Abstract

In this dissertation, we investigate a supercooled phase transition (PT) in the early Universe. Using high-temperature dimensional reduction, we calculate the next-to-leading order (NLO) nucleation rate, paying particular attention to the proper inclusion of higher-order corrections and renormalisation scale (RG-scale) dependence. In our approach, we explicitly evaluate fluctuation determinants, providing a state-of-the-art description of thermal bubble nucleation at NLO. We study a concrete model called SU(2)cSM, which extends the conformal Standard Model by an additional $SU(2)_X$ gauge sector. The new sector also contains an additional scalar field that acquires a non-zero vacuum expectation value via radiative symmetry breaking. This breaking of the $SU(2)_X$ proceeds through a supercooled first-order phase transition.

The first part of this thesis provides the theoretical background. We begin by introducing effective actions and discussing RG-scale dependence of effective potentials. Then, we turn to quantum field theory at finite temperature. We describe the relation between free energy and the effective actions. We review perturbation theory at finite temperature and perturbative expansions within the framework of effective field theory (EFT). Finally, we discuss the thermal bubble nucleation.

In the second part of this dissertation, we apply the theoretical framework we have introduced to describe the SU(2)cSM model and supercooled phase transition. We establish a power-counting scheme and construct the leading-order (LO) effective potential. Next, we discuss the symmetry breaking and show the available parameter space of the model. To address the issue of RG-scale dependence, we derive an RG-improved effective potential for the dark SU(2) sector. Then, we include the thermal corrections and show how high-temperature effects can influence even a supercooled transition. Then, for the first time, we apply high-temperature dimensional reduction to a model with classical scale-invariance (and supercooled PT). We discuss the resulting 3d EFT and its validity. Finally, we derive an NLO thermal nucleation rate. We examine various approximations to this nucleation rate and their limitations. In particular, we numerically evaluate fluctuation determinants, which leads to a state-of-the-art description of bubble nucleation. We also present a direct comparison of different methods for a chosen benchmark.

In the last part, we collect the phenomenological results. We describe all phase transition parameters and perform parameter space scans for SU(2)cSM. Then, we discuss the gravitational wave (GW) signals resulting from the phase transition, and show quantitative predictions of the signal in SU(2)cSM. We compute the signal-to-noise ratio for Laser Interferometer Space Antenna (LISA) and find that the supercooled phase transition in SU(2)cSM leads to a strong signal observable in LISA across the entire parameter space considered. This makes the model experimentally testable in future experiments. Finally, we discuss the impact of theoretical uncertainties on the GW predictions. We show that inclusion of higher-order corrections can substantially modify both the PT parameters and resulting GW spectra.