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Analysis and modeling of small-scale turbulence

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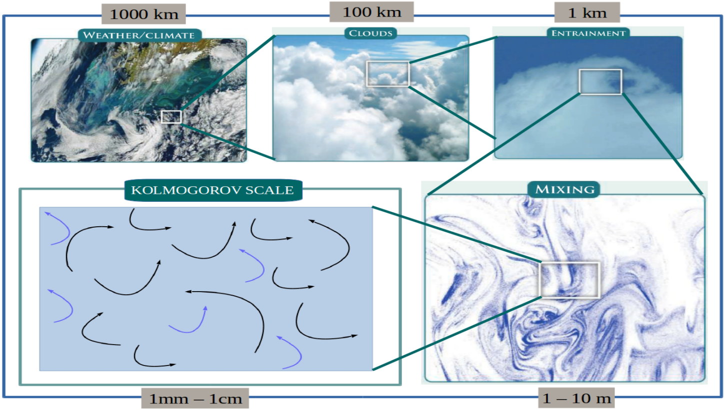
Supervisors:
Prof. dr hab. Szymon Malinowski
Dr inz. Marta Wacławczyk

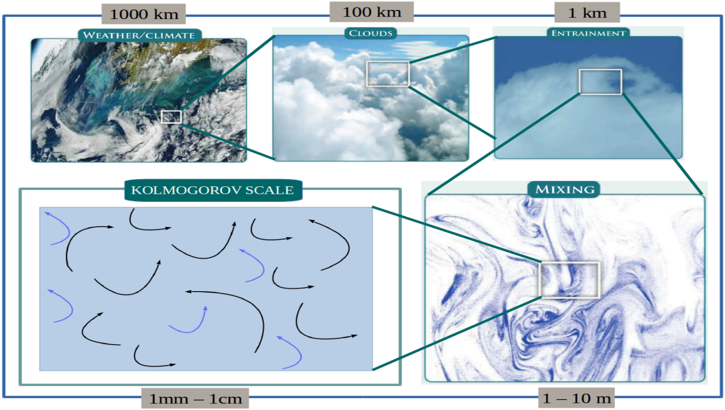
26th June, 2020

Atmospheric turbulence



E. Bodenschatz et al. [Science, 327:970-971, 2010]

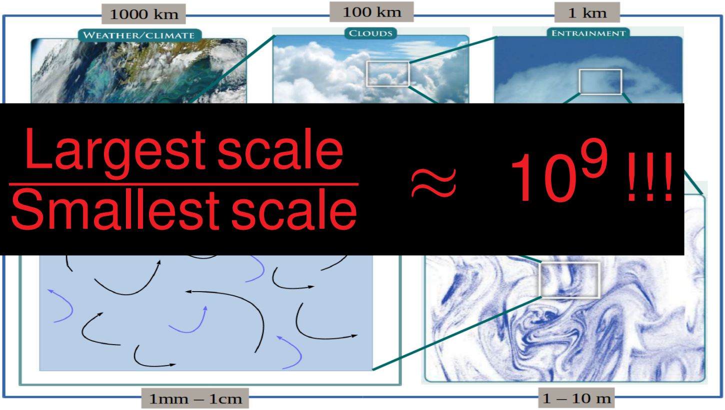
Atmospheric turbulence



E. Bodenschatz et al. [Science, 327:970-971, 2010]

Complexity of turbulence is caused by its non-linear scale interaction

Atmospheric turbulence



$$\frac{\text{Largest scale}}{\text{Smallest scale}} \approx 10^9 \text{ !!!}$$

E. Bodenschatz et al. [Science, 327:970-971, 2010]

Complexity of turbulence is caused by its non-linear scale interaction

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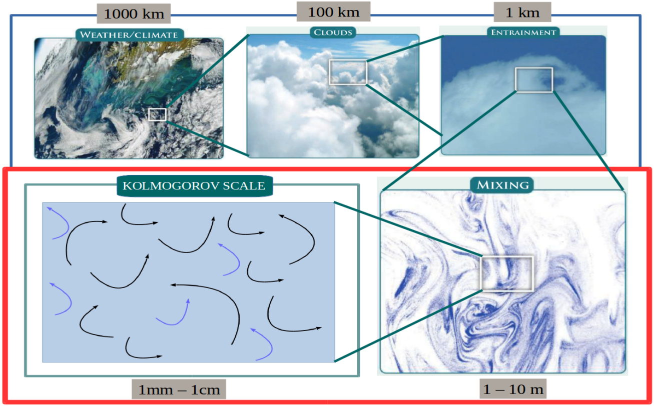
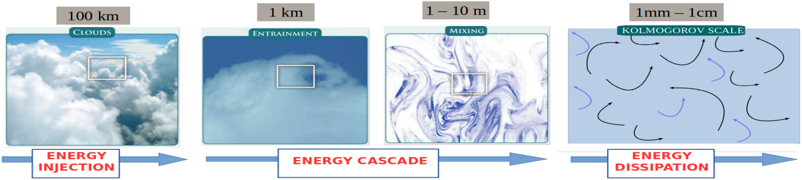
Atmospheric turbulence

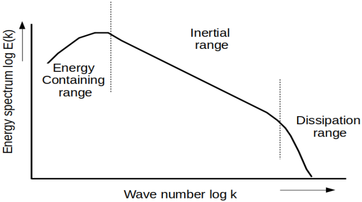


Small scales influence cloud lifecycle and droplet collision rates in clouds, precipitation

E. Bodenschatz et al. [Science, 327:970-971, 2010]

Richardson-Kolmogorov's similarity hypothesis



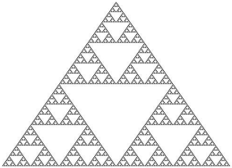


$$E(k) \sim \epsilon^{2/3} k^{-5/3}$$
$$\eta = (\nu^3/\epsilon)^{1/4}$$

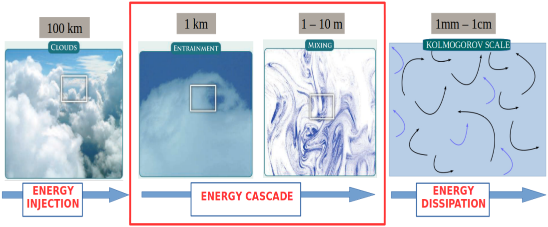
L - characteristic lengthscale
ν - viscosity
ϵ - energy dissipation rate
η - Kolmogorov length scale

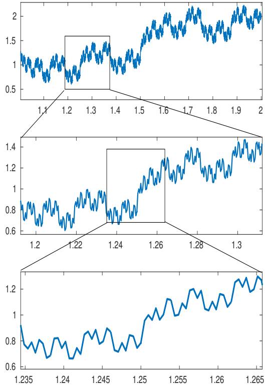
Self-similarity and fractals

Self-similarity is connected to a geometrical construct called fractals.



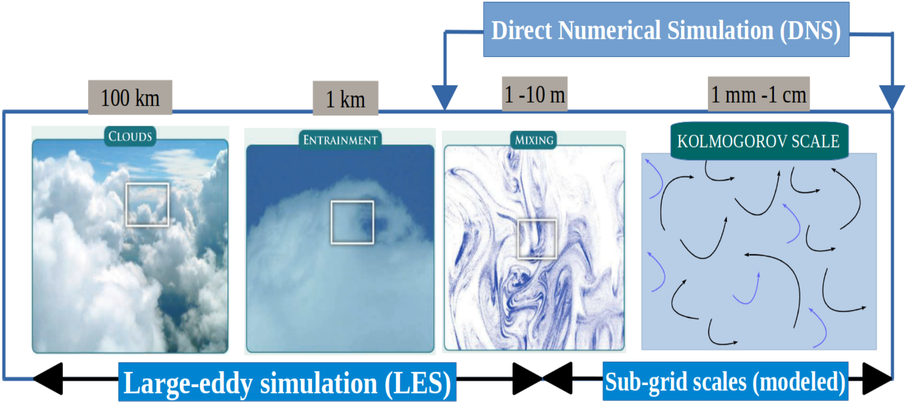
The Sierpinski Triangle. (Photo: Wikimedia Commons)





Fractal signal

Simulation



E. Bodenschatz et al. [Science, 327:970-971, 2010]

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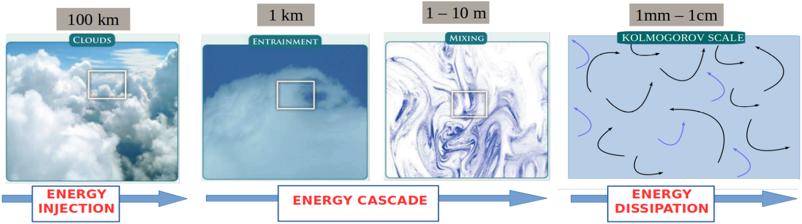
Objectives

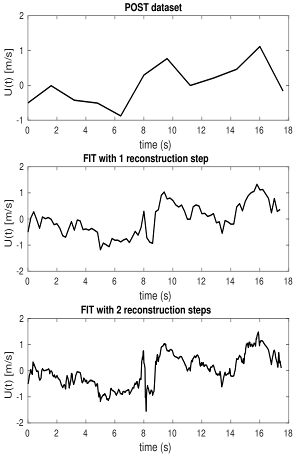
Part 1 - Retrieving information of small-scale turbulence

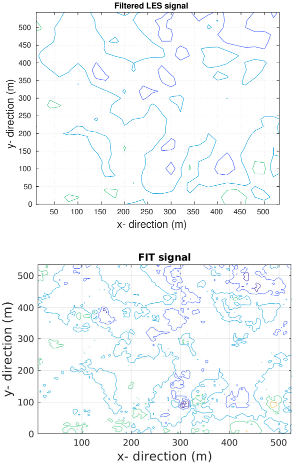


- the comparison of different methods for retrieving ϵ from atmospheric measurements
- investigates how the presence of anisotropy (due to buoyancy and external intermittency) affects the various retrieval techniques of ϵ in the atmospheric configurations.

Objectives

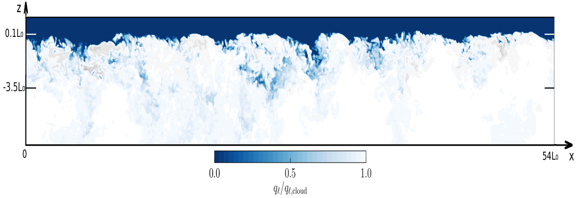
Part 2 - Numerical reconstruction of small-scale turbulence



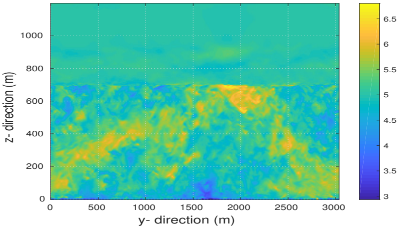


- develop an improved fractal interpolation (FIT) model for the reconstruction of small-scales in large-eddy simulations (LES)
- use the improved FIT model in Lagrangian particle tracking

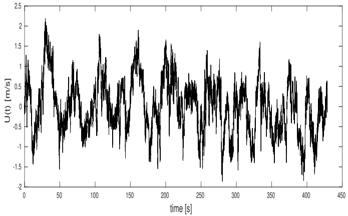
Case study - stratocumulus cloud boundary layer



Vertical cross section of the liquid water specific humidity in the cloud-top mixing layer [Schulz and Mellado 2018]. Courtesy: Prof. J.-P. Mellado [MPIM, Hamburg]

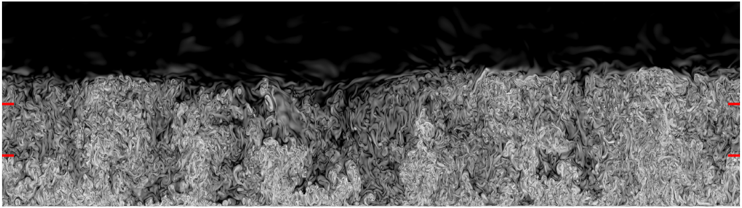


Vertical cross-section of *u*-component of LES velocity of the stratocumulus cloud boundary layer [Pedersen et al. 2018]



u-component of the physics of stratocumulus cloud (POST) airborne dataset [Gerber et al. 2013, Malinowski et al. 2013]

Case study - convective boundary layer



Vertical cross section of the logarithm of the enstrophy in the convective boundary layer (CBL) [Mellado et al., *Bound.-layer Meter.,* 159:69–95, 2016]. The horizontal bars at the side of the figures indicate a height equal to the CBL depth *h* and equal to half of it.

Part 1

Retrieving information of small-scale turbulence

Energy dissipation rate ϵ

The quantity, which plays a crucial role in the study of small-scale turbulence is the turbulence kinetic energy (TKE) dissipation rate ϵ.

$$\epsilon = 2\nu \langle s_{ij} s_{ij} \rangle , \quad s_{ij} = \frac{1}{2}\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right) ,$$

s_{ij}- fluctuating strain rate tensor,

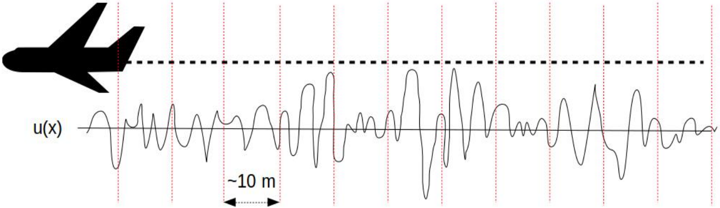
$u_i' = u_i - \langle u_i \rangle$ - i-th component of fluctuating velocity,

$\langle \cdot \rangle$- ensemble average operator

Tennekes and Lumley [Cambridge, Mass.: MIT Press, 1972]

Indirect method for estimating ϵ

- Airborne datasets are usually one-dimensional (measured along research aircraft track) and not fully resolved (approx. 10 m).

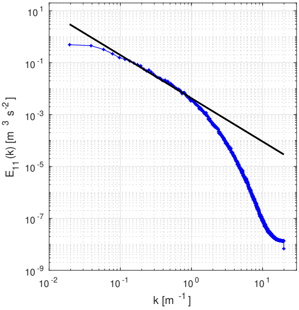


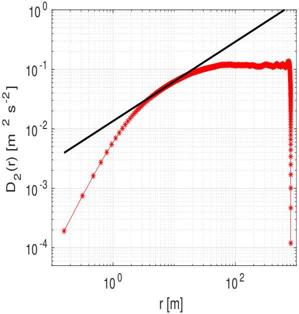
$$E(k) \sim \epsilon^{2/3} k^{-5/3}$$

Methods for ϵ retrieval

Inertial-range scaling of power spectral density and 2^{nd}-order structure function $C_1 \approx 0.49$, $C_2 \approx 2$.

$$E_{11}(k) = C_1 \epsilon_{PS}^{2/3} k^{-5/3} \qquad D_2(r) = C_2 \epsilon_{D_2}^{2/3} r^{2/3}$$

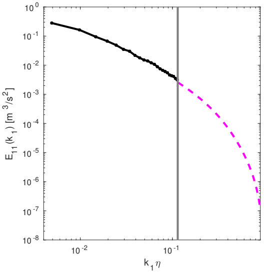




Methods for ϵ retrieval

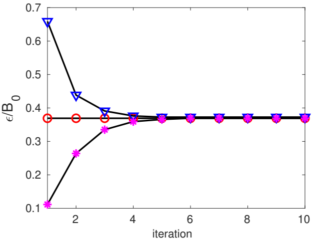
Iterative method

$$\epsilon_{\lambda R} = 15\nu \left\langle \left(\frac{\partial u'_{cut}}{\partial x} \right)^2 \right\rangle \mathcal{C}_{\mathcal{F}}(\eta, k_{cut}).$$



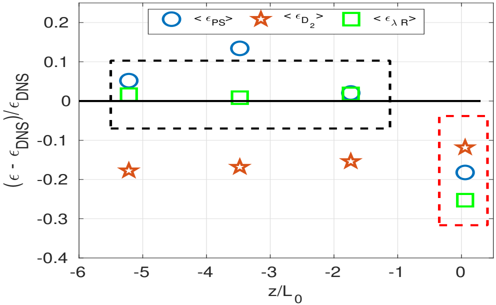
Procedure:

- *Assume an analytic form of* $E(k)$
- *initial guess of* η_0
- *calculate* $\mathcal{C}_{\mathcal{F}}$ *and* $\epsilon_{\lambda R}$
- *repeat procedure with the previous* $\epsilon_{\lambda R}$ *to get the new one until the procedure converges*

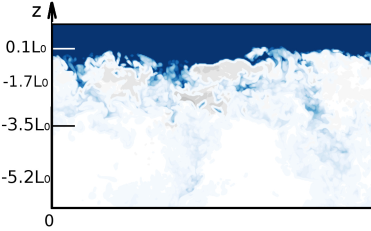


E.O. Akinlabi et al. [J. Atmos. Sci., 2019]

Analysis - stratocumulus cloud-top simulation



Normalized TKE dissipation rate estimates' errors from both direct and indirect methods with k_{cut} placed in the inertial range. Fitting ranges for ϵ_{PS} were based on ϵ_{D_2}.



Vertical cross section of the liquid water specific humidity in the cloud-top mixing layer showing horizontal plane $z = -5.2L_0, -3.5L_0, -1.7L_0, 0.1L_0$.

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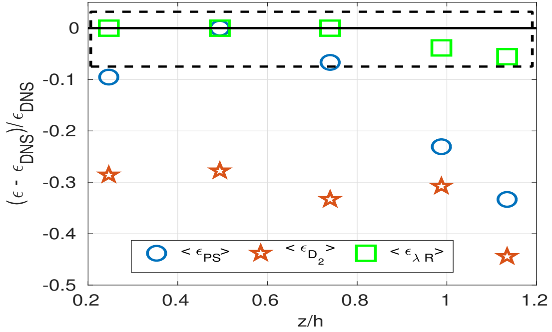
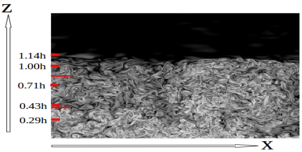
Analysis - convective boundary layer simulation



Normalized TKE dissipation rate estimates' errors from both direct and indirect methods with k_{cut} placed in the inertial range. Fitting ranges for ϵ_{PS} were based on ϵ_{D_2}.



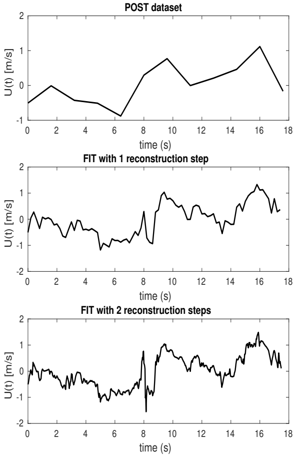
Vertical cross section of the logarithm of the enstrophy in the convective boundary layer showing horizontal plane $z = 0.29h, 0.43h, 0.71h, 1.0h, 1.14h$.

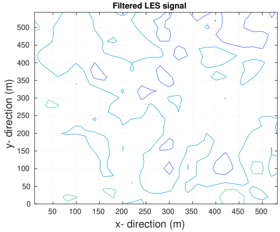
Part 2

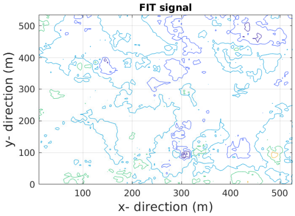
Numerical reconstruction of small-scale turbulence

Objectives

Part 2 - Numerical reconstruction of small-scale turbulence







- develop an improved fractal interpolation (FIT) model for the reconstruction of small-scales in large-eddy simulations (LES)
- use the improved FIT model in Lagrangian particle tracking

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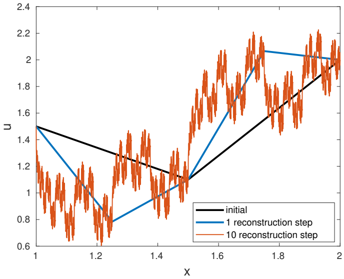
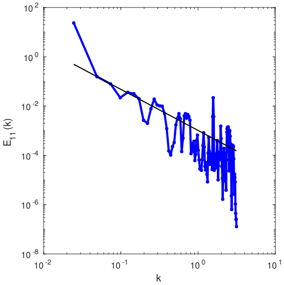
Method

Fractal Interpolation technique (FIT)

The FIT is an iterative mapping procedure to construct synthetic small-scale eddies of any field (e.g $\mathbf{u}(\mathbf{x}, t)$) from the knowledge of its filtered field.

Scotti and Meneveau [Physica D, 1999]





a) Different stages during the construction of a fractal function after 0,1 and 10 iterations with stretching parameter $d = \pm 2^{-1/3}$
b) Energy spectrum of the constructed signal.

Method

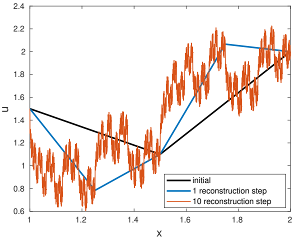
Stretching parameter

Stretching parameter *d* is the vertical stretching of the left and right segments of three interpolation points at each iteration. It also determines characteristics of the reconstructed signal.

The stretching parameter can be computed from the scaling exponent spectrum *D* as:

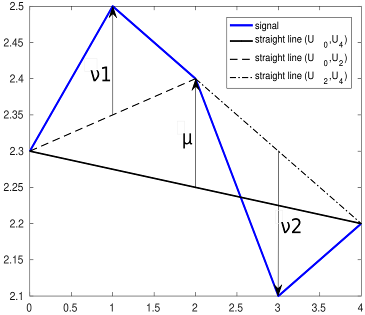
$$D = 1 + log_N \sum_{n=1}^{N} |d_n| \quad \approx \quad 5/3$$

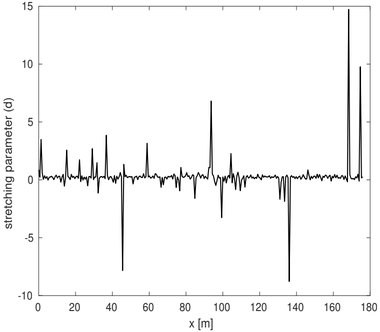
where $N =$ the number of anchor points $- 1$.



[Orey (1970), Praskovsky et al. (1993), Scotti et al. (1995)]

Computing stretching parameter





(a) Mazel and Hayes' algorithm for computing the local stretching parameter d_i of any given arbitrary dataset.
(b) Variability of local stretching parameters d in 1D DNS velocity signals.

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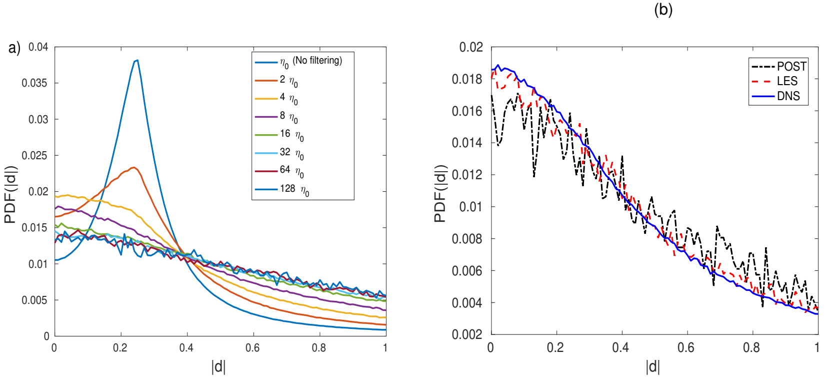
Probability distribution of stretching parameter



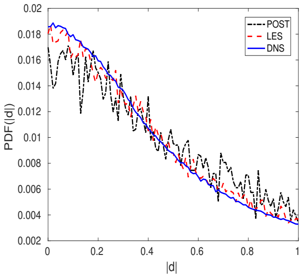
(a) Probability distribution (PDF) of local stretching parameter *d* within the interval [0, 1]
(b) PDF of the absolute value of the stretching parameter $|d|$ from filtered DNS, LES velocity signals of stratocumulus cloud-top and physics of stratocumulus top (POST) airborne data. DNS and LES velocity fields were filtered with wavenumber within the inertial range.

Reconstruction of sub-grid velocity signal

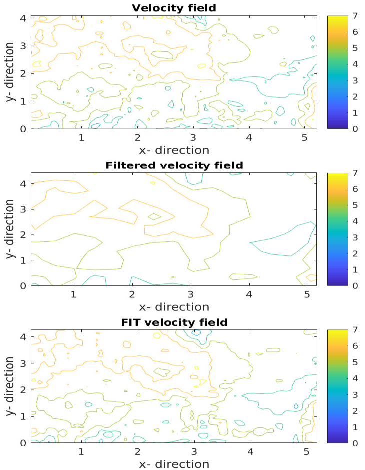
Previous method applied constant stretching parameter. For example

- Scotti and Meneveau [Physica D, 1999] - $d = \pm 2^{-1/3}$
- Basu et al. [Phys. Review, 2004] - $d = -0.887, -0.676$

Random values of the stretching parameter are chosen from its PDF



Reconstruction of sub-grid velocity signal



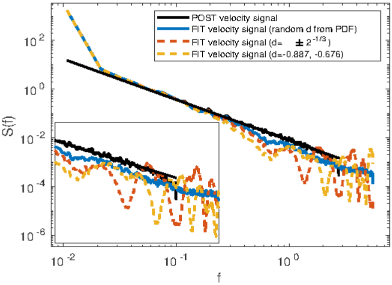


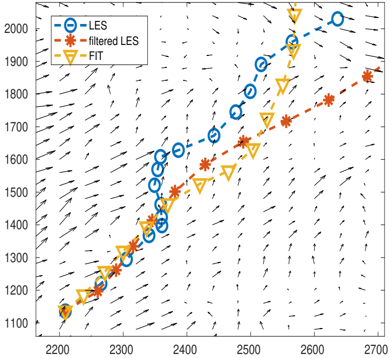
Table of contents

Reconstructing inertial range scales for Lagrangian simulation of droplets - preliminary test

The motion of each droplet is governed by

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{U}(\mathbf{x}_i, t), \qquad \frac{dA_i}{dt} = 0,$$

where $\mathbf{U}(\mathbf{x}_i, t)$ is the fluid velocity field (LES, filtered LES or FIT) at the droplet position $\mathbf{x}_i$ and A_i is the attribute.



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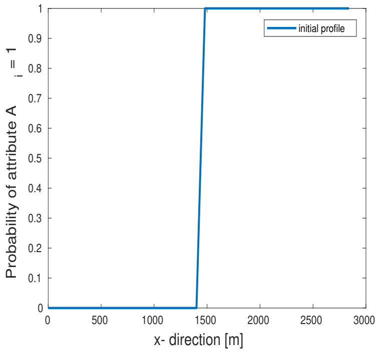
Tracking of Lagrangian particles in the FIT reconstructed field

The motion of each droplet is governed by

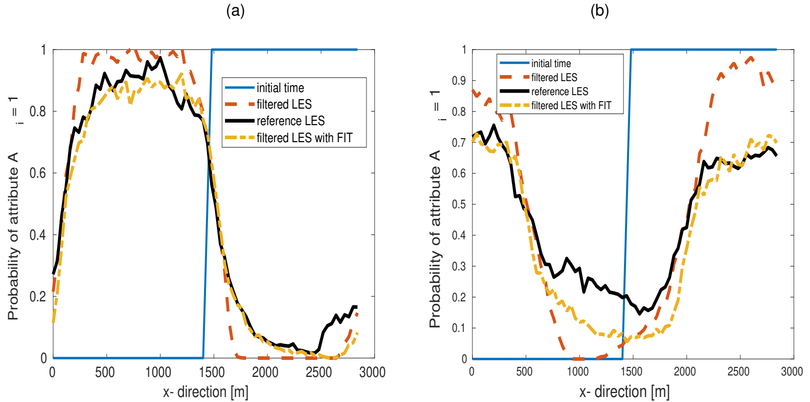
$$\frac{d\mathbf{x}_i}{dt} = \mathbf{U}(\mathbf{x}_i, t), \qquad \frac{dA_i}{dt} = 0,$$

where $\mathbf{U}(\mathbf{x}_i, t)$ is the fluid velocity field (LES, filtered LES or FIT) at the droplet position $\mathbf{x}_i$ and A_i is the attribute.



Tracking of Lagrangian particles in the FIT reconstructed field

(LES of stratocumulus cloud)



The attribute probability $A_i = 1$ in the x- direction, averaged over y- and z- directions in the in-cloud region only at (a) 60 minutes (b) 120 minutes of simulation time.

Conclusion

- This thesis focuses on the analysis and numerical reconstruction of small-scale turbulence.
- The scaling of energy spectra of turbulent flows in atmospheric configurations and different methods for TKE dissipation rate retrieval from $1D$ intersections of the flow domain is investigated.
- The reconstruction of sub-grid scales in large eddy simulation of turbulent flows is addressed.
- A new fractal sub-grid model is developed using random value of the stretching parameter $|d|$.
- Based on the preliminary test, the LES velocity field used in the Lagrangian tracking of droplet could be improved with FIT.

Acknowledgement

This project has received funding from the European Union Horizon 2020 Research and Innovation Programme under the Marie Sklodowska-Curie Actions, Grant Agreement No. 675675.

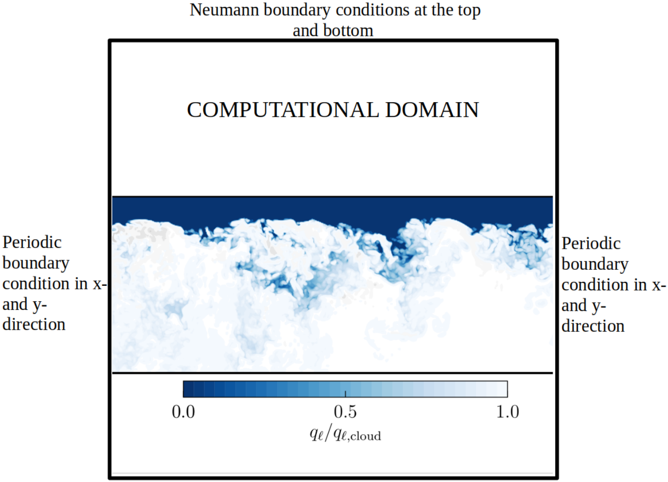
Thank you for listening

Reply to reviewers

Question 1

One would welcome that Sec. 4.2.1 (stratocumulus cloud-top simulation), important for the thesis, be more self-contained. In particular, I have not grasped where the reduction in *Re* (factor of 300) comes from? Is it due to rescaled L_0 ? Or viscosity? Is the size of dissipative eddies correctly reproduced in the DNS? What are the boundary conditions (BC) of the simulation? I guess, these are periodic BC in x and y (?). But what about z? Are the BC the same as in the LES case of Sec. 7.3.2? Probably not, as the velocity statistics at the extremes of z differ between Fig.4.3 and 7.7. Please explain.

Reply to reviewers



[J. P. Mellado *priv. comm.*]

Reply to reviewers

Question 1

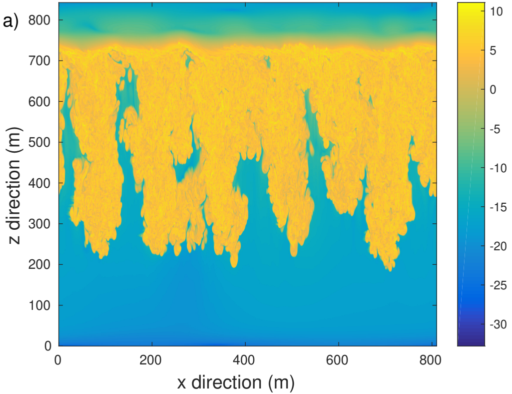


Figure 4.2a - Vertical cross section of the logarithm of enstrophy ($\Omega = 1/2|\omega|^2$ where ω is the vorticity field) at $y = 405m$.

Reply to reviewers

Question 2

Concerning Sec. 4.2.2: one would wish some quantitative results to be recalled here, as in Fig. 4.3. As amply discussed in the following, the flow anisotropy is an important factor that deteriorates the estimation of ϵ, right? And the profiles of rms(u_i) would provide some idea about at least the large-scale anisotropy. As the Reynolds number is rather low, the anisotropy may persist down to the smallest resolved scales.

Continue on next page

Reply to reviewers

Question 2

From this point of view, given the non-homogeneity of turbulence in the CTL case, the results reported for example in Tab. 5.2 are expected. On the other hand, the Author states (concluding on the estimates of ϵ based on different velocity components) that “this makes the isotropy assumption questionable” (p.44). Actually, already given the r.m.s. velocity profiles in Fig. 4.3b, one should not expect the turbulence to be isotropic. Please explain. Perhaps even, a better estimation of ϵ may be imagined, as supported by the data on anisotropy expressed by the respective levels of u’, v’ and w’? Not sure, but this might be a path worth some reflection and further exploration.

Reply to reviewers

Question 2

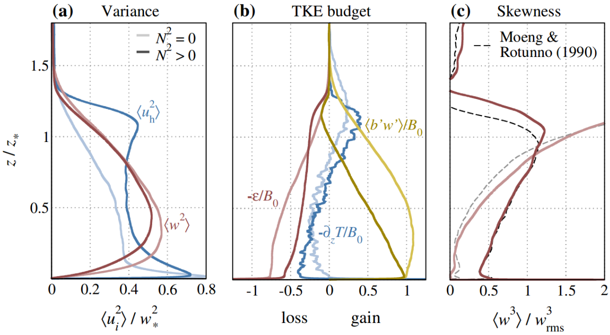


Fig. 2 Large-scale properties at $z_*/z_\kappa \approx 680$. $u_h = (u_1^2 + u_2^2)^{1/2}$ is the magnitude of the horizontal velocity. $T = \langle w'u_i'u_i'/2 + p'w' - u_i'\tau_{i3}' \rangle$ is the vertical turbulent flux of TKE and $\varepsilon = \langle \tau_{ij}' \partial_i u_j' \rangle$ is the viscous dissipation rate, $\tau_{ij} = \nu(\partial_i u_j + \partial_j u_i)$ being the components of the viscous stress tensor

Extracted from Mellado et al. [Bound.-layer Meteorol., 159:69–95, 2016]

Reply to reviewers

Question 2

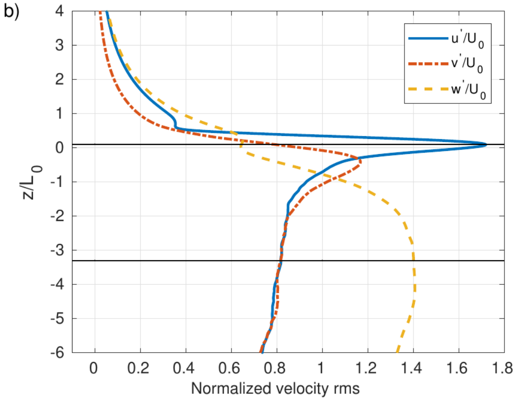


Figure 4.3b - Normalized root-mean-square of velocity in the cloud-top mixing layer. The upper horizontal line indicates the height of minimum buoyancy flux (horizontal plane $z = 0.1L_0$) while the lower horizontal black line indicates the height of maximum buoyancy flux (horizontal plane $z = -3.5L_0$)

Reply to reviewers

Question 3

Fig. 5.15 seems to be quite rich in information (and perhaps usable in future work?). Any general comment on it?

Reply to reviewers

Question 3

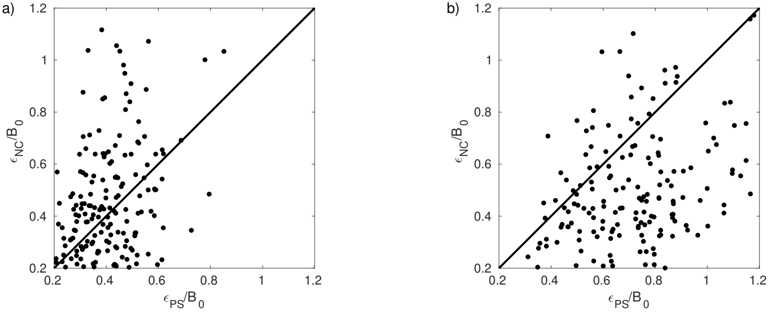


Figure 5.15 - Results of ϵ_{NC} vs. ϵ_{PS} normalised with B_0 for signals with a) $k_{cut} = 5[m^{-1}]$ b) $k_{cut} = 0.62[m^{-1}]$.

More analysis at Wacławczyk et al. [Atmosphere, 11:199, 2020]

Reply to reviewers

Question 4

The role of external intermittency in estimating the dissipation rate is well explained (the correction factor γ in Sec. 5.2.3, etc.). But then, the concept of internal intermittency, due to Kolmogorov (1963), is also invoked in the text. However, I have not found it in Chapter 3. What is in practice the role of internal intermittency when working with the estimates of ϵ?

Rice [Bell. Syst. Tech. J., 24:24–156, 1945], Sreenivasan et al. [J. Fluid Mech., 137:251–272, 1983], Katul et al. [Phys. Fluids, 6:2480–2492, 1994]

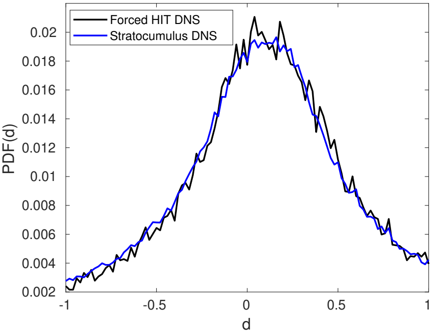
Reply to reviewers

Question 5

Concerning the studies reported in Chapter 7, in particular the determination of statistical properties of d, using not-so-isotropic (neither homogeneous) data: perhaps, these properties, and the PDF($|d|$) in particular (and not just the autocorrelation computed in Sec. 7.5) could have been determined from the data on forced isotropic turbulence as well (unless this have been done in the literature already?).

Reply to reviewers

Question 5



Probability distribution of the absolute value of the stretching parameter $|d|$ from filtered DNS of stratocumulus cloud-top and filtered DNS of forced homogeneous isotropic turbulence data. DNS velocity fields are filtered with wavenumber within the inertial range.

Yeung et al. [J. Fluid Mech., 700:5–15, 2012], Salvetti et al. [Proc. Conference on Turbulence and Interactions, 2006], Ding et al. [Phys. Review E, 82:036311, 2010]

Reply to reviewers

Question 6

Sec. 7.2: it is unclear how the 5-point algorithm (yielding d_1 and d_2 , Fig. 7.4) is applied to the whole velocity signal to infer d values from it (Fig. 7.5a)? Then, I have some concerns about the procedure: it seems from Fig.5 that $< d >$(the mean value) is larger than 0, is it? If so, then the PDF is certainly not symmetric, so why then one should introduce PDF($|d|$)? Also, it is not clear why "the ensemble average $< |d| >$should be comparable to the channel flow data of Ref.[118]" (p.91) which is a strongly non-homogeneous case? What are these mean values?

Reply to reviewers

Question 6

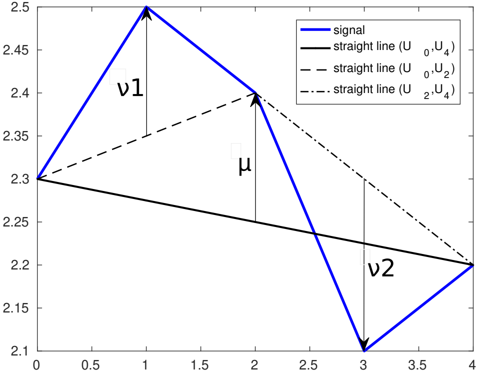


Figure 7.4 - Mazel and Hayes' algorithm for computing the local stretching parameter d_i of any given arbitrary dataset.

Reply to reviewers

Question 6

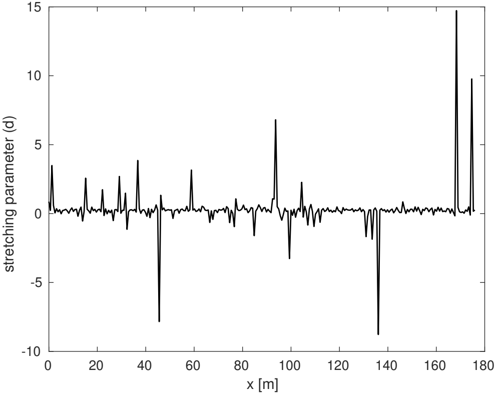
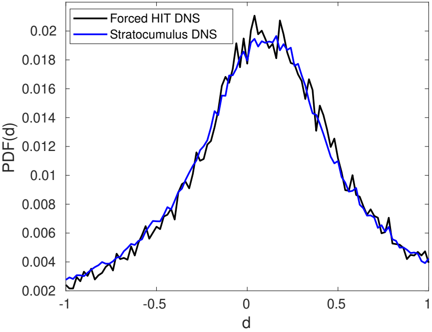


Figure 7.5a - Variability of local stretching parameters *d* in 1D DNS velocity signals.

Reply to reviewers

Question 6



Probability distribution of the absolute value of the stretching parameter $|d|$ from filtered DNS of stratocumulus cloud-top and filtered DNS of forced homogeneous isotropic turbulence data. DNS velocity fields are filtered with wavenumber within the inertial range.

Burattini et al. [Exp Fluids, 45, 523-535, 2008]

Reply to reviewers

Question 6

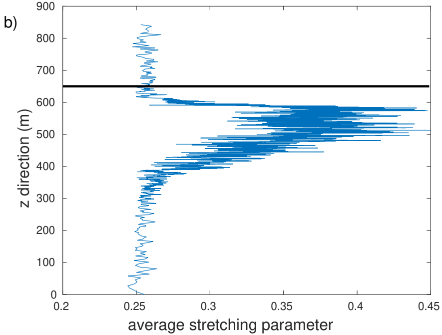


Figure 7.9b - The vertical profile of the average stretching parameter for DNS velocity. The black line indicates approximately the cloud-top region.

Salvetti et al. [Proc. Conference on Turbulence and Interactions, 2006]

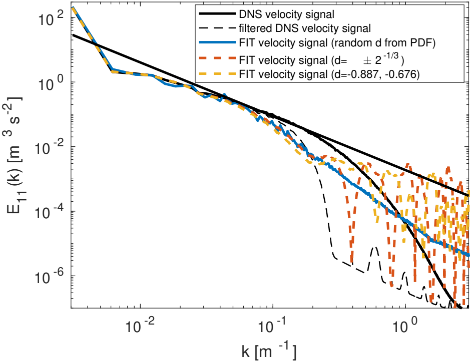
Reply to reviewers

Question 7

Continuing on the proposed improvement of the fractal interpolation technique (p.86, p.91): judging from the plot of local stretching parameters (Fig.7.5a) and from the PDF (Fig.7.8a), when the values d such that $|d| < 0.5$ are discarded, then not much seems to be left (?). Actually, this is my most serious concern about the proposed improvement of the method. The argument (p.86) that "only then a fractal signal will dissipate energy" does not seem to be relevant as the reconstructed field does not evolve dynamically (as in the N-S eq.). Rather, as in the applications shown in Chapter 9, it is used to advect a passive scalar ("the attribute") or to provide an "enriched" local fluid velocity for particle motion. Please explain.

Reply to reviewers

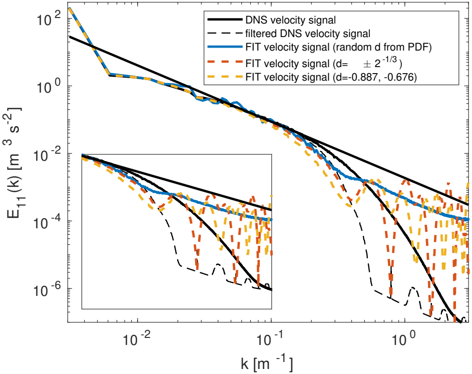
Question 7



Longitudinal energy spectra of *u* velocity component for DNS and FIT with constant stretching parameter $d = \pm 2^{-1/3}$, with constant stretching parameters $d = -0.887$ and $d = -0.676$ and with random stretching parameters $|d| < 1$ from calculated PDF

Reply to reviewers

Question 7



Longitudinal energy spectra of *u* velocity component for DNS and FIT with constant stretching parameter $d = \pm 2^{-1/3}$, with constant stretching parameters $d = -0.887$ and $d = -0.676$ and with random stretching parameters $0.5 < |d| < 1$ from calculated PDF

Reply to reviewers

Question 8

In the a priori LES test, the residual kinetic energy k_r is introduced through Eq. 9.4. When the DNS solution is filtered, the fluid kinetic energy decreases, so $k_r > 0$. Here, the residual energy may be negative (which is, NB, a priori unusual), as the filtering (or tilde) operator is not idempotent (unlike the ensemble average in RANS). As a consequence, the PDF of k_r is endowed with a quite large negative part. Please comment on these unusual features.

Reply to reviewers

Question 8

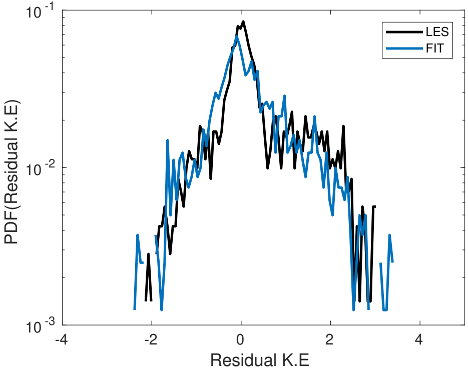


Figure 9.1 - PDF of the residual kinetic energy for reference LES and filtered LES with FIT in the in-cloud region at 20 m $\leq z \leq$ 650 m.

Reply to reviewers

Question 9

Concerning the autocorrelation $R(\tau)$ of the stretching parameter (Sec. 7.5): it is not clear why are we interested in this quantity? Is it meant for future applications of FIT? Which ones? Then, it would be perhaps worthwhile to also compute the decorrelation time out of the Lagrangian function R. Or even from the Eulerian one with the application of frozen turbulence hypothesis to see whether the decorrelation times substantially differ among these three cases. Also, this might be of future use for FIT applied to particle/droplet motion.

Reply to reviewers

Question 9

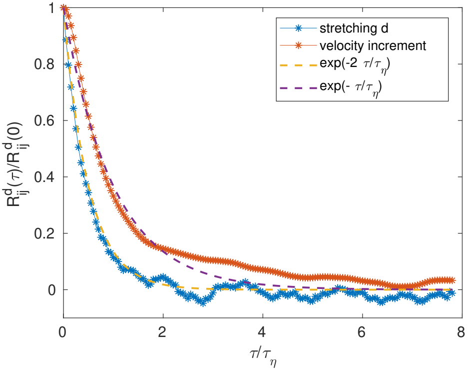


Figure 7.11 - Autocorrelation function $R^d(\tau/\tau_\eta)$ for the stretching parameter and autocorrelation function $R^{\delta \mathbf{u}}(\tau/\tau_\eta)$ for the velocity increment $\delta\mathbf{u}$.

Reply to reviewers

Question 1

In the abstract, the Doctorate states that the analysis of fine-scale structures can contribute to better parameterization of climate models. In general this is true, but the parameterization of dynamics in small scales is much greater for modelling weather forecasts.

Reply to reviewers

Question 2

In Page 13, the statement "In general, the turbulence flows can be described as a random field" is a big simplification. In a typical turbulence flow there are structures that have a certain spatial organisation. This has a direct impact on the dynamics of cloud processes, e.g. the average speed of falling droplets in turbulent flow is higher than in air without turbulence. And in a random field, the speed of the drops does not change.

Fung and Vassilicos [Phys. Rev. E, 57:1677–1690, 1998], Pinsky et al. [J. Atmos. Sci., 65, 2064-2086, 2008], Pope [Cambridge, 2000]

Reply to reviewers

Question 3

The doctoral student examines the differences in energy dissipation values calculated using different methods in quite a detailed way, while the analysis of statistical uncertainties of the obtained results is treated more cursory.

Reply to reviewers

Question 3

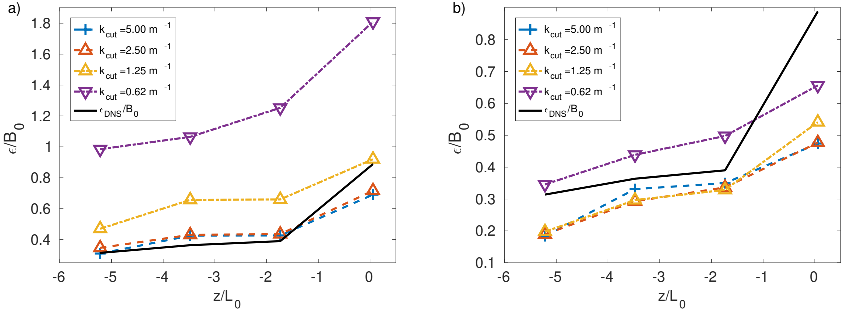


Figure 5.14 - Normalized TKE dissipation rate estimates from signals with effective cut-off's k_{cut} as a function of vertical coordinate z/L_0 (dimensionless) a) ϵ_{PS}, b) ϵ_{NC}.

More analysis at Wacławczyk et al. [Atmosphere, 11, 199, 2020]

Reply to reviewers

Question 3

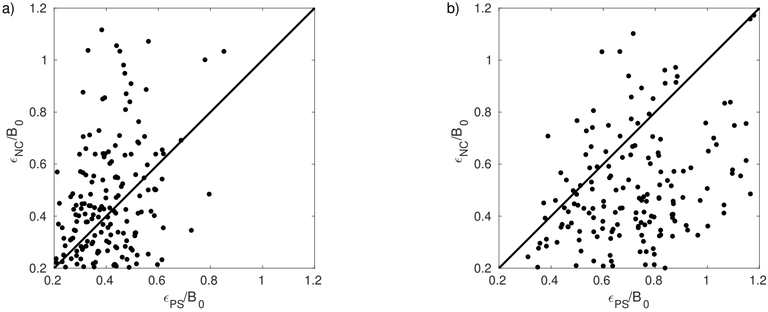


Figure 5.15 - Results of ϵ_{NC} vs. ϵ_{PS} normalised with B_0 for signals with a) $k_{cut} = 5[m^{-1}]$ b) $k_{cut} = 0.62[m^{-1}]$.

Reply to reviewers

Question 3

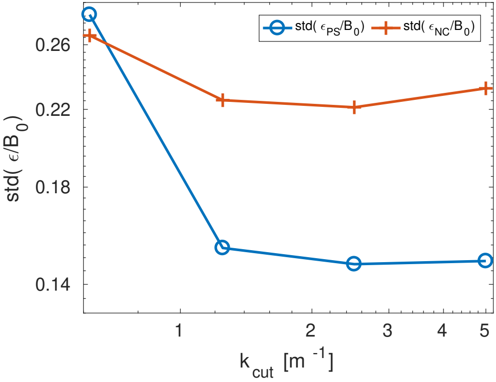


Figure 5.16 - Standard deviations of ϵ_{NC} and ϵ_{PS} estimates normalized with B_0 as a function cut-off k_{cut}.

Reply to reviewers

Question 4

It is difficult to deduce from the text of the dissertation whether DNS data from one time step were used to determine energy dissipation or whether the data were averaged over a certain period of time. This may be important for the accuracy and generality of the results obtained.

Reply to reviewers

Minor remark 1

p.11: the statement “sound waves are completely neglected... since (they) propagate via density variation” seems inaccurate in the presented context (or even wrong – why?).

Reply to reviewers

Minor remark 2

rigorously, the integration limits in Eqs. 2.18 and 2.19

$$\langle \mathbf{U}(\mathbf{x}, t)\rangle \approx \langle \mathbf{U}(\mathbf{x}, t)\rangle_T \equiv \frac{1}{T}\int_t^{t+T} U(\mathbf{x}, \tau)d\tau.$$

$$\langle \mathbf{U}(\mathbf{x}, t)\rangle \approx \langle \mathbf{U}(\mathbf{x}, t)\rangle_{x,y,z} \equiv \frac{1}{L_x L_y L_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} \mathbf{U}(\mathbf{x}, t)dxdydz$$

are incorrect;

Reply to reviewers

Minor remark 2

The integration limit is changed to:

$$\langle \mathbf{U}(\mathbf{x}_0, t_0)\rangle \approx \langle \mathbf{U}(\mathbf{x}_0, t_0)\rangle_{\Delta T} \equiv \frac{1}{\Delta T}\int_{t_0-\Delta T/2}^{t_0+\Delta T/2} \mathbf{U}(\mathbf{x}_0, \tau) d\tau.$$

and

$$\langle \mathbf{U}(\mathbf{x}_0, t_0)\rangle \approx \langle \mathbf{U}(\mathbf{x}_0, t_0)\rangle_{\Delta \mathcal{V}} \equiv \frac{1}{\Delta \mathcal{V}}\int_{\Delta \mathcal{V}} \mathbf{U}(\mathbf{x}, t_0) d\mathbf{x}$$

Reply to reviewers

Minor remark 3

Fig. 7.5b: to better appreciate the final effect of FIT reconstruction, it would be instructive to add a 2Δ-filtered velocity signal to the picture, even just in the zoom-in chunk (the inset plot);

Reply to reviewers

Minor remark 3

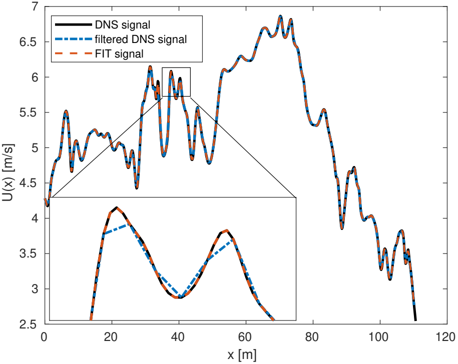


Figure 7.5b - 1D DNS velocity signals showing the original, filtered and the FIT reconstructed signal using local values of d in figure 7.5a.

UNIVERSITY OF WARSAW | INSTITUTE OF GEOPHYSICS

Emmanuel O. Akinlabi

Analysis and modeling of small-scale turbulence

Supervisors:
Prof. dr hab. Szymon Malinowski
Dr inz. Marta Wacławczyk