Abstract

All in all, it's just another brick in the wall

Pink Floyd, *Another Brick in the Wall, Pt. 2*

In modern physics, there are few field theories more versatile and ubiquitous than the $O(N)$ models. For several decades they have proven invaluable in describing a myriad of physical phenomena. Despite being conceived many years ago, the $O(N)$ models are still subjects of significant developments. This thesis describes a few of the most recent discoveries regarding the classical, bulk phase transitions of the $O(N)$ models and their anisotropic extensions.

The topics discussed in this thesis can be broadly classified as relating to the pure $O(N)$ models or their anisotropic extensions. Regarding the pure $O(N)$ models, we reexamine an old perturbative analysis by Cardy and Hamber [1] predicting a fixed-point collision scenario leading to exotic consequences. Above two dimensions, the collision is expected to induce nonanalytic behavior of critical exponents, while below two dimensions it constitutes a mechanism for the disappearance of the phase transition at the lower critical dimension. We revisit their perturbative analysis and showcase some of its poorly appreciated consequences. We confront these predictions with the results of our nonperturbative renormalization group calculations at the $O(\partial^2)$ order of the derivative expansion. Our calculations confirm the presence of the fixed-point collision below two dimensions and offer robust arguments against the collision scenario above two dimensions. The discussion of this subject, presented in Chapter 3, is based on two articles: *Analyticity of critical exponents of the O(N) models from nonperturbative renormalization* [2] and *Low-temperature behavior of the O(N) models below two dimensions* [3].

The other area explored in this thesis concerns the effects of weak cubic perturbations in the $O(2)$ model. We investigate how the leading scaling exponent of the anisotropic field y_4 varies with the dimension. We offer an accurate determination of y_4 in three dimensions and, subsequently, observe the evolution of y_4 while reducing the dimension towards two. In the vicinity of two dimensions, we observe y_4 approaching zero marking the onset of the nonuniversal behavior related to the Kosterlitz-Thouless physics. We also discuss how the comparative performance of two alternative approaches to the derivative expansion varies with the dimension. These topics are explored in Chapter 4 based on the article *$\mathbb{Z}_4$-symmetric perturbations to the XY model from functional renormalization* [4]. This article is a product of a collaboration with Carlos A. Sánchez-Villalobos, Paweł Jakubczyk, and Nicolás Wschebor; in the conclusion of the chapter, we disclose the authors' contributions.