Warsaw, 14.08.2014

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Abstract of the PhD thesis

Applications of the spin networks and spin foam models in quantum gravity

The Spin Foam models are a path integral picture of Loop Quantum Gravity approach to quantisation of gravitational field. This PhD thesis presents a study of four issues of Spin Foam models.

The first problem addressed is the question of the class of 2-complexes, that ensure that Spin Foam models are compatible with the kinematic sector of Loop Quantum Gravity. A framework of diagrammatic representation of spin foams was developed while researching this issue. This diagramatic representation is called Operator Spin-network Diagrams (OSDs). The OSDs allow to express a spin foam as a collection of graphs, connected by certain relations. Each graph captures the local structure of one of spin foam vertices, i.e. nodes of a graph correspond to edges and links of a graph correspond to faces incident to a spin foam vertex. The relations between graphs in OSDs represent the way, in which edges and faces connect vertices. It is proven, that for each OSD there is an unambiguous way to construct a 2-complex with cells labelled by a spin foam coloring, so that one can calculate the spin foam transition amplitude. A clear procedure to glue OSDs along their boundaries was developed. Such gluing is an equivalent of composing quantum processes. All possible OSDs are characterised in terms of gluing of basic diagrams representing zero or one interaction vertex each. The proposition of the answer to the first question is that the appropriate class of 2-complexes for Spin Foam models is given by all the 2-complexes that can be obtained out of OSDs.

The OSDs was applied to find a solution of so called *boundary problem*: to find all spin foams which have boundary given by certain initial and final states of Loop Quantum Gravity. An algorithm finding a series of all OSDs with a given fixed boundary is presented. The series is ordered by the number of internal edges of the corresponding spin foam. The algorithm is tested by applying it to Dipole Cosmology model (introduced in 2010 by E. Bianchi, C. Rovelli and F. Vidotto). All the diagrams contributing to Dipole Cosmology amplitude, which have the minimal number of internal edges, are found. The contribution to transition amplitude coming from these diagrams is studied. It appears that in this order of expansion all the diagrams except from one gives amplitudes that are exponentially suppressed in the semiclassical limit, thus their presence does not spoil the result of authors of Dipole Cosmology model.

The third issue addressed in this thesis were the divergent amplitudes in Spin Foam models caused by *bubbles* in spin foam 2-complexes (i.e. subcomplexes forming closed surfaces). Within the framework of 2-complexes it is relatively hard to find the bubble part of a spin foam, whereas the framework of OSDs provides a simple procedure that unambiguously identifies the bubble subdiagram. A notion of the rank of a bubble is introduced. The rank counts the number of elementary bubbles that the considered bubble consist of. A method to calculate the rank for each given OSD is presented. Several simple cases of diagrams containing bubbles, that illustrate the algorithms, are presented and studied.

The fourth question posed and answered within this thesis is related to detailed study of one particular case of a spin foam bubble, called *melonic bubble*. The melonic bubble is a spin foam analogue of self-energy renormalization in Quantum Field Theory. Recent research led to a conclusion, that in the first order the self-energy correction is proportional to some operator $\mathbb{T}$, however the operator $\mathbb{T}$ was not known. In the thesis this operator is studied in semiclassical limit. After some elaborate calculations the exact form of the leading order of $\mathbb{T}$ is found: for fixed eigenvalues of the area operators it is proportional to the identity operator, with the proportionality constant dependent on the eigenvalues.