Gabriel Molina-Terriza
Ikerbasque Research Professor
gabriel.molina.terriza@gmail.com
Ph: 943018925
Manuel Lardizabal 5
Donostia-San Sebastian
20018 Gipuzkoa (SPAIN)

Centro de Física de Materiales
Materials Physics Center

Aprin 21st 2025

The Academic Council of Physical Sciences

Chairman Professor Jacek Szczytko

Dear Professor Szczytko,

Thank you for allowing me the chance of reviewing the Doctoral Thesis of Ms Bohnishikha Ghosh, entitled "Observations of anomalous transverse local momenta in spatial wave-functions". It was a pleasure to read it and I better understood some concepts of the subject that I was not aware of. After reviewing the Thesis and considering the conditions set in your invitation letter I have no hesitations in concluding positively (a) positive) on this Thesis for the reasons I will mention below.

The Thesis of Ms. Ghosh delves into the phenomenon of quantum backflow. This is an interesting effect which was studied in quantum mechanics where a quantum particle (electron, photon, …) can be set in a superposition state such that even if the particle's momentum wavepacket is directed to a given direction, say right, part of the wavepacket in real space can go in the opposite direction (say left). In this way, the probability of finding the particle in the "wrong" direction can increase with time in some conditions,

In Chapter 1, the candidate gives a very concise and instructive description of the phenomenon, linking it to wave phenomena in optics. In particular I enjoyed very much the discussion on the links to superoscillations and superresolution imaging. In this way, the candidate opens a venue of exploration where the backflow in quantum mechanics can have applications which go beyond the fundamental nature of this Thesis. I would have liked to see in this Chapter any indication of how to "engineer" the conditions of backflow. It is quite clear from this Chapter that backflow will occur when there is a local momentum of the field that goes in the opposite direction (as defined by wavefunction in momentum space). Nevertheless, from the Thesis it seems as if the conditions for optimal backflow are difficult to find and, in the Thesis, this is done with iterative algorithms. Maybe there aren't any general methods to analytically ascertain the conditions for obtaining backflow for a given wavefunction in momentum space, but it would have been interesting to mention it in this otherwise very pedagogical chapter.

In Chapter 2 Ms. Ghosh presents a very beautiful discussion regarding the different local momenta which appear in optics, in particular the canonical momentum and the Poynting vector. Moreover, in this chapter the layout of the main experimental set-up is presented where the phenomenon of backflow can be measured as well as the nature of the canonical momentum. This last part is the one presented in this Chapter.

The experimental set-up is not very complicated in terms of optical elements. The most critical part of the experiment is the measurement of the local momenta with a microlens array and a camera. The candidate explains very well what are the difficulties of working with this system and how to proceed with a calibration of the system. This is done by finding the centroid of the spots focused from the microlenses. In the Thesis it is explained how this algorithm is implemented and there is even a link to a opensource repository for using it. I have tried a couple of times and could not access the repository, but maybe it is due to the fact that the thesis has not been published yet. In any case, I find it vey good for Science in general and for our community that those programs can be freely used.

I have just a couple of minor comments on this part. The first one is about the noise in the measurements of Figure 2.5 and the deviations from the expected behaviour. This is briefly mentioned in a couple of lines and, in my opinion, it would have deserved a much lengthier discussion. I understand that some details can be found in the manuscript published in the scientific journal, but given that the limitations of space in the thesis are less stringent, the candidate could have added an extended discussion here. Moreover, part of this discussion, at least the one related to the crosstalk from the different spots, appears in Section 4.2. In my opinion, to the very least there should be a call to that Section, but I would have appreciated more details on the subject. The other missing piece in this Chapter is related to the breaking of the paraxial approximation in the microlenses. If all the experiment is performed in the paraxial approximation it is clear that the polarization of the field does not play a role, but I wonder if there could be deviations from these expectations if the microlenses have a very large Numerical Aperture. Even if a bit out of the scope of the Thesis, I think that briefly mentioning this in the Conluding remarks of this Chapter would have added value to the thesis.

Chapter 3 presents a very beautiful experiment and theoretical analysis on the measurement of backflow in optics using Gaussian beams. The theoretical analysis is quite exquisite taking into account that Gaussian beams extend to the infinity and then, in principle, some of their tails could have a bit of backflow *per se*. This is explained in Section 3.3, although there is a discussion I don't really understand well, so probably it could have a bit more detail: when comparing Eqs. (3.8) and (3.9). Eq. 3.9 is supposed to give the amount of "back-momentum" expected from purely measuring the wavefunction in momentum space. As such, it takes all the probability of the wavefunction up to a kref and divides it by the total weight of the function in the full space. On the other hand, Eq. 3.8 provides a value of the region of the fringes where the local momentum is smaller than the set reference, what I don't understand is why in this case, the region under study is a single fringe, maybe in other fringes there is not as much "back-momentum", and then the ratio of PBF would be worst.

Besides this small point, I have really enjoyed in this Chapter the care to detailing the model including lens imperfections, finite size, etc. The comparison between the experimental values and the theoretically obtained ones is very beautiful and the inclusion of the real-life situations provides a very good intuition on why in some situations where one could expect a bigger back-momentum, it would be difficult (or impossible) to measure.

Here, in my review, I would like to include a discussion which also applies to Chapter 4. Something that I found surprising is that in the whole Thesis what is really measured is the effect of what I call "back-momentum", i.e. a local momentum flowing in a direction opposite to what is expected. As I mentioned before, the candidate does a very good job in the first Chapters of tying this with the phenomenon of "back-flow", i.e. an increase of the power/densitity/intensity/probability in a direction which is not expected. However, this phenomenon is not checked directly and I could not find a discussion in the Thesis on why this is so. I am not sure if this is because it is very difficult to measure, because the percentage of increase is very small o what. I understand again that this could be out of the scope of the thesis, which from the very title indicates that it is going to focus on this back-momentum (anomalous momentum), but I think that a discussion on the experimental nuances of measuring that expected increase in local intensity would have been interesting.

Passing to Chapter 4, again I have to express my admiration for the beauty of the experimental set-up and the care the candidate has put in the Thesis to explain the experimental problems encountered. This Chapter focuses on experiments dealing with structured light and orbital angular momentum, and explains in detail how an azimuthal back-mometum has been measured. This, as far as I know, is a novel concept. The Thesis also explains how the experimental calibration could be performed and explains the problem that was already mentioned in Chapter 2 of the cross-talk between different microlens spots.

Something that surprised me about the experimental set-up and that I think that it goes a bit against the "concept" of backflow, is that the superposition of the two vortex beams is alredy produced in the Spatial Light Modulator. I understand that this relaxes a lot the experimental complexities and actually it is going to produce the same result (if not better) than if the experiment would have been performed by preparing two different beams and making them interfere. The thing that I find a bit "dirty", if I may use this expression, is that, if I understand well the experiment, the local phase gradients are already present in the SLM, so that the "surprise" of their appearance in the measurement is not as much. I think that the candidate should have mentioned this in a bit of detail and, above all, why they chose this preparation instead of the one used in the previous Chapter.

Chapter 5 was a bit confusing in my first reading of the Thesis. There is no doubt that the science and experiments discussed there are very nice, and it is really impressive how the candidate has managed to work in all these different systems to explore the possibilities of backflow: classical optics with linear momentum, classical optics with azimuthal momentum, quantum optics (single photon regime) and in the next Chapter even starting some research with electrons. However, I say that this Chapter in particular is confusing because it could have been structured in a different way. The chapter describes an experimental set-up using down-conversion in order to use triggered photon states to measure the backflow in the quantum regime. Unfortunately, this experimental set-up did not work as expected and the Chapter describes quite well what were the problems encountered and how to overcome them in future refinements of the experiment. This kind of "failed" experiments often interest me more in a Thesis than the "successful" ones, because they are seldom found in the scientific journals and it is clear that in an excellent Thesis like this one, some experiments must not provide the expected results because the research is cutting-edge.

My confusion stems from this not being explicitly stated as far into the Chapter. I think that this could be plainly stated in the introduction of the Chapter because otherwise it is not understood certain sentences there: there is a the preparation of a SPDC source, but then the results are numerically simulated and then mimicked with an attenuated laser. Once we arrive to the part where it is explained why the experiment did not work as expected then, everything makes more sense. However, I would have enjoyed a bit more details on the operation of the camera and the efforts made to measure the single photon regime. Maybe providing histograms of the noise of the camera, the background and the histograms when single photons where expected to appear, would have helped to visualize the problems with the measurements.

Chapter 6 closes the Thesis with an investigation of the possibilities of observing backflow in future experiments with TEM systems. The Chapter is short and basically presents a set of calculations and also the lay-out of possible experiments. I think that having a dissertation where all these different systems are investigated experimentally is rare, and, nevertheless, the candidate shows a good understanding of the physical principles of all the systems.

The Conclusions Chapter I found particularly interesting because the Discussion on future experiments and possibilities shows the maturity of the candidate in the research and that she could pursue a research career independently, pursuing interesting topics of research.

Overall, as I said in the beginning I have found this Thesis very interesting to read. There are a few typos and some visualization problems that could be corrected in the future. In particular, I think that the fonts in most figures are way too small and would be difficult to read in he paper version of the Thesis. In the pdf I had to use the magnification that computers offer in order to be able to understand some figures. Notwithstanding this minor problems in the presentation of the dissertation, it is in my opinion an excellent research work that has advanced the knowledge in this area of optics, paving the way to more complex experiments using electrons or other matter waves.

Again, I reiterate that I think that the Thesis can approved as it is, maybe if possible, by implementing the optional suggestions that I have been laying out in this report.

Kind regards,

April 21st 2025

Prof. Gabriel Molina Terriza
Ikerbasque Research Professor
Quantum Nanophotonics Laboratory
Materials Physics Center