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Review of Doctoral Dissertation of Maciej Kolanowski

The thesis "Stationary perturbations of near-horizon geometries" by Maciej Kolanowski presents a detailed study of the geometry near the horizon of generic extremal black holes. The thesis is based on three published papers written in collaboration with Gary Horowitz and Jorge Santos. It is impressive that Maciej - as a doctoral student - has managed to establish successful collaborations with such leading figures in the field of gravity and holography, branching out from his earlier work in General Relativity on isolated horizons and de Sitter spacetimes with his doctoral advisors Jerzy Lewandowski and Wojciech Kaminski. It is worth noting that Maciej's earlier doctoral work consists of a further 10 or so published papers (some single author). In my opinion, it is a shame that Maciej decided to not include some of his interesting earlier doctoral work, especially the work on the geometry near smooth extremal horizons which would have fitted nicely in the thesis (although it is mentioned in the Introduction). Having said this, I understand that focusing on the recent work gives a very focused and unified thesis with a clear narrative. In any case, it is very unusual for a student in this field to amass such a large and diverse body of work, and Maciej is clearly already operating at the level of a postdoc. Based on this alone Maciej deserves to be awarded a distinction.

The thesis itself is well written in an informal and engaging style, demonstrating the candidate has an excellent general knowledge of the field and related areas, indeed, the thesis includes a very comprehensive bibliography. The introduction, while fairly concise, further supports this breadth of knowledge. The main results presented in the thesis are as follows.

In Chapter 2 it is argued that generic extremal black holes in AdS_4 are singular. This is achieved by a study of stationary perturbations of the near-horizon geometries in four-dimensional Einstein-Maxwell theory using a variety of techniques. First, the study of perturbations of the near-horizon geometry of extreme Reissner-Nordstrom is performed by employing the Kodama-Ishibashi formalism (which is suited to static spacetimes). Next, the study of perturbations of the near-horizon geometry of extreme Kerr-AdS is performed using the Teukolsky equation. In all cases these are organised in terms of eigenfunctions of the (generic) dilation symmetry of the near-horizon geometry and reveals that the associated scaling dimension γ is generically non-integer and for low harmonics (with respect to the sphere) one can have $0 < \gamma < 2$. This indicates that while the near-horizon geometry is intact, the perturbation may not be C^2 at the horizon and the curvature blows up. Strangely, this effect becomes more pronounced the larger the black hole (in the sense that more harmonics are singular). In the absence of a cosmological constant γ is a positive integer and this effect is not present. Thus a main result of this chapter is that generic stationary perturbations of extremal horizons in AdS_4 are singular.

In order to make the stronger claim that generic extremal black holes in AdS_4 are singular, a couple of numerical constructions are employed to construct full non-linear solutions to the Einstein equations that correspond to the back-reaction of such perturbations (for technical numerical reasons, this is done for near-extremal solutions). The "perturbations" now correspond to inhomogeneous boundary conditions on the AdS boundary. Therefore, the term 'generic' here refers to a choice of the AdS boundary condition. The classification of AdS spacetimes with nontrivial boundaries is a very complicated and unexplored area which has no counterpart in asymptotically flat spacetime and therefore this work should be of great interest in holography. It is quite a striking claim that extremal black holes in AdS spacetimes with inhomogeneous boundary conditions will generically suffer from such singularities. A question I have, which is not addressed in the thesis, is if breaking the symmetry of the boundary is the only mechanism to generate such singularities. In particular, if one insists on asymptotically globally AdS_4 spacetimes, would extremal black hole horizons be generically smooth at in flat space? This is an important question as uniqueness theorems even for globally AdS black holes are unknown in AdS gravity.

In Chapter 3, it is first shown that generic stationary perturbations of the near-horizon geometry $\mathrm{AdS}_2 \times S^3$ in five-dimensional Einstein-Maxwell theory with a negative cosmological constant have negative scaling dimensions and hence blow up on the horizon. In AdS/CFT this is interpreted an an unstable IR fixed point. Motivated by this, a new five-dimensional $SO(3)$ invariant static near-horizon geometry solution is constructed perturbatively and numerically, which is stable to $SO(3)$ symmetric perturbations. Independently of application to AdS/CFT this is an interesting result since the classification of static near-horizon geometries in 5d Einstein-Maxwell theory – even with no cosmological constant – is an open problem. Indeed, in the absence of a cosmological constant non-trivial near-horizon geometries are already known (albeit with toric rather than spherical symmetry). It is therefore not surprising that new static near-horizon geometries are lurking in 5d Einstein-Maxwell theory, especially with a cosmological constant. This is therefore a valuable example for the classification programme for near-horizon geometries. It turns out this new near-horizon geometry is unstable to non-spherical perturbations, which may be hinting that even less symmetric geometries remain to be found. This is certainly an interesting avenue that needs further study.

In Chapter 4 - the final chapter with new material - it is argued that extremal black holes in higher derivative effective field theories of gravity are generically singular, even in the absence of a cosmological constant. Conceptually this is similar to the results of Chapter 2 on extremal black holes in AdS_4. Technically, the inclusion of higher derivative terms in the action adds substantial complications. It appears that even the near-horizon geometries were not known, so a non-trivial sub-result of this chapter are the generic higher-derivative corrections to the near-horizon geometry of the extreme Kerr horizons. It is then found that stationary perturbations of these corrected near-horizon geometries are generically singular in the sense that the scaling dimension $\gamma < 2$. Furthermore, the presence of charge (higher-dimensional RN or 4d Kerr-Newman with charged matter) seems to be more severe than having a negative cosmological constant: if the weak gravity conjecture (WGC) is true the scaling dimensions are negative. It is curious - perhaps not surprising - that these IR instabilities have something to do with the WGC.

It is worth noting that in contrast to Chapter 2, the full non-linear (numerical) black hole solutions that correspond to the backreaction of the perturbations are not constructed (presumably the higher-derivative terms render this technically intractable at the moment). Hence the claims in Chapter 4 are on a less firm footing, in the sense that it is unclear what mechanisms would source such perturbations at the nonlinear level. A particular question I have here is as follows. Does one really expect that asymptotically flat extremal black holes in higher-derivative theories to be generically singular? Or, like in AdS, is the expectation that the generic singular perturbations are sourced by non-trivial boundary conditions which in this case would presumably correspond to some external gravitational field (source by some matter)? In other words, could it be that the fine tuning required for a near-horizon perturbation to integrate to a full non-linear asymptotically flat solution render the horizon again generically smooth? Again, as in Chapter 2, this would help clarify what the term 'generic' is referring to. Nevertheless, the results obtained are significant in studies of black holes in higher derivative gravity theories, hinting that extremal black holes may generically have singular horizons.

To summarise, overall the thesis is of high quality, contains a large amount of work and makes significant advances in the understanding of extremal black holes in settings more generic than those normally studied, i.e. stationary perturbations arising from non-trivial AdS boundary conditions or higher-derivative corrections. As such, it certainly merits the award of PhD. Furthermore, as mentioned above, I would also recommend it is awarded a distinction.

Your sincerely,

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