

Statistical physics of coevolving networks

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PhD thesis abstract

Statistical physics has introduced key concepts for analyzing systems consisting of a large number of elements. It showed that problems seemingly out of reach for quantitative description can be actually treated in a strict manner. The essential achievement was a shift in the notion of prediction from deterministic to stochastic ground. The approach of statistical physics has been successfully applied in numerous branches of science, of which the most important one for this thesis is the theory of complex systems.

Networks create the core of complexity science. When a studied object contains many interacting parts and the pattern of interaction is not trivial, networks are the most natural tool to describe it. Some recent publications even identify complex systems as coevolving multilayer networks. One can argue with such a rigorous definition, but the coevolution of structure and state on its own has proved to be a crucial feature in complex systems.

Studies on the coevolution's impact on the behavior of particular models are relatively new. Moreover, a general theory of coevolving networks is so far out of reach. In order to get closer to universal laws we need to understand single problems first. Thus, I seek to explore the effects of coevolution in different models, both analytic (or equilibrium) and algorithmic (or non-equilibrium) ones.

The thesis explores outcomes of introducing coevolving mechanisms in three models, namely the voter model, the Axelrod model and the Ising model. The first one is a reference point in the quantitative description of social systems, however it was successfully applied also in physics, biology, and finance. The voter model is extended in this work to integrate coevolution, triadic closure, nonlinear interactions, and noise. The Axelrod model has a purely social interpretation – it's a model of social interactions or dissemination of culture. It is studied in the thesis how different types of rewiring, during the coevolution of the network's structure and state of the nodes, influence the final topology of the system. Results obtained using the Axelrod model are compared with empirical data. The model was improved for better agreement with the empirically observed scaling behavior. Notably, previous extensions of the model didn't resolve the contradiction with the empirical data, which is solved here. The Ising model was originally constructed to explain ferromagnetism. However, it gained much bigger attention than the first application might have suggested. It became a reference point for network models and has been studied in many variations, not only empirically implied but also theoretically thrilling. In this abstract context, the spin dynamics from the Ising model is combined in the thesis with topological traits of the nodes to analyze the outcome of a coevolving equilibrium model with structural traits included in the Hamiltonian.

In all models studied throughout this thesis new results are obtained. The most important ones are the following. In the nonlinear coevolving voter model with triadic closure a new shattered phase is observed together with high values of the clustering coefficient. When the noise is included two new phases are obtained. One of these phases persists in the thermodynamic limit. The other one contains topological communities driven by state of the nodes, what was not observed before for the coevolving voter model with noise. Additionally, a new analytical description of the model is proposed. As this model is the most general one, it contains previously studied limit cases like the coevolving voter model or the nonlinear coevolving voter model. Similarly, in the Axelrod model high clustering is generated, as well as a power-law degree distribution. But most importantly, due to the implemented changes the model displays a new scaling of the number of domains with the system size. This result, in contrast to results obtained with the original model, is consistent with empirical data. Finally, an equilibrium model of coevolving networks is proposed for the first time. More precisely, the Hamiltonian of the model includes not only states of the nodes and their mutual

interactions, but also degree of the nodes, as a local topological trait. A rich phase diagram obtained in this way is described analytically. The observed configurations coincide with those obtained in non-equilibrium models, suggesting a possibility of their equilibrium description.

The results of the thesis provide a new insight into the behavior of coevolving networks from a statistical physics perspective. The theory of complex systems is not yet complete and new building blocks are being discovered. Hopefully, after analyzing and understanding enough separate parts a more universal theory will emerge. The content of this thesis contributes to the development of such theory by providing several new building blocks. Above all, these blocks take into account coevolution of network's structure and state, as one of the crucial properties of complex systems is their adaptive behavior. The obtained results surely represent advancement in particular models that were studied within the scope of the thesis, but can be also seen as another step towards the theory of complex systems.