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**Report on the PhD thesis by Bartłomiej Bąk:**
***Theory of Gravity as Theory of Local Inertial Frames***

## 1. General background

This dissertation develops a novel and systematic extension of gravitational theories formulated within the purely affine framework.

Purely affine means that, like in General Relativity (GR), spacetime is modelled on a four-dimensional real differentiable manifold, but that, unlike in GR, no metric is initially given. Rather, the fundamental mathematical object representing the unified "gravito-inertial-field" is an affine connection $\Gamma$ on the tangent bundle (extendable to the whole tensor bundle and also densities) over spacetime. The only kinematical constraint the connection has to *apriori* satisfy is that it is of vanishing torsion, which is the same as saying it is "symmetric". This requirement is necessary in order to ensure the local existence of inertial frames, i.e. local coordinate systems in which inertial motion is represented by straight lines. For that, one needs the antisymmetric part of a connection, which is the tensor of torsion, to vanish. In this way, theories of gravity are considered to be theories of local inertial structures rather than of metric relations.

This line of thought has a long tradition, starting with Hermann Weyl in 1918 and later continued by other eminent scientists, such as Erwin Schrödinger. It is also the starting point of the fundamental axiomatic approach developed by Jürgen Ehlers, Felix Pirani, and Alfred Schild in 1972, that influenced many researchers in the community of General Relativists, amongst them Andrzej Trautman and Jerzy Kijowski in Warsaw.

## 2. Aim of the dissertation

As I understand it, the dissertation by Bartłomiej Bąk is written in the same spirit and continues this tradition. It aims to advance our understanding of what a theory based on a symmetric but otherwise kinematically unrestricted connection could be, thereby going beyond what has hitherto been achieved. This means to classify possible Lagrange densities, formulate appropriate (and consistent!) variational formulations, identify the metric as a derived variable (conjugate momentum), and, last but not least, interpret the physics implied by the field content of the theory and the dynamical laws they satisfy. The last step also means attempting a translation of one's findings in terms of the more familiar metric picture. The affine fields may then look like a metric plus extra fields which then assume the interpretation as "matter". It is one of the main aims of this dissertation to do precisely that for those more general affine theories based on the *full* curvature tensor, rather than just its Ricci part.

I consider the conceptual basis of this line of research well motivated within the realm of classical field theories. It takes the characteristic features of our current understanding of gravity seriously and attempts to provide a fresh and unbiased derivation from first principles. Naturally, this meets considerable analytic problems to be overcome. Some of the longer and more tedious calculations in this dissertation give ample evidence of that. It is therefore no surprise that the author had to compromise on some of the aims. But I should also add at this point that the dissertation also contains many examples where the standard arguments and calculations that one finds in textbooks have been given here in an improved and logically more coherent form. A good example is the discussion of the variational principle for hyperbolic systems, which is often stated inconsistently—but not here!

## 3. The technical task

As clearly stated in the dissertation, it partially rests on previous work by its author and his supervisor. Its main contribution is to significantly generalise previous results in the following sense: The class of affine theories considered here is required to be derivable from a variational principle. The corresponding Lagrangian, which must be a scalar density (or, equivalently, a four-form) has to be constructed from the only field available, which is the connection. Restricting to first derivatives of that field, the scalar density has to be somehow constructed from the curvature tensor. The curvature tensor allows for an intrinsic trace, resulting in the Ricci tensor. For non-metric theories, the Ricci tensor acquires next to its usual symmetric part $K$ an antisymmetric part $F$. Hence the curvature can be decomposed into its trace part, encoded in the pair $(K, F)$, and a traceless part, called $W$ (that here is the analog of what would be the Weyl tensor for metric connections). Since there is no metric, one cannot form further metric-induced traces like in GR. Hence the Lagrangian density must be formed from the triple $(K, F, W)$ directly. A Ricci scalar does not exist. Natural first-guess candidates of the right weight are the square roots of the determinants of $K$, and of $R = K + F$. The first case is well known as "Eddington gravity" and goes back to Arthur Eddington (early 1920s). It is equivalent to GR with cosmological constant, as shown in the present dissertation in section 2.4.1. and apparently also in the author's Bachelor's thesis. The second case marks the first generalisation of GR and corresponds to gravity coupled to an antisymmetric tensor field, somewhat analogously to gravity coupled to Born-Infeld theory. This has also been discussed before and is reviewed in section 3.1 of this dissertation. Genuinely new aspects enter when going beyond the first-guess candidates listed so far, i.e. those which are based on the *full* curvature including $W$. The task is to classify possible Lagrange densities built from these quantities, study their equations of motion, possibly perturbatively in $F$ and $W$, and, last not least, interpret the physics behind it in terms of the new fields as representing "matter".

## 4. Positive achievements

The dissertation contains many remarks and technical details that clearly demonstrate a thorough engagement with the subject and a strong command of both the formalism and the conceptual foundations, including the more subtle aspects that are often avoided in textbooks or, worse, misrepresented.

Positive and new results are the classification of the possible Lagrangians $V_1, \cdots, V_6$ beyond the pure Ricci theory (represented by $V_0$) on p. 45, the perturbative expansions in Sec. (2.5), the discussion of the field equations beyond the Ricci-theory (that is dealt with in Sec. (3.1)) for the $V_1$-theory in Sec. (3.3) and for the $V_6$-theory in Sec. (3.4). Moreover, as the analytic complexity added by a dynamical $W$ field turns out to be significant and not easy to handle in full generality, the author also considers an approximate strategy in which $W$ is dynamically frozen, i.e. considered just as a background field. This is done in Sec. (3.5). Finally, as an essential part of the physical interpretation, the affine to metric transition is discussed in Sec. (4.2) for all the cases explicitly dealt with in the previous sections, i.e. the pure Ricci theory, the $V_1$ theory, the $V_6$ theory, and the Ricci theory with $W$ background. In each case various parameters, like the mass of the vector (Proca) particle, are related to the non-vanishing cosmological constant.

## 5. Points for improvement

There are a few points where I missed a more detailed discussion. A first mathematical example is the following: On p. 45 the author seems to state without proof that $V_0, \cdots, V_6$ are a complete set of independent curvature invariants of the right weight. That reminded me of the corresponding classification in Weyl's old theory and the extensive work of Roland Weitzenböck in his book “Invariantentheorie” from 1923. Quite generally, stating a form of “independence” and “completeness” of a set of invariants should be accompanied by some unambiguous statement of what operations are allowed; e.g., just polynomials? A second mathematical example concerns the way the spacetime metric is identified in the affine setting, where it becomes a derived quantity. It is stated several times throughout the dissertation that one “assumes” its signature to be Lorentzian. It does not become clear whether this is like an initial condition or a constraint, whether, once imposed, it will be preserved by the dynamics, or whether non-trivial extra assumptions need to be made. Finally, a physical example is the belief expressed repeatedly by the author (p. 9,78,113,114) that the $W$-part of the curvature, if slowly varying, “appears to be a promising approach” to the problem of dark matter (p. 114). Is this just a speculation, or is it backed up by some other investigations?

## 6. Evaluation

This thesis is written in a conceptually and technically very challenging part of physics. It contains new results and lasting new insights in the field of affine gravity. It is very well organised and logically coherent. Furthermore it is written with visible expertise, dedication, and strong command of the underlying mathematical concepts. The analytic complexity is at times enormous and sets limits to the degree with which this ambitious programme could be completed within the given time. I detected no errors in language or formulae. The list of references in the bibliography is complete and, as far as I have checked them, correct.

Overall, this is an excellent dissertation that shows without doubt the author's own ability to independently conduct scientific research and present an original solution to a scientific problem. The points for improvement mentioned above do not detract from the overall impression. My conclusion is

***positive with distinction***

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