Spin-foam dynamics of Loop Quantum Gravity states

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Abstract

This thesis studies the dynamics of the Loop Quantum Gravity states defined by the spin-foam models of Euclidean 4D Quantum Gravity. A link between the 4D spin-foam theory and the kinematics of the (3+1) Loop Quantum Gravity (LQG) was proposed by J. Engle, R. Pereira, C. Rovelli and E. Livine [1]. Their model, called the EPRL spin-foam model, is a promising candidate for the spin-foam model of the dynamics of the Loop Quantum Gravity states. In the original formulation, the EPRL spin-foam model is defined for triangulations and is applicable to specific LQG states. A generalization of the model to all the LQG states was proposed in [2] by W. Kamiński, J. Lewandowski and myself. Some properties of the generalized model were studied in [2, 3, 4, 5]. In particular, in [5] a general framework for studying symmetries of spin-foam models was proposed. The heart of the generalization is the generalized EPRL vertex amplitude. E. Bianchi, D. Regoli and C. Rovelli proposed another spin-foam model of 4D Quantum Gravity with the generalized EPRL vertex amplitude [6]. E. Bianchi, C. Rovelli and F. Vidotto used the model [6] to construct the first model of Quantum Cosmology based on the spin-foam formalism [7]. They calculated a transition amplitude between coherent states peaked on homogeneous, isotropic geometries using certain approximations. The approximations were justified a posteriori by a correct semiclassical limit of the transition amplitude. One of them was a truncation of the transition amplitude to a contribution from a single foam with one internal vertex, four internal edges and a certain boundary, which we will call a BRV foam. F. Hellmann discussed contributions from other foams with these properties, which a priori cannot be discarded [8]. All the possible foams were listed in [9] by J. Lewandowski, J. Puchta and myself. The class of the foams considered was defined by graph diagrams, which we introduced in [10]. We expect that the contributions from the foams we have found can be neglected in the limit of large universe.

In chapter 1, I present the construction of the Loop Quantum Gravity states and introduce the spin-foam formalism. Since I do not take under consideration the matter couplings in this thesis, I consider the combinatorial Hilbert space [11] to be the Hilbert space of the Loop Quantum Gravity states.

In chapter 2, I briefly summarize the state of art of the research in the spin-foam models of 4D Quantum Gravity before my contribution and the key results of this thesis.

In chapter 3, I define and compare the two models generalizing the EPRL model. First, I present the generalized EPRL intertwiners and the generalized EPRL vertex amplitude that were constructed in [2] by W. Kamiński, J. Lewandowski and myself. After this, I present the two spin-foam models with the generalized EPRL vertex amplitude [2, 6] and compare them.

Each EPRL intertwiner is labelled with an SU(2) invariant tensor (SU(2) intertwiner). A map from a space of SU(2) intertwiners to a space of Spin(4) intertwiners that maps an SU(2) intertwiner into its corresponding EPRL intertwiner is linear. It will be called an EPRL map and its image will be called a space of EPRL intertwiners. In chapter 4, I show that an EPRL map is 1-1 if its co-domain is non-trivial. I also give an example of non-isometric EPRL map. The results were obtained by W. Kamiński, J. Lewandowski and myself and published in [1, 3, 4].

Since an EPRL map can be mapping an orthonormal basis into non-orthonormal one, there is an ambiguity in defining the sum over the intertwiners: in the model [6] the sum is over orthonormal basis of the SU(2) intertwiners and in the model [2] the sum is over a basis of the EPRL intertwiners orthonormal in the scalar product inherited from the space of Spin(4) intertwiners. In chapter 5, I present an approach to spin foams where instead of labelling the internal edges with intertwiners and summing over them, one labels the internal edges with operators. This approach is called operator spin foams [5]. I study moves on the (operator) spin foams changing the orientations or refining the (operator) spin foams. The model [2] is symmetric with respect to the moves and the model [6] is not symmetric. This chapter is based on a paper by B. Bahr, F. Hellmann, W. Kamiński, J. Lewandowski and myself published in [5].

In chapter 1 the foams are defined as certain oriented piecewise linear 2-complexes. However, the complexes refer to auxiliary affine structures, which are not compatible with the diffeomorphism invariance of General Relativity. In chapter 6, I present another class of foams, introduced in [10] by J. Lewandowski, J. Puchta and myself, defined combinatorially by using certain diagrams, called graph diagrams. For each graph diagram I construct the corresponding foam and the boundary graph, which are oriented CW-complexes. I define operator spin-network diagrams as graph diagrams with suitable coloring. The coloring induces a coloring of the corresponding foam, making it an operator spin foam. I show that the construction of the operator spin foam corresponding to an operator spin-network diagram is not needed to calculate the spin-foam operator and thus to calculate the transition amplitudes. The spin-foam operator can be read directly from the operator spin-network diagram. As a result the formalism of operator spin-network diagrams can be used independently from the formalism of operator spin foams. I use this technical advantage in the following chapter.

In chapter 7, I construct all graph diagrams such that the corresponding foams have one internal vertex, four internal edges and the same boundary as the BRV foam. I discuss contributions to the transition amplitude from some of the graph diagrams. I show that in the limit of large volume of the universe they can be neglected in comparison to the contribution from the BRV foam. I expect that this property holds for every graph diagram constructed in this chapter. This chapter is based on a paper by J. Lewandowski, J. Puchta and myself published in [9].

References

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