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Kraków, 05.01.2026

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Report on Ph. D. thesis

"Dynamics of bubble walls in cosmological first-order phase transitions"

by Mateusz Grzegorz Zych

The submitted thesis concern the question of the understanding of dynamics of the bubble of a true broken vacuum expanding in a false unbroken vacuum. Such bubbles are expected to be produced in the cosmological first order phaser transition in the early epoch of the Universe. As the bubbles expand, they interact with the surrounding plasma particles as well as collide with other bubbles. All of this generates inhomogeneities in the energy momentum tensor, which eventually translates into gravitational waves (GW). Obviously, detailed understanding of the bubble dynamics is essential for search of their imprints in the future GW observations, which may prove (or disprove) some scenarios of the evolution of the early Universe.

The first order phase transition is also relevant in the context of fundamental field theories. Indeed, an Electroweak first order phase transition was postulated to explain the matter-antimatter asymmetry. Importantly, baryogengesis takes place at the surface of the bubble. Again, a comprehensive understanding of phase transition dynamics and in particular the real-time evolution of the bubbles is essential for prediction of baryon asymmetry.

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A very important quantity, which is still less under control, is the velocity of the bubble wall. Obviously, due to the energy difference between the true and false vacua, there is a dragging force that naively makes the bubble accelerate to the speed of light, $v \to c$. However, the bubble does not live alone. It is surrounded by plasma of (fundamental) particles and other bubbles. Interaction with them effectively exerts a velocity dependent friction that can balance the vacuum pressure and lead to a steady expansion of the bubble with a constant velocity $v < c$.

Notably, the velocity of the bubble has nontrivial phenomenological consequences. Bubbles with small and moderate velocities are efficient sources for baryogenesis, whereas bubbles with $v \to c$ give a stronger GW signal. Therefore, a comprehensive understanding of the dynamics of the bubble and especially the velocity of the babble wall is crucial both from a theoretical as well as phenomenological point of view. This is the main aim of the thesis to compute this quantity.

The thesis has 112 pages divided into five sections and is supplemented by two appendices and the bibliography, which includes 138 positions. The thesis is based on three published articles: *„Hydrodynamical constraints on the bubble wall velocity"*, Phys. Rev. D 108 (2023) 103523, *„Bubble-wall velocity in local thermal equilibrium: hydrodynamical simulations vs analytical treatment"*, JHEP 05 (2024) 011 and *„Steady-state bubbles beyond local thermal equilibrium"*, JHEP 06 (2025), 118. Moreover, four other papers give additional support for the claims of the thesis.

The thesis begins with an Introduction where the Author describes the motivation of the research program, focusing on the importance of wall velocity in the cosmological and electroweak phase transition.

The first section is devoted for a theoretical description of the first order phase transitions. It starts with a presentation of false vacuum decay, both due to quantum tunneling (at $T=0$) and thermal fluctuations (at $T>0$). Here, the static bubble is defined. In fact, it is a 3-dim (or 4-dim) *sphaleron*, i.e., a saddle point solution. If perturbed and evolved neglecting the surrounding thermal plasma (and other bubbles), it expands and replaces the energetically less favorable false vacuum by the true vacuum.

Thermal fluctuations modify the effective potential via the temperature-dependent corrections, which allows us to extract physical parameters that describe the dynamics of the phase transition. They are e.g., nucleation rate, nucleation temperature, percolation time, and temperature, duration, and strength of the phase transition. Finally, there is the bubble wall velocity that plays a central role in predicting the phenomenological imprints, especially in the baryogengesis and in gravitational waves.

There are three primary mechanisms for GW production: (1) collisions of the bubble walls; (2) sound waves in the primordial plasma, and (3) turbulent motion. Which of the mechanisms is the dominating source of GW depends on the dynamics of the bubble in the phase transition. In stationary expansion the energy (latent heat) is transferred to the plasma and sound waves and turbulences are the main sources of GW. In the run-away scenario, where $v \to c$, produced GW comes mainly from the bubble collisions. The resulting GW stochastic spectra, and their possible overlap with the sensitivity of the LISA experiment, are discussed at the end of this section.

In the second section, mgr Mateusz Zych discusses cosmological first order phase transition generated by the real scalar singlet extension o the Standard Model. It is a well established fact that the electroweak symmetry breaking is a smooth crossover transition and, consequently, is insufficient to trigger the baryon asymmetry. This requires a rather strong out-of-equilibrium process as a first order phase transition. This can be realized via extensions of the Standard Model, where the scalar singlet is the simplest and the most straightforward candidate. This not only induces the first order Electroweak phase transition and put it in a cosmological context but simultaneously provides a natural candidate for Dark Matter. In this section the Author studies the resulting thermal effective potential and the vacuum structure (and how it changes with the temperature) defining the acceptable parameter space and the properties of this first order phase transition.

In the next, third section, M. Zych focuses on the hydrodynamic description of expanding bubbles, where the surrounding plasma is treated as a perfect fluid.

In principle, the dynamics of the bubble is encoded into the scalar field hyperbolic equation of motion, where in addition to the scalar-field force (potential difference between the vacua) the back-reaction of the thermal plasma particles should be included, see eq. (3.1). It contains the usual equilibrium part as well as a novel non-equilibrium, dissipative-like contribution. Assuming a stationary

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expanding bubble with a constant velocity v, a balance formula is found, eq. (3.5). Neglecting the non-equilibrium part gives the local thermal equilibrium (LTE) expansion, while taking into account this part (via a phenomenological parameterization) leads to the beyond LTE analysis.

In the pure hydrodynamic approximation, the dynamics is encoded into hydrodynamic equations - the conservation of the energy-momentum tensor. Assuming a stationary motion of the bubble bubble results in a reduction to self-similar regime where hydro PDEs are replaced by two ODE (3.10), (3.11) supplemented by two matching conditions (3.13), (3.14). To close the system, an Equation of State (EoS) has been added. Here, the bag model is used. Solutions of a hydrodynamical system depend solely on the transition strength and the bubble-wall velocity (which enters the matching conditions). There are three qualitatively distinct regimes:
(1) deflagration, which is the slowest solution, where the expanding bubble pushes the heated plasma in front of the bubble-wall.
(2) detonation, which is the fastest solution, where the bubble-wall expands like a shock wave and hits the fluid remaining at rest in front of the bubble.
(3) hybrid, which is a mixture of these two.

This picture is further refined by introducing entropy. In the LTE approximation, where non-equilibrium effects are neglected, the crucial observation is that no entropy is produced. This implies the conservation of the entropy current and leads to the third matching condition that allows us to get rid of the wall-velocity as a free parameter. One result of this LTE approach is the appearance of no nucleation regime (to weak phase transition) and run-away regime (no stationary solution).

To apply this framework to a realistic situation, one maps the thermal effective potential to EoS. This leads to an estimate of the hydrodynamic backreaction (3.29). Of course, the stationary solution emerges if it is equal to the vacuum dragging force. However, the back reaction is a non-monotonic function of the wall velocity. In conclusion, in some parameter space, deflagration or hybrid solutions come with a corresponding detonation solution. The detonation solution is not a stable solution and transmutes to the run-away scenario.

To go beyond the LTE regime it is obligatory to assume entropy production at the babble wall. This modifies the third matching condition and is parametrized by a non-equilibrium friction parameter η. This allows for studying of the near-LTE regime, where $\eta \to 0$. In this limit, the run-away region (η=0) consists of stable detonation solutions. Below, there is a region with a deflagration/hybrid and (simultaneously existing) detonation solution (reflecting the previously mentioned non-monotonic property of the backreaction). For even weaker phase transition one finds a deflagration/hybrid regime and finally no nucleation regime.

The main question arising from the hydrodynamical approach is which solution, the slower or faster, is dynamically realized. This issue is addressed in the fourth section, where the real-time simulation of the bubble growth is investigated. Here, the thermodynamic equations of the plasma are supplemented by a field theoretical equation of motion of the *single*, *spherically symmetric* expanding bubble. Both dynamical systems (field theory and the fluid) are coupled via the non-equilibrium friction term (and via EoS). Furthermore, the equilibrium effective potential is assumed to be the field theoretical potential with a temperature-corrected coefficients, eq. (4.16). This is the most innovative part of the thesis, providing a novel insight into the true dynamics of the bubble. Initially, the bubble is assumed to be in its sphaleron-like state with the spatially uniform static plasma at the nucleation temperature.

The initial sphaleron evolves to a stationary self-similar solution. Depending on the friction, it is detonation or deflagration/hybrid, which shapes excellently agree with the bag model predictions. In general, a higher friction leads to a lower bubble wall velocity.

One important result is the existence of a velocity gap, which means that not all velocities of the bubble-wall are realized. As the nucleation temperature grows to the critical temperature, the gap widens with more significant exclusion of deflagration/hybrid solutions. This is explained as hydrodynamic obstruction. As we are close to the critical temperature, the increasing bubble velocity leads to an increase in maximal temperature, T_max, in the temperature profile across the bubble. Hence, for sufficiently large wall velocity, T_max is bigger than the critical temperature and no stationary solution can be formed.

The size of the velocity gap can be predicted by incorporating the entropy production matching condition.

In the LTE limit, the real-time dynamics reveal significant deviations from the (hydrodynamical) matching method. Basically, the detonation solution (which for $\eta=0$ becomes a run-away solution) replaces all the deflagration/hybrid solutions. The real-time dynamics selects the fastest solution if there are two available solutions. This can be explained as an effect of rapid bubble growth. At an early stage of the evolution, the perturbations of the plasma are small, and therefore the backreaction on the bubble is weaker (than in the stationary regime). Thus, the bubble accelerates further to the point where no stationary solution can be formed any longer.

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In a consequence, *the detonation scenario emerges as the more generic outcome*. This is the first main finding of the thesis. In the beyond LTE regime, M. Zych derives a selection rule for the existence of a stationary solution. Namely, if there is no detonation solution for η >0, then the LTE gives a steady motion solution. Otherwise, if there is a detonation solution, then in the LTE limit a run-away scenario is realized.

The second main observation, perhaps even more important, is that „*the assumption of immediate stationarity, inherent to the matching method, overlooks the crucial role of early-stage dynamics in determining the long-term behaviour of the bubble wall".* This should be underlined: **the early stage of bubble evolution** (during cosmological phase transitions) **has a significant impact on the long-time evolution,** resulting in phenomenologically important modifications.

The last five chapter briefly summarizes the results underlying the phenomenological consequences. For example, many regions of the parameter space, previously considerable viable, do not lead to an efficient baryogenesis, for which a slow evolving bubble wall is needed. On the other hand, such fast-moving bubble walls may be a strong source of GW signals.

The thesis is very well written, with only a few typos, and I read it with great pleasure. Definitely, it contains several results with a great phenomenological impact. The result that the short-time evolution of the bubble is so crucial for its long-time evolution and the formation (or not) of a particular stationary solution is extremely important. It also led to the following questions:

1. In the real-time computation (section IV), a particular initial condition is assumed. It is a static bubble solution (sphaleron). The natural question is how the results of this section depend on this choice?
2. One could for example consider a gaussian initial condition for the profile of the field. Such „dirty" initial conditions can generate the bubble in a kind of excite state. Excitation can mean excitation of an internal degrees of freedom of the bubble, i.e., a normal (or quasi-normal) mode. It can also mean radiation trapped inside the bubble. The point is that early, medium and even late time dynamics of localized solutions, typically called solitons - to which one can include the bubble itself - are known to be very sensitive to such excitations.

An example of such an impact can be provided by head-on collisions of two vortices in the Abelian Higgs model at critical coupling. Unexcited vortices scatter under 90° degree, while excited vortices reveal a much more complicated behavior. Depending on which mode is excited one finds also backscattering (repulsion) or even multiple bouncing solutions (temporal attraction) with a chaotic dependence on the initial data (amplitude and phase of the mode and velocity of the colliding vortices), see:

[1] Steffen Krusch, Morgan Rees, and Thomas Winyard, *„Scattering of vortices with excited normal modes"*, Phys. Rev. D110, 056050 (2024), arXiv:2406.04164

[2] A. Alonso Izquierdo, N. S. Manton, J. Mateos Guilarte, and A. Wereszczynski, *„Collective coordinate models for 2-vortex shape mode dynamics"*, Phys. Rev. D110, 085006 (2024), arXiv:2405.20249 [hep-th].

[3] A. Alonso-Izquierdo, N. S. Manton, J. Mateos Guilarte, M. Rees, and A. Wereszczynski, *„Dynamics of Excited BPS 3-Vortices"*, Phys. Rev. D111, 105021 (2025), arXiv:2502.15087 [hep-th].

Undoubtedly, the late time dynamics, and even the final state, depends on the early-time evolution and small changes in the initial data. This effect is known to exist in many other theories with different types of solitons - also in the case of domain walls, which are very closely related to the bubbles, see

[4] A. Alonso Izquierdo, L. M. Nieto, and J. Queiroga-Nunes, *„Scattering between wobbling kinks"*, Phys. Rev. D103, 045003 (2021), arXiv:2007.15517 [hep-th].

3. A related question is the possible formation of a broken-phase bubble even before the phase transition, that is, for temperatures close to but above the critical value. Although unstable such bubbles can be easily stabilized by the above mentioned excitation of the modes or trapped radiation, see section V in

[5] D. Canillas Martinez, P. Dorey, T. Romanczukiewicz, P. Saffin, K. Slawinska, A. Wereszczynski, *„Oscillons and bubbles in Q-ball dynamics"*, JHEP12 (2025) 154; [arXiv:2509.03192].

In principle, such a bubble could be formed before the phase transition. Then, as the temperature further drops and the broken vacuum becomes the true vacuum, it can have a sufficiently large size to prevent the rapid growth period. This might offer a mechanism in favor of the stationary bubble solutions.

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Overall, I am very impressed with the candidate and I strongly support him in obtaining his Ph. D.. Furthermore, I think that the thesis deserves to be honored with a distinction.

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