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Subject: Report on PhD thesis
Name of candidate: Ms. Deeksha Kanti
Title of thesis: Radiative Recombination in Strong Laser Pulses
Supervisor: Dr. hab. Katarzyna Krajewska

To whom it may concern,

The dissertation of Ms. Deeksha Kanti is devoted to theory development, with the aim of investigating laser-assisted radiative recombination in short laser pulses. The topic is strong-field physics and attoscience, an area of physics that has received significant attention over the last 30 years. Two different scenarios have been considered in the dissertation, i.e., laser-assisted electron-atom radiative attachment and laser-assisted electron-ion radiative recombination, as discussed in Chap. 2 and 3, respectively. In both cases, focus was set on the development of approximate and analytic models for the processes at hand. This thesis work is both timely and relevant, in view of the fact that radiative recombination represents an essential step in high harmonic generation, a process that has played a crucial role in the experimental development of attosecond laser pulses.

The thesis, which is written in English, consists of in total 101 pages and is divided into 4 chapters. Chapter 1 gives a five-page long detailed introduction to the topic. It is well written and provides an adequate and useful introduction to basic concepts. The introduction also includes as many as 64 well chosen references, demonstrating that the candidate is able to put the work into a broader context. Chapter 2 and 3 define the main body of the thesis and will be elaborated on below. The work is summarized in Chap. 4. In addition, the dissertation contains two appendices with supplementary material. The thesis, which is rather short and condensed, is clearly structured, well written and contains only very few typographical and grammatical errors.

The PhD work has resulted in two scientific papers published in Physical Review A, which is an internationally leading journal within atomic, molecular and optical (AMO) physics. The candidate is first (main) author on both papers, and as such, has made a major contribution to the research and writing process. Furthermore, the candidate has presented the work in several international conferences and workshops.

The two publications are listed below:

- Deeksha Kanti, M. M. Majczak, J. Z. Kamiński, Liang-You Peng, and K. Krajewska, *Laser-assisted electron-atom radiative recombination in short laser pulses*, Phys. Rev. A **104**, 033112 (2021).

- Deeksha Kanti, M. M. Majczak, J. Z. Kamiński, Liang-You Peng, and K. Krajewska, *Laser-assisted radiative recombination beyond the dipole approximation*, Phys. Rev. A **110**, 043112 (2024).

Chapter 2 in the thesis is devoted to theory development around *Laser-Assisted Radiative Attachment* (LARA). The impact of a short laser pulse on the attachment process, where an electron combines with the hydrogen atom to form the hydrogen anion H^-, was considered. This process results in the emission of a photon, whose spectrum is subjected to theoretical analysis. The content of Chap. 2 is basically the same as was published in the candidate's first paper in Phys. Rev. A **104**, 033112 (2021). In effect, this part of the thesis work has already been subjected to peer review and found worthy of being published. Nevertheless, I will endeavor to make an independent assessment.

The formulation of radiative attachment in Chap. 2 is based on the strong-field approximation (SFA) ansatz, i.e., the influence of the atomic potential is neglected in the initial electron scattering state, and further, the influence of the laser field on the final bound state is not accounted for. For the short range atomic potential considered here, and for not too strong laser fields, the approximation seems adequate. It is also assumed that the electric dipole approximation is valid, which seems plausible for the laser parameters considered in the numerical examples. As such, much of Chap. 2 is devoted to theory development and the mathematical formulation of LARA, arriving at an explicit formula for the angular and energy distribution of the emitted photon. The mathematical derivations seem correct, given the approximations introduced.

Although laser-assisted radiative recombination has been studied before, as also mentioned in the introduction to Chap. 2, what is really new here as compared to previous works, is that both field-free, field-modified and field-induced recombination are treated on an equal footing, with the possibility of influence among them. By calculating emission spectra from given examples, it is demonstrated that the present model performs better than previous ones, in that unphysical effects are eliminated. These effects were identified to be remnants of the field-free and/or field-modified contributions that were not properly accounted for in earlier formulations.

Yet this is a point of confusion to me and my physical understanding of the process:

- How possibly can the field-free process that occurs on an infinite timescale interfere with the field-induced recombination that only occurs on a very limited period of time?
- Is it the field-free and/or field-modified contribution that gives rise to the unphysical oscillations in the energy spectra of LARA as seen in, for example, Fig. 2.2?
- If it is the field-modified component that is responsible, should this component not simply be subtracted from the LARA amplitude in order to remove unphysical effects? Is it possible to remove this component from the theory? If so, how would Fig. 2.2 then appear?
- Varying the laser intensity, how would this impact the results in, for example, Figs. 2.2 and/or 2.3?

I hope the candidate could clarify my misunderstanding on these matters during the defense of the thesis.

Chapter 2 in the thesis also contains a study of the impact of trains of pulses on the LARA energy spectra, effectively leading to frequency combs that could find potential use in spectroscopy and metrology. Furthermore, and very interestingly, performing a time-frequency analysis of the resulting spectra, it is demonstrated that the radiation from LARA could possibly be used as a new experimental tool for temporal reconstruction of the laser pulse, simply by analyzing the corresponding spectrum of the radiation.

Chapter 3 in the thesis is devoted to theory development on *Laser-Assisted Radiative Recombination* (LARR) beyond the dipole approximation. Here the recombination of electrons with hydrogen-like positive ions is considered, and the effect of the long-range Coulomb potential is accounted for via the approximate Coulomb-Volkov scattering state. As such, the model developed in Chap. 3 differs from the one in Chap. 2 in two important ways in that both Coulomb and nondipole effects are included into the formulation. Beside that, the two models resemble each other. Furthermore, much like in Chap. 2, the recombining wave packet is assumed to be a near monochromatic electron wave. The main body of Chap. 3 was already published in the candidate's most recent (second) paper in Phys. Rev. A **110**, 043112 (2024), which again means that external review has already been undertaken.

I have some questions to the work in Chap. 3 that I hope the candidate could elaborate on during the defense:

- It is made an effort in developing a model that takes Coulomb effects into account, but ultimately it seems that Coulomb effects only play a minor role. What was the role of Coulomb field in the present work? When exactly are Coulomb effects expected to become important, and how would they affect the results?
- In Eq. (3.29) the effect of the nondipole field is manifested in the Volkov phase by two terms, i.e., the $e\boldsymbol{A} \cdot \boldsymbol{p}$ and $-e^2\boldsymbol{A}^2/2$ terms, respectively. Which (if any) of the two terms are most important for the nondipole effects studied in the present work?
- In the model, beyond dipole dynamics comes in both via the Volkov phase in the scattering state, cf. Eq. (3.29), and via the light-matter interaction in the recombination step, cf. Eq. (3.39). Which of the two contributions are most important for the present results, and what happens if one of them is left out?
- In the light-matter interaction Hamiltonian Eq. (3.39), the effect of the nondipole field is present in two terms, i.e, the $e\boldsymbol{\mathcal{E}} \cdot \boldsymbol{r}$ and $e\boldsymbol{\mathcal{E}} \cdot \boldsymbol{r}\ (\boldsymbol{n} \cdot \boldsymbol{p}/mc)$ terms, respectively. Which (if any) are important for the nondipole effects studied here?
- Only one laser intensity was considered here, and the dependence of the results on the laser intensity was not paid much attention. By increasing/decreasing the laser intensity, how would this alter the results in, for example, Fig. 3.2?
- Finally, on the role of nondipole effects, how would Fig. 3.2 appear in the dipole approximation?

In summary, this thesis represents a valuable new and independent contribution to the strong-field physics domain. The thesis is well written, and the work has resulted in two original publications in Physical Review A, where the candidate is the first (main) author. The development of the radiative-recombination model is considered being the main achievement of the candidate, alongside with several applications of the model on relevant problems. This model represents an original solution to a scientific problem. Although I have raised several questions during my assessment, these are by no means meant as any criticism of the thesis, but rather as a starting point for scientific discussions during the thesis defense. Therefore, I would like to emphasize that no changes to the thesis are necessary at this point. Finally, my conclusion is positive, and I find that the PhD thesis of Ms. Deeksha Kanti is eligible for conferral of the PhD degree and thus recommend that Ms. Deeksha Kanti is given the opportunity to defend her written dissertation in an oral examination.

Sincerely yours,

Morten Førre