Abstract of the PhD dissertation

The aim of this thesis is to find a mathematical framework providing a clear and conceptually simple description of classical gauge field theories. In particular we are interested in the reduction with respect to internal symmetries of gauge theories and in the relation between Lagrangian and Hamiltonian description of this class of fields.

Our first goal is to construct a geometrical picture of the Lagrangian and Hamiltonian description of gauge theories, which will allow a convenient description of the dynamics. This part of the thesis is primarily based on a geometrical structure known in the literature as a *Tulczyjew triple*, and which is described in detail in the section 3. One of the main advantages of the Tulczyjew formalism is that it does not require any regularity of the Lagrangian to find the dynamics of the system and to pass to Hamiltonian description. By a regular system we mean a system, for which a Legendre map is a local diffeomorphism. This feature is particularly important in classical field theory where most of the interesting systems coming from physics is irregular. The starting point for the construction of the Tulczyjew triple is a rigorous analysis of the geometry of the bundle of connections and its configuration and phase bundle.

In the next step we perform a reduction of the derived Tulczyjew triple with respect to the internal symmetries of gauge theories. The general formalism of classical field theory is based on jet bundles over a fibration, but in physics Lagrangian usually does not depend on the full jet of the connection but only on its value and curvature. It is a natural question in that context, whether the projection of the first jet of the connection onto its value and curvature can be described geometrically. Such a projection implies the reduction of the entire structure of a given gauge theory and it is described in detail in sections 5 and 6.

The last chapter of our work is devoted to the reduction with respect to gauge symmetry. Our main tool in this part is the so-called dressing field method, which is described in detail in the section 7. The important result of this section is the geometric description of the dressing field method in a language of jet bundles and principle bundles within both partial and full reduction of the gauge symmetry. As a consequence we obtain a reduced Lagrangian formalism for gauge field theories.

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