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Dimitrios Patramanis' PhD report

Dimitrios Patramanis' PhD thesis concerns the topic of Krylov complexity in the context of quantum-many body systems, quantum field theories and gravity.

It consist of a two-step introduction (respectively, very broad and broad, but topical), **four chapters, each being the original peer-reviewed research article with a short commentary**, and a brief summary of all results, as well as brief outlook.

The PhD candidate in his PhD studies supervised by Prof. Paweł Caputa made interesting and noticed contributions to a very active contemporary area of research. The latter is reflected in **impressive, 300+ citations to the candidate's works**. In broad terms, this area concerns quantum information aspects of high energy and gravitational physics.

About 12 years ago first proposals due to Susskind and later Susskind with collaborators appeared proposing that so far overlook gravitational quantity, spacetime volume, is related to quantum circuit complexity. This idea was convincing and potentially powerful on one hand and vague on the other, which triggered significant interest (measured in several hundreds of papers) in studying the potential connection.

An offspring of this activity was not only progress in understanding quantum circuit complexity in quantum field theory and quantum-many body systems, but also discovering new notions of complexity and understanding their properties and relevance. The latter is precisely what the thesis is about: the notion of Krylov complexity appeared in the context of operator dynamics about seven years ago and about four years ago was generalized to state dynamics by the PhD advisor and co-authors.

Furthermore, Ref. [190] showed that Susskind's black hole volume in the simplest model of quantum gravity, the JT gravity, is the Krylov complexity of the underlying SYK model. This, and later developments tie the story of Krylov complexity with gravity and provide additional strong motivation to study Krylov complexity.

As a result, it is fair to state that the thesis concerns a frontier topic of significant importance.

Let me now go one by one through the research papers in chapters three to six.

The paper in chapter three, coauthored by the advisor and Javier Magan and published in Physical Review Research, concerns associating Krylov complexity with the evolution associated with simple symmetry algebra. This work should be regarded as a set of rare exact results where there are closed-formulas for Krylov complexity and its ingredients. In particular, some of these results appeared elsewhere and the authors recognized their origin in the underlying symmetry. The candidate points to several new directions, in establishing which the results described in chapter three played an important role. **The paper was cited close to 200 times, which is a significant number.**

In chapter four, the results of the preceding paper are used to shed light on the dynamics in systems with bi-partite Hilbert space giving rise to various entanglement-based probes. The paper is a single-authored work by the PhD candidate published in Progress of Theoretical and Experimental Physics, which indicates scientific maturity. The main insight of this paper is elucidating new physical information in other than Krylov complexity quantities characterizing the occupation in the Krylov bases.

In chapter five, the underlying paper was written with Watse Sybesma and published in SciPost. The paper has a lot of flavour of the paper from chapter three with the exception that the group know is more complicated (the Schrödinger group in 2 dimensions), yet is a semi-direct sum of simpler groups. The authors cleverly tackle the problem by not constructing the Krylov basis directly and argued their trick applies to possible other groups being semi-direct sums. The journal report for this paper is openly accessible

https://scipost.org/submissions/2306.03133v4/#report_1

and I agree with it.

In chapter six, the PhD candidate included his work with the advisor, Javier Magan and Erik Tonni published in Physical Review D. The key idea of this work is to define and analyze the properties of Krylov complexity for time evolutions generated by modular Hamiltonians. The latter has been an important research topic in the past 15 years or so and it was a very natural and interesting idea to understand the Krylov complexity associated with it. The authors realize it in a number of exactly soluble examples. This work is especially interesting as it opens a new direction in the studies of Krylov complexity.

In chapter seven, the PhD candidate included a brief recap of the results and some general open questions for the field, which I found acceptable.

Regarding the judgement of the thesis, it should be clear that it is a positive one. Its strengths lie in timeliness, impact, scope, one single-authored paper that corroborates scientific maturity of the candidate and diverse collaborators. If I were to seek weaknesses, given that the bulk of the thesis are original research articles, I would be even more pleased in my role of a referee if the rest of the thesis was more tight and connected. By this I mean in particular clarifying the logic for inclusion of various topics in the introductory chapters (for example, path integral optimization is relevant for some remarks made in conclusions, but otherwise one may question why it appears in the introduction so prominently) and interconnecting the thesis. Also, the conclusions are quite broad and written from the perspective of the discipline rather than the perspective of the results in the thesis.

I am happy to accept the thesis as it is and recommend to proceed with the PhD defence.

Prof. Michal P. Heller

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