Summary of PhD thesis

Refined Topological Vertex and Supersymmetric Gauge theories

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String theory and M-theory are one of the most exciting candidates for unification of gauge theories and gravity. Apart from fundamental strings, they also describe behavior of various types of branes. To describe physics in four-dimensional spacetime, string theory needs to be compactified on a Calabi-Yau manifold. Supersymmetric gauge theories can be constructed from the string theory perspective in various ways, using either Calabi-Yau manifolds or branes. For instance, 5d $\mathcal{N} = 1$ gauge theories can be engineered by compactifying M-theory on noncompact Calabi-Yau threefolds, or by 5-brane webs in type IIB string theory [1]. It is important to understand properties of gauge theories from both such perspectives.

In this thesis, we consider 3d $\mathcal{N} = 2$ supersymmetric gauge theories that can be engineered by toric Calabi-Yau manifolds without compact four-cycles (i.e. so called strip Calabi-Yau manifolds) in the presence of a Lagrangian brane [2]. Interestingly, these Calabi-Yau manifolds can be represented by toric diagrams, which are related to brane webs through the duality between M-theory and type IIB string theory [3]. Moreover, topological string theory provides computational tools to study gauge theories. In particular, the topological vertex formalism, originally derived to compute amplitudes in topological string theory, enables to compute also partition functions of gauge theories. These partition functions encode contributions from BPS particles, which from M-theory perspective arise as M2-branes wrapped on two-cycles in the corresponding Calabi-Yau manifold.

In this thesis, we generalize the refined topological vertex formalism [4], and then use it to obtain refined open partition functions, which are equal to the vortex partition functions of some 3d $\mathcal{N} = 2$ gauge theories. We also find that refined 3d vortex partition functions have a quiver representation. To understand these quivers, we study the corresponding 3d gauge theories using 3d mirror symmetry. Finally, we discuss the corresponding brane webs for these gauge theories.

More precisely, the refined topological vertex computes closed topological string partition functions, however it is not very clear how to compute refined open topological string amplitudes. To overcome this problem we use the geometric transition [5] in the topological string A-model, which enables us to obtain refined open amplitudes associated to strip Calabi-Yau threefolds in the presence of one Lagrangian brane. We find that there are four types of such branes and we describe relations between them. We also find corresponding Ooguri-Vafa formulas and extract refined open BPS invariants that they encode, which (as expected) are positive integers. These results are presented also in [6]

Analogously as in the knots-quiver correspondence [7, 8], it can be shown that refined open partition functions can be written in terms of quiver generating series. We find quivers that define such series; among others, this automatically proves integrality of associated refined open BPS numbers. Furthermore, in order to understand these quivers physically, we study the underling three dimensional $\mathcal{N} = 2$ gauge theories determined by strip Calabi-Yau threefolds, using sphere partition functions and 3d mirror symmetry [9]. It turns out that these quivers encode the effective mixed Chern-Simons levels of a particular type of mirror dual theories. We also verify this conclusion from vortex partition functions, which are equal to open topological string partition functions. This part is based on [10].

The last part of this thesis is devoted to the description of 3d $\mathcal{N} = 2$ gauge theories using brane webs. We discuss the real mass deformations of brane webs and show how to compute vortex partition functions using topological vertex formalism. The real mass deformations of brane webs give rise to many seemingly different brane webs, but their vortex partition functions are equivalent. In addition, we find that nonabelian theories also have quiver representations. This part is based on [11].

To sum up, we describe properties of 3d gauge theories engineered by toric Calabi-Yau manifolds or branes. We show that open topological string partition functions encode BPS numbers and prove their integrality. Refined geometric transition and quiver representation play an important role in understanding of 3d $\mathcal{N} = 2$ gauge theories.

As indicated above, the results of this thesis are also presented in publications [6, 10, 11]. In addition, I have written two other publications on related topics [12, 13].

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