Referee Report: *"Observations of anomalous transverse local momenta in spatial wave-functions"*

PhD Candidate: Bohnishikha GHOSH

Summary

The thesis investigates the phenomenon of backflow, occurring in quantum mechanics when wave-functions composed exclusively of states with positive momentum yield negative local probability currents. This counterintuitive effect manifests through negative local phase gradients despite the positivity of the underlying momentum components.

The research primarily explores backflow within the framework of paraxial optics. In this optical analogy, the transverse optical field amplitude takes on the role of the wavefunction, and the evolution of the field along the main propagation axis mimics the temporal evolution of quantum wavefunctions. Consequently, optical backflow is studied through the transverse phase gradients of propagating optical fields.

Structured into six chapters, the thesis addresses both experimental and theoretical aspects of backflow. Chapter 1 provides an extensive overview, illustrated by the example of a quantum particle initially confined to the negative half-line ($x<0$) with strictly positive momentum. Paradoxically, the particle demonstrates an increasing probability of remaining within this region over time. The chapter further establishes a conceptual link between quantum backflow and optical phenomena, particularly focusing on transverse optical momentum. The author acknowledges ongoing discussions on whether backflow fundamentally arises from quantum mechanics or can be explained purely as a wave phenomenon.

Chapter 2 details the experimental methodologies applied throughout the thesis, focusing primarily on optical measurements using a Shack-Hartmann wavefront sensor (SHWFS). A key contribution is the precise differentiation between the Poynting vector, local momentum, and canonical momentum, experimentally demonstrating that local transverse momentum aligns specifically with canonical momentum. The principal findings from this chapter are presented in Ref. 67, with Ms. Ghosh as the first author.

In Chapter 3, the thesis describes experimental observations of anomalous transverse momenta in classical optical configurations involving the superposition of two broad Gaussian beams. The central discussion revolves around whether these phenomena should strictly be categorized as backflow or as superoscillation, given that locally measured momenta exceed the initial frequency components. The primary results of Chapter 3 are published in Ref. 31, authored first by Ms. Ghosh.

Chapter 4 explores azimuthal backflow by superimposing optical beams, each carrying positive orbital angular momentum (OAM). Notably, experiments reveal regions of reversed local azimuthal momentum despite all constituent beams possessing positive OAM. Key outcomes of this chapter are documented in Ref. 32, again with Ms. Ghosh as the first author.

Finally, Chapter 5 extends these studies into the quantum domain, reporting on experiments performed with single photons and photon-sensitive cameras. These findings have been presented at several scientific conferences, highlighting broad interest in quantum manifestations of optical backflow and related effects.

Questions

Q1. How were experimental uncertainties systematically treated and quantified throughout the classical optical experiments, particularly when using wavefront sensors, considering error bars are typically absent in experimental plots?

Q2. In Chapter 2, Fig. 2.5 exhibits experimental oscillations absent from theoretical predictions. Could these deviations stem from imperfections in generating Laguerre–Gaussian beams?

Q3. In Chapter 3, why is the phenomenon classified explicitly as backflow rather than superoscillation, given that local momenta exceed the frequencies present in the original wave components, as shown in Fig. 3.3?

Q4. In Chapter 4, does the observed backflow region rotate along the propagation axis due to differing Gouy phases between the individual OAM components?

Q5. In Chapter 5, how exactly was Fig. 5.2 generated? Does it represent numerical simulations or experimental data derived from classical optical experiments, augmented with simulated Poissonian statistics?

Q6. Why, in Chapter 5 (Fig. 5.5), is a spatial filter used alongside single-mode fiber coupling, which typically already ensures spatial mode purity?

Q7. Is the operation of cameras explicitly in a photon-counting regime critical for the quantum experiments presented in Chapter 5, or could alternative solutions (e.g., EMCCD or intensified CCD cameras) without time-resolved detection suffice?

<u>**Comments**</u>

C1. Chapter 2, Fig. 2.1(c) could benefit from ensuring beam shifts clearly align with local phase gradients.

C2. Chapter 4, Figures 4.2(a) and 4.4(a) would significantly improve with a consistent 1:1 aspect ratio.

C3. Explicitly referencing Hypergeometric-Gaussian (HyGG) modes in Chapter 4 (Eq. 4.1 and Eq. 4.8) could enhance theoretical clarity. See Optics Letters 34 (8), 1225-1227 (2009).

C4. Chapter 5, Fig. 5.8 would be clearer if the color scale were adjusted to enhance visibility of individual photon detection events.

<u>**Overall evaluation**</u>

The thesis is well-written, clear, and demonstrates high scientific rigor. Most content has already been shared with the scientific community through scientific publications (Refs. 67, 31, and 32), with Ms. Ghosh as the first author, and via presentations at national and international conferences. The results presented are of significant interest, highlighting a remarkable counterintuitive phenomenon observed in relatively simple optical setups. The listed comments and questions are primarily minor and aim to further enhance the final dissertation.

Finally, let me say that the PhD thesis justifies the request for distinction. It makes a significant contribution to the understanding of backflow in quantum mechanics and its optical analogues. The results presented offer deep insights into fundamental aspects of physics and are of clear interest to the wider scientific community. They are likely to inspire further investigations, not only in the field of optics but also in matter-wave systems. Furthermore, the candidate's role as first author on multiple publications in high-impact journals attests to the originality and relevance of the work, and to its recognition by the research community.

Naples, 2025 April 17th

Dr. Filippo Cardano