Abstract

This thesis is devoted to the following question: what does the neighborhood of a *generic* extremal horizon look like? We are interested in all possible (stationary) perturbations that may be induced either by a far-away matter distribution or by non-trivial boundary conditions at infinity (in the case of asymptotically AdS spacetimes). Thus, we want to go beyond explicitly known solutions such as Kerr-Newman (AdS). To this end, we consider the behavior of a small perturbation that is supposed to die off at the horizon. Its smallness allows us to linearize appropriate (e.g. Einstein-Maxwell) equations. Our considerations will mainly focus on spacetimes with a negative cosmological constant.

We found out that in four dimensions, *generically* the perturbations are not C^2. As a result, the horizon is replaced by a null singularity. Perhaps counter-intuitively, the larger the black hole, the worse the singularity can get. At the same time, all curvature scalars remain finite and thus analytic continuation of those solutions to the Euclidean signature is smooth. The singularity vanishes when the cosmological constant $\Lambda = 0$. It exists for $\Lambda > 0$ albeit it is no longer so robust. Moreover, in that case, the tidal forces are always integrable through the horizon and thus the effect is not so big.

At finite temperatures, the singularity leaves significant observational imprints. The tidal forces at the horizon, though finite, may be arbitrarily large as $T \to 0$. Moreover, the specific heat is changed by an anomalous term that, for large and cold black holes, dominates over the usual (linear in T) contribution.

In five (and higher) dimensions the singularity for RN AdS becomes even worse and leads to *RG instability* - a small perturbation of the boundary conditions dramatically changing the infrared (it means near-horizon) region. New near-horizon geometries were constructed. However, none of them is RG stable. In dimensions higher than five, the same conclusion also holds without a cosmological constant. Moreover, for toroidal black holes, we have a phase transition. Small ones are stable and only large ones are not. At the phase transition threshold, we expect the resistivity to approach a constant.

As a result, we predict that the boundary theory put on a non-homogeneous background could flow to a different infrared fixed point. Unfortunately, the endpoint of that instability (either in the bulk or on the boundary) is currently unknown. In particular, it is not clear that it is described by only one horizon.

Finally, we show that the higher-curvature corrections generated by UV physics may render even an asymptotically flat extremal Kerr black hole singular. This is an example of a scenario in which quantum effects qualitatively change the system despite the fact that the curvature is very small. The scaling dimensions of RN_4 do not get modified. However, they are shifted in higher dimensions. In particular, quantum corrections render RN_5 RG-unstable provided that the Weak Gravity Conjecture is satisfied. Moreover, we found that this very conjecture implies that Kerr-Newman (AdS) is unstable with respect to stationary perturbations by massive, charged scalar fields.