## **Abstract**

This dissertation presents an in-depth study of the relativistic aspects of time in quantum mechanics, focusing on two primary themes: quantum time dilation and the concept of indefinite temporal order, including the implications of temporal Bell inequalities.

The research investigates quantum time dilation, a phenomenon where time dilation, well-known in relativity, also occurs at the quantum level. Our findings reveal that quantum time dilation is universal, similar to classical time dilation, meaning it does not depend on the specific mechanisms of the clocks used. This universality suggests that quantum time dilation is a fundamental property of quantum systems, expanding our understanding of time in the quantum realm. However, the dissertation also explores gravitational quantum time dilation and finds that it is not universal. Different clocks experience varying amounts of time dilation due to gravity, depending on their internal structures. This discovery mirrors the classical understanding of gravitational time dilation and highlights the complex interplay between quantum mechanics and gravity.

The second focus of this dissertation is the concept of indefinite temporal order. In classical physics, events occur in a definite sequence, but in the quantum world, events can exist in a superposition of different orders. This means that the sequence of events is not fixed and can be indefinite. We developed a scenario where accelerating particles interact with quantum fields to demonstrate this indefinite order. By using special relativistic time dilation, we established a protocol that shows a violation of Bell's inequalities, traditionally used to test definite temporal order. Our results indicate that events can indeed occur in an indefinite order, challenging classical perceptions of time.

Our research highlights that the assumptions necessary for proving Bell's inequalities for temporal order are not always satisfied in practical scenarios. This insight led to a critical reassessment of these assumptions, revealing the complexities of disentangling the free dynamics of a system from local operations. These findings are significant for both theoretical and experimental investigations into the nature of time. This dissertation advances our understanding of quantum time dilation and indefinite temporal order. It challenges conventional notions of time and causality, providing new perspectives on the fundamental nature of the universe.