix analysis, and I believe that this is a very strong first paper for a PhD candidate.

The second paper, *"Biased random access codes"* considers a well-explored 'prepare-and-measure' quantum information processing protocol called 'random access codes', and generalises it in a substantial manner. In particular, it considers cases in which the inputs to the protocol are not selected uniformly randomly (as is commonly studied) but are sampled from some non-uniform distribution. The candidate (and co-authors) finds that this input distribution has significant effect on the optimal quantum strategy (states and measurements) of the protocol. In some cases, certain self-testing-like statements can also be made, under the dimension assumption. The candidate also developed an extensive numerical package based on semidefinite programming for analysing arbitrary 'biased' random access codes, and this package has been highly useful in deriving some of the optimality and certification results in the paper.

The third paper, *"Machine Learning meets the CHSH scenario"* applies machine learning techniques to characterising correlations one can observe in the simplest Bell scenario (the 'CHSH scenario'). An even simpler variant of this scenario (the 'correlation space') has already been completely characterised analytically, providing a solid benchmark for the machine learning tools. The candidate (and co-authors) then apply various machine learning techniques to the task of classifying is a given correlation can be realised in quantum theory or not. They also address the problem of generating training data, and use the aid of well-known (in the community) semidefinite programming hierarchies. The authors find that constructing machine learning models that perform well on average in the classification problem is feasible. On the other hand, they don't yet know how to enhance existing methods especially for the 'difficult to characterise' regions of the correlation space. Perhaps it would have been a strong contribution to try and explore a bigger correlation space, or some previously less explored 'corners' of the CHSH scenario. Nevertheless, this paper is a highly valuable and thorough first account (to my knowledge) on machine learning techniques for characterising Bell scenarios, and researchers looking into this possibility should certainly consult this paper first.

4. Conclusions

The papers together with the introductory material clearly demonstrate the candidate's high level of knowledge of the theoretical concepts in the field of quantum theory and quantum information. Specifically, a deep understanding of the concept of Bell non-locality, and an ability to employ sophisticated mathematical methods as well as develop complex numerical tools for its analysis. It is a rare combination to show high-class intuitive, mathematical and numerical abilities. It is also clear that the results presented by the candidate provide original solutions to problems in quantum information science, significantly generalising previously developed techniques in certification, as well as initiating the study of machine learning tools in classifying non-signalling correlations. In conclusion, in my view the thesis positively fulfills the conditions for a PhD degree award at the University of Warsaw.

5. Specific technical comments and questions

This section contains a list of comments and some questions about details of the introductory material. Some of these are (a non-exhaustive list of) typos or examples of arguments that could be improved on, while some of these are genuine scientific questions. The comments are in order of appearance in the introductory material.

(1) In the abstract and the preface, various acronyms (and corresponding concepts) are not introduced: NPA hierarchy, CHSH quantum set, SDP.
(2) In the second paragraph of page 6, the connection of local realism to the measurement problem is not quite clear.
(3) The last paragraph of page 6 is confusing to me. It seems to claim that the experiment considered by EPR is problematic if we take a non-realistic perspective. Whereas, to my understanding, the problem arises if one takes a realistic perspective together with locality.
(4) Similarly, on the top of page 7 it is stated that EPR "argue that, in the absence of local-realism, quantum mechanics does not provide a complete description of physical reality". I find it difficult to understand this statement. I would maybe expect something like EPR argued that quantum theory is not complete (i.e. not real) because if one assumes it's real then it violates locality (which they found problematic). Perhaps this is what is meant here, but the phrasing was confusing to me.
(5) On the same page, the Copenhagen interpretation is not explained (just mentioned).
(6) The account on von Neumann's impossibility theorem is very intriguing, but perhaps doesn't highlight that this impossibility theorem is fundamentally different from Bell's (they are mathematically but perhaps even conceptually trying to prove different things).
(7) The term 'locality' and 'locally' are used in a somewhat free fashion. Initially, it is used in the context of local realism, but on the bottom of page 7, it's apparently used more in the sense of 'local' (classical) correlations, so essentially to mean both locality and realism.
(8) The explanation of a local hidden variable model is very intuitive. However, once we get to Eq. (1.4), the roles of x and y are not really mentioned (previously only A and B are mentioned). This maybe leaves a bit of a gap in the reader's understanding.
(9) Altogether, the presentation of a local hidden variable model doesn't make an explicit connection to local realism. This could have been done through the λ parameters, which are the 'elements of reality' (well-defined properties of a physical system, as described on page 6).
(10) The lack of mathematical preliminaries make statements like "evaluating the above realisation" below Eq. (1.11) very knowledge-dependent and somewhat vacuous.
(11) Statements like "the locality loophole concerns the experimental requirement that the measurement process be subject only to causal influences local to each part" on page 10 are perhaps overly brief and could benefit from slightly more detailed explanation.

(12) The right-hand-side of the last equation on page 10 doesn't depend on a and b, while the left-hand-side does. It looks like effects instead of observables should have been used here.
(13) The description of Ekert's protocol on pages 11–12 leaves quite a few details out. For example, it only talks about checking the violation of CHSH, which only uses settings 0 and 2. How does then one infer anything about the outcomes of the settings 1, which are used in the key generation?
(14) On page 12 in the sentence "It is important to note, however, that certification has a broader meaning and is used to designate validation protocols based on weaker assumptions than those typically used by a device-independent scheme.", I assume "weaker" should be "stronger"? The assumptions in a device-independent scheme are usually as weak as possible.
(15) Between Eqs. (1.17) and (1.18) the phrase "convex hull of inequalities" is used, and it's unclear to me what this means.
(16) Eq. (1.18) presents 8 independent parameters describing a correlation in the CHSH scenario. Perhaps a link to the fact that this set is 8-dimensional could have been made here.
(17) 'Facet' is not defined on page 13.
(18) On page 14 it is mentioned that the set $\mathcal{Q}$ "cannot be finitely generated", but it's unclear to me what this means.
(19) The subsequent paragraph mentioning the commuting paradigm and Tsirelson's problem is vague to the extent that it's probably impossible to understand without prior awareness of the problem.
(20) On the bottom of page 14 it is written that the TLM inequalities provide the only case in which the boundary of the quantum set is known analytically. How does this relate to recent results analytically characterising the extremal points of the full 8-dimensional CHSH quantum set in `arXiv:2406.09350`?
(21) On Fig. 2.1, the notation $[d]$ is not defined.
(22) Near the top of page 16 it's written "While non-facet inequalities do not provide necessary and sufficient criteria for non-locality", which is true, but nor do facet inequalities (neither of them are necessary).
(23) The subsequent paragraph, starting "To write these measurements...", assumes a lot of prior knowledge about measurements, subspaces and incompatibility.
(24) Looking at Eq. (2.1), I'm wondering whether a quantitative connection to (biased) QRACs can be made. Re-writing the terms $p(a = \bar{y}, b = x_y|x, y) = 1 - p(a = y, b = x_y|x, y) - p(a = \perp, b = x_y|x, y)$, once we add the penalty terms in Eq. (2.2), the Bell inequality is expressed in terms of only $p(a = y, b = x_y|x, y)$ and $p(a = \perp, b = x_y|x, y)$, the first of which is very much of a QRAC flavour. It is perhaps a missed opportunity (or a potential future research question) to make this connection between the first two papers.
(25) Motivating the penalty term in Eq. (2.2) could perhaps be done more convincingly, using arguments from Ref. [45] (the penalty is forcing the optimal realisation)
(26) I believe the inequality below Eq. (2.4) in $\mathcal{O}_{x_1x_2} \leq 1$ should be strict.
(27) The footnotes are not numbered consistently in the introductory material. E.g. both the footnotes on pages 8 and 17 are numbered 1.

(28) Above Eq. (2.6) it is written, "an isometry which acts locally in the Hilbert space of Alice must be a unitary operator", but clearly some isometries are not unitaries.

(29) Below Eq. (2.6) it is not clearly stated that the self-testing statement is made from the maximal violation of the inequalities.

(30) Above Eq. (2.7) it is written "A semidefinite program is an optimisation problem in which the objective function is optimised within the intersection of the positive semidefinite cone of matrices in the problem space". It's not said what the objective function could be (it's not an arbitrary function) and it's not clear what we intersect the positive semidefinite cone with.

(31) Below Eq. (2.7) it's written that the A, B_i and C_j matrices are square, but they don't have to be in general.

(32) In the footnote on page 19, the notation $\succ$ and $\prec$ is not explained.

(33) The description of the NPA hierarchy on top of page 20 is quite vague.

(34) Above Eq. (2.8), it's written "As we do not wish to impose any constraints on the dimension of the Hilbert space, the state can be assumed to be pure and the measurements, projective, without any loss of generality", but there's no mention why (purification and Naimark dilation).

(35) The '1' in Eq. (2.8) should be the identity operator.

(36) Below Eq. (2.8) 'Gram matrix' is not explained.

(37) In the following paragraph there is a sentence "Although the solution to this optimisation problem does not fully resolve the quantum set, it defines the set $\mathcal{Q}_1$ of which $\mathcal{Q}$ is a subset". From the current description, however, it's not quite possible to discern what $\mathcal{Q}_1$ is.

(38) On top of page 21, it is said that "the optimisation is conducted within a direction in the probability space rather than a Bell expression". I'm wondering why these are not equivalent (as any Bell expression is a linear functional on correlations)?

(39) In the first paragraph of section 2.2.2. it is written "the problem becomes significantly easier when either the state or the measurements are fixed, allowing the problem to be tackled using SDPs". If the measurements are fixed, the problem is indeed an SDP, but if only the state is fixed and both measurements are variables then it is not.

(40) In Eq. (2.9) the summation has a lower limit (0) but not an upper one.

(41) It's not quite clear what is optimised in Eq. (2.10). The sum only runs over a, b and y, but x is fixed, so this isn't maximising the Bell inequality, just a part of it. It would already be an SDP if we include the sum over x and all $M_{a|x}$ are variables.

(42) The QRAC section is adequately written, but suffers from similar problems as the other ones, in terms of self-containment and mathematical rigour.

MA'TE' FARKAS, YORK, 12/11/2024