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## Report on the dissertation of Maciej Kierkla

Dear Colleagues,

The topic of the thesis are cosmological phase transitions in extensions of the standard model of particle physics, and their gravitational wave (GW) signatures. The thesis focuses on one particular scenario, a conformal extension of the standard model (cSM), which is well motivated and an important benchmark for general new physics scenarios. Quantitative studies of cosmological phase transitions are a formidable task, due to poor convergence of perturbation theory, gauge dependence in incomplete higher order calculations, and due to the presence of several dynamical scales which become important in various steps. The cSM scenario furthermore allows for a large amount of supercooling, which calls into question the validity of the standard approaches to these problems.

The thesis is based on a series of publications by M. Kierkla and collaborators, which systematically address these problems, and results in one of the first calculations of the false vacuum decay rate with full next-to-leading order accuracy in a suitable expansion parameter (the gauge coupling  $g$ ). Along the way, several approximation schemes (such as the loop expansion, gradient expansion and high temperature approximation) are evaluated and their limitations are clearly highlighted. The methods developed in this context can be applied to other models and serve as a blueprint for how to achieve this level of precision. The results are used to update the predictions for the GW signal of this model, and confirm that all of the viable parameter space can be probed by future GW missions.

In the introductory chapters (2 and 3) the candidate clearly demonstrates that he has **mastered the general knowledge in the subject area of the thesis**. Not only are the basic methods of computing the effective potential at zero and finite temperature reviewed, but also their shortcomings and limitations are highlighted – this is in particular the gauge dependence of the so called Daisy-resummation method which leads to large uncertainties in signal predictions obtained with that method. As an alternative, the dimensional reduction method is introduced, which allows for a systematic resummation at fixed order in perturbation theory, such that gauge dependencies are eliminated (or pushed to higher orders). Further subtleties arise in the computation of the vacuum decay rate, which probes the effective potential at different scales, and requires the evaluation of the so called bounce action as well as the corresponding fluctuation determinant. The general formulas

and several approximation methods are discussed in this section, based on state-of-the-art developments in the field.

These tools are then applied to the cSM in chapters 4, 5 and 6. The results presented there are novel, and as already mentioned above, represent **an original solution to a scientific problem**, namely to obtain accurate predictions for phase transition parameters and the GW signal in models with classical scale invariance. This also clearly demonstrates that the candidate has obtained **the ability to independently conduct scientific work**.

I will now discuss the results and their importance in more detail. In chapter 4, the basic structure of the cSM is presented. Due to the classical scale invariance, the model features a large separation of scales and a phenomenon called dimensional transmutation, where a mass scale is generated from the renormalisation group evolution (RGE) of the couplings. This implies that computations at fixed scales suffer from large logarithms which can spoil the perturbative expansion. Therefore the RG improved effective potential is constructed, which also serves as basis for the next chapters.

Chapter 5 is the heart of the thesis. In 5.1, the high temperature effective theory (EFT) is obtained using the methods explained in chapter 3, and the nucleation EFT is obtained. A crucial issue which is resolved here is the following: Since the phase transition is supercooled, the temperature drops below the field value at the zero temperature minimum of the potential, thus potentially spoiling the validity of the high temperature expansion. The important insight is then that the vacuum decay rate only depends on the effective potential in the region of the potential barrier, where the high temperature expansion is valid. The final result of subsection 5.1 is the consistent, gauge invariant leading order estimate of the vacuum decay rate, together with the demonstration that all approximations taken to obtain it are valid, or that contributions from regions where validity is not guaranteed are suppressed. This is already novel in the sense that most work in the literature is based on a mixture of LO and NLO results, which are then typically gauge dependent.

Starting with subsection 5.2, the question then is how to achieve NLO precision for the transition rate, which requires the evaluation of several fluctuation determinants, at the one loop level. The result then has to be re-expanded according to the actual soft expansion, to obtain consistent fixed order results. In subsection 5.3, the fluctuation determinants are evaluated in a derivative expansion, resulting in the master formula Eqn. 5.71, which is a result for the NLO tunneling rate which is RGE and gauge invariant up to higher order corrections. However there is a caveat, since it still needs to be checked whether the derivative expansion is valid in all regimes. One of the main concerns are the existence of so called scale-shifters, i.e. fields whose mass changes significantly along the profile of the tunneling field configuration. This is carefully evaluated in subsection 5.4, which is another remarkable part of the thesis. The conclusion is that for the LO result the derivative expansion is valid, while for NLO the approximation starts to fail, and it is shown that it becomes completely unreliable at the next (NNLO) order. However it is also shown that several parts of the computation at NLO can reliably be obtained in the derivative expansion. Since this is technically easier, a viable strategy is presented according to which this is supplemented by an explicit computation of the fluctuation determinant for those contributions where the validity could not be demonstrated.

Therefore a second method to calculate the fluctuation determinants is employed, the so called Gel'fand-Yaglom method. The final result is then Eqn. 5.109, which is the final expression for the NLO tunneling rate which is now demonstrated to be NLO accurate, gauge and RGE invariant, and consists exclusively of results derived within the thesis.

Since results in several approximations are derived throughout the thesis, it now becomes possible to compare them also in practice, which is done in subsection 5.6. The result is that also in practical applications, only the full NLO expression and one slight simplification of it concerning the calculation of the prefactor yields reliable results. Other approximations also give meaningful results, however they don't approximate the NLO result better than the simplest Daisy resummed approach. This is another important insight: Partial inclusion of higher order corrections do not necessarily lead to an improved understanding of the theory.

In chapter 6 the results are used to perform several phenomenological studies of the model. In particular the GW signal is computed, and the importance of the higher order corrections is demonstrated.

The thesis concludes with a thorough outlook, listing a few possible avenues for future directions and applications. It shows that the candidate has obtained a good overview of the field and keeps track of current developments.

The thesis itself is well written. A pedagogic introduction is followed with a concise but clear presentation of the results, while lengthy mathematical expressions and intermediate steps are reported in the appendix. There are a few typos that could be fixed, but they don't affect my overall excellent impression of the thesis. I find it comparable to the best theses that I got to read and grade here in Mainz, and therefore I recommend it for a **distinction**. I find that the breadth and depth of the results clearly goes beyond what can be expected from a PhD thesis on average. The key result of the thesis is a blueprint for how to obtain NLO precise phase transition parameters and gravitational wave predictions in new physics scenarios, that can easily be applied to other models, and will be crucial input for experimental searches for these models. And the thesis represents an important bridge from theoretical papers, where methods and approximations are discussed in principle, to concrete applications in realistic models, with all the resulting complications.

Sincere regards,



Prof. Dr. Pedro Schwaller