Abstract

Strongly correlated quantum systems, characterized by competing interactions, play a central role in emergent phenomena across quantum materials. Motivated by the need for robust models and reliable simulation techniques, this thesis develops a rigorous theoretical framework alongside novel efficient methods to study hybrid quantum systems that combine light–matter and electron–phonon interactions.

We begin by deriving the fundamental Hamiltonians governing these interactions. For light–matter systems, the second quantization of the electromagnetic field leads to the Dicke model, which is then extended by incorporating direct spin–spin interactions. In parallel, the Holstein model for electron–phonon systems is derived and further generalized to include electron–electron interactions, thereby capturing the essential physics of competing processes in complex materials.

Building on these theoretical developments, efficient numerical techniques are introduced by representing bosonic degrees of freedom through Gaussian and non-Gaussian states and applying variational methods that culminate in a hybrid numerical approach. This method integrates a variational ansatz for the bosonic sector with many-body numerical techniques for fermionic and spin components, enabling precise simulations of strongly correlated systems.

The developed framework is then applied to both sectors. For electron–phonon systems, a quantum simulation platform is proposed and the hybrid numerical method is validated on the Hubbard–Holstein model, serving as a precursor to its extension to more complex scenarios. For light–matter systems, the extended Dicke–Heisenberg model is systematically investigated under the hybrid numerical method to elucidate the interplay between spin–photon and spin–spin interactions and to clarify the conditions underlying superradiance and magnetic ordering.

Overall, this study provides a clear and versatile framework for exploring the interplay of competing interactions in hybrid quantum systems, with direct implications for both theoretical analysis and experimental design in quantum materials.