External Reviewer Report
Doctoral Dissertation: Denis Dobkowski-Ryłko

Overview

The thesis "Isolated horizons in spacetimes with cosmological constant" by Denis Dobkowski-Ry łko reports on several results concerning the classification of isolated horizons. The thesis is based on five published papers with various collaborators including his PhD advisor Prof. Jerzy Lewandowski.

The main results consist of a classification of the Petrov types of isolated horizons with a cosmological constant. First, this requires identifying appropriate assumptions which ensure that the Petrov type is a property that can be ascribed to the intrinsic geometry of the horizon. Then a detailed study of the type D isolated horizons is performed, including the derivation of a single PDE which characterises this (Petrov type D equation), which is then solved for axisymmetric horizons with 2-sphere spatial topology and higher genus surfaces (with no symmetry assumptions). The analysis in the higher genus case is particularly elegant revealing no nontrivial solutions. The result is that all such non-extremal horizons correspond to the horizons of the non-extremal Kerr-de Sitter metrics.

This is a very satisfying result that is reminiscent of the theorem that extremal isolated horizons solve the near-horizon geometry equation, which is an equation for its intrinsic geometry; this has been previously looked at in the literature where it was found the extremal horizons with axisymmetry and spherical sections correspond to those of the extremal Kerr-de Sitter metrics. Curiously, the thesis also shows that any solution of the near-horizon geometry equation for extremal isolated horizons also solves the Petrov type D equation. This is then used to solve the near-horizon geometry equation in the higher genus case with no symmetry assumptions, which fills in an important gap in the classification of extremal horizons (previously the best that was achieved was ruling out the toroidal case with axisymmetry).

There is also an analysis of isolated horizons with nontrivial bundle topology (circle bundle over the spatial cross-section). Since the Petrov type D equation is local these can be solved for in an analogous fashion. In particular, the general axisymmetric solution with a 2-sphere base is found, and this arises in the accelerating Kerr-de Sitter Taub-NUT family of metrics (although this is not shown in this thesis in generality).

The final chapter is on the gravitational quadrupole formula for radiation in a de Sitter background. The radiation is computed on the cosmological horizon, which is also an isolated horizon, hence in this sense it fits with the rest of the thesis. The main result is to extract the first order correction to the quadrupole formula in an expansion in the cosmological constant from a general formula due to Ashtekar et al.

The thesis well written and a coherent piece of work that makes clear advances in the field. I have some minor comments and optional suggestions for improvements.

Comments and questions

- The definition of the rescaling freedom of ℓ should really appear before eq 1.25, rather than in eq 1.31 which is later.
- Perhaps a comment on the existence of the function U in eq 1.54 should be made. That is, for compact S standard Hodge theory guarantees existence. Also, it should be stated why it is useful to consider U.
- The Theorems 1.3.3 and 1.3.4 at the end of Chapter 1 could benefit from being illustrated by examples. Do these situations arise in any interesting spacetimes?
- Trivial typo in eq 2.3 (indices).
- A justification for eq 2.16 would be helpful.
- End of chapter 2, page 48. Reference [39] in point (iii) only considered vanishing cosmological constant; I think that should reference [38]. Indeed, the first reference that found the near-horizon geometry of extremal Kerr de Sitter and showed that it is the unique axisymmetric solution to the near-horizon geometry equation was [64].
- In Section 3.1 after eq 3.2 (and 3.6) it is stated that "coordinates ψ, ϕ are globally defined on S, although not continuous.". This is not strictly correct, the coordinates on a torus (or circle) are not globally defined since they are not smooth (or even continuous) functions on the torus. However, it is true that the vector fields $\partial_\psi, \partial_\phi$ and dual 1-forms are globally defined and non-vanishing on the torus, which is all that is used in the analysis.
- The right bracket in eq 3.16-3.17 I think should be to the right of f. Also the P appearing in the text above eq 3.19 I think is meant to be Q. Similarly below eq 3.33 and in eq 3.37-3.38.
- The decomposition in eq 3.39 and discussed below it is just using the complex structure to define the holomorphic $(1,0)$ part of the vector field. This is of course a standard decomposition so perhaps this more global definition should be mentioned.
- Theorem 3.3.2 refers to rotating isolated horizons. Presumably it is meant that $\Omega = 0$, i.e. the rotation 1-form ω is locally exact? This should be defined. Also, presumably Corollary 3.57 referred to below eq 3.61 should be Corollary 3.3.1?
- It may be helpful to clarify what is meant by "the Petrov type D equation is local" at the start of sec 4.1. In particular, since it is crucial for this section a more explicit reason could be given why its derivation does not care about the bundle structure and is only defined on the base. Presumably invariance under the vertical vector ℓ is crucial.
- Typos in eq 4.22 (repeated dx) and 4.23 I guess should be $\Psi_2^{-1/3}$, eq 4.24 should be $(c_1 x + c_2)^{-3}$? Also, is eq 4.26 is the same as 2.28 in the trivial bundle case? If so this perhaps should be emphasised.
- The last sentence in sec 4.2.1 states that the embedding is not unique and depends on the choice of symmetry. It is not so clear what symmetry this is referring to, it would help to be more explicit about this.

Conclusion and Recommendation

Overall this is a high quality thesis that contains several interesting results and is a valuable addition to the literature (indeed, it is based on five published papers). Furthermore, it is well written and includes a good introduction to the topic of isolated horizons. I only have minor suggestions for improvements and clarifications as detailed above. My conclusion is therefore positive and in my opinion this work meets the standard for the award of a doctorate.

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