Quantum coherence and correlations in cold atom system

The main purpose of the thesis is to better understand the non-classical correlations in ultra-cold atom systems. The main difficulty in achieving this goal lies in the general and discriminative definition of entanglement. One of the solutions to this problem, which was presented in the thesis, is to examine the entanglement form the point of view of its usefulness for certain tasks, like precise metrology. The efficiency at which this task is performed, when the quantum state is used as a resource, can serve as a measure of the degree of the entanglement.

The atomic interferometry, a branch of quantum metrology, exploits the wave nature of matter and the particle entanglement to attain precise measurements of the phase beyond capabilities of classical devices. It turn out that ultra-cold atom systems are most well suited for this type of task.

In modern experimental realizations of such interferometers the Bose-Einstein condensate (BEC) used as a probe can be described as a collection of qubits. Due to their bosonic nature, the state of the BEC can be described as a state of a single pseudo-particle with the spin equal to the half of its particle number N. It follows that any (unitary) interferometric transformation is mapped onto a sequence of rotations of the spin, with one of the angles being the unknown phase. Moreover, it is important to note that all possible unitary transformations of a single qubit are equivalent to rotation, and it follows that any coherent transformation of the collection of qubits (i.e. the transformation which acts on each qubit in the same way) is also mapped onto global rotation of the system. Since the indistinguishable qubits cannot be addressed individually, coherent rotations constitute the entirety of possible local operations - transformations which do not introduce or destroy the entanglement between particles.

The efficiency of the interferometer is tied to the precision of the phase estimation. According to the Cramér-Rao theorem, which is known from classical theory and was later supplemented by Braunstein and Caves with quantum considerations, the precision is bounded by the quantum Fisher information (QFI). The QFI quantifies the susceptibility of the state to change induced by a given transformation, which in this case is the interferometric sequence.

When the transformation is coherent, the QFI cannot surpass the threshold of the shot-noise limit unless the state is entangled. Therefore, for a given interferometric sequence, the QFI can serve as a criterion for the entanglement that is useful for enhancing the precision of this particular interferometer. For another choice of the interferometer, the QFI would deem different set of states as usefully entangled.

On one hand, the freedom of choice of the transformation is essential for embedding the question of entanglement nature in the context of a real experiment. On the other hand, this ambiguity might obscure some of the important structures and relations which could help in understanding the non-classical correlations, which is the main aim of this thesis. This problem is solved by introducing the dynamical susceptibility, which is defined as the QFI for ``standard'' interferometer and is a function of the state alone. The dynamical susceptibility allowed to show that states which are usefully entangled for one interferometric sequence are related to states useful for different interferometer through coherent transformation, which conserves the degree of particle entanglement.

By exploiting the equivalence between bosonic qubits and the spin system, it can be showed how particle indistinguishability can enhance the overall degree of the entanglement. The susceptibility to change due to coherent transformation (quantified by the dynamical susceptibility) of the spin N/2 system is in general greater than the susceptibility of the ensemble of N individuals with spins 1/2. If the qubits constituting the state were distinguishable, the spins of the individuals could add-up to a whole range of total angular momenta, thus diminishing the state's susceptibility. For identical qubits, the only possibility is to add the individual spins to the maximal total spin of N/2, allowing for potentially maximal degree of entanglement.

The entanglement criteria based on the QFI, including the dynamical susceptibility, establish a direct correspondence between the particle entanglement and the susceptibility of the state to rotations. In classical physics such susceptibility is measured by the moment of inertia of the system, which is given by the distribution of the mass in respect to the axis of rotation. The dynamical susceptibility is indeed closely related to the quantum analog of the classical moment of inertia. In the quantum case the axis of rotation is given by the quantization axis and the classical mass is replaced by the components of the density matrix of the state decomposed in the basis of spherical tensor operators.

The final result of the thesis is the formulation of the classical model of the quantum state and its entanglement. Within this model the state of N bosonic qubits is represented by nested massive spheres of increasing radii. The quantum nature of the system is manifested by enforcing the mass of each sphere to only occupy parallels located at quantized distances from the common rotation axis. The model embraces the analogy between the dynamical susceptibility and the classical moment of inertia of a rigid body and identifies the degree of correlations between qubits with the distribution of the fictional mass on the surfaces of the spheres. The most entangled states are represented by the system with the majority of its mass located at the equator of the largest sphere so that the moment of inertia is the greatest.