Nobel Prize in Physics 2025: Schrödinger’s Cat gets bigger

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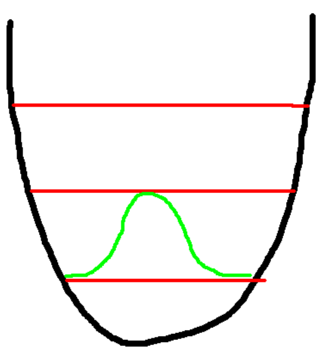
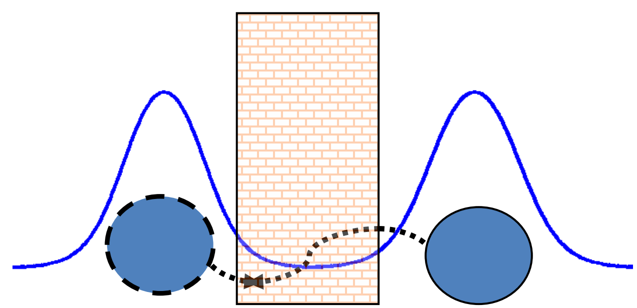
Konwersatorium im. J.Pniewskiego i L.Infelda, Faculty of Physics, UW, Warszawa, 24/11/2025





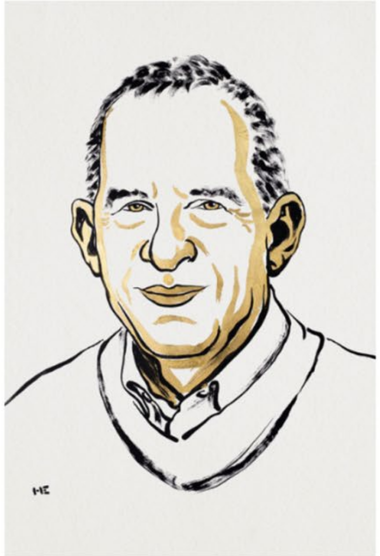
Nobel Prize 2025 was awarded for:

"for the discovery of
macroscopic quantum mechanical tunnelling
and energy quantisation in an electric circuit"

Ill. Niklas Elmehed © Nobel Prize Outreach

John Clarke

Prize share: 1/3



Ill. Niklas Elmehed © Nobel Prize Outreach

Michel H. Devoret

Prize share: 1/3

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John M. Martinis

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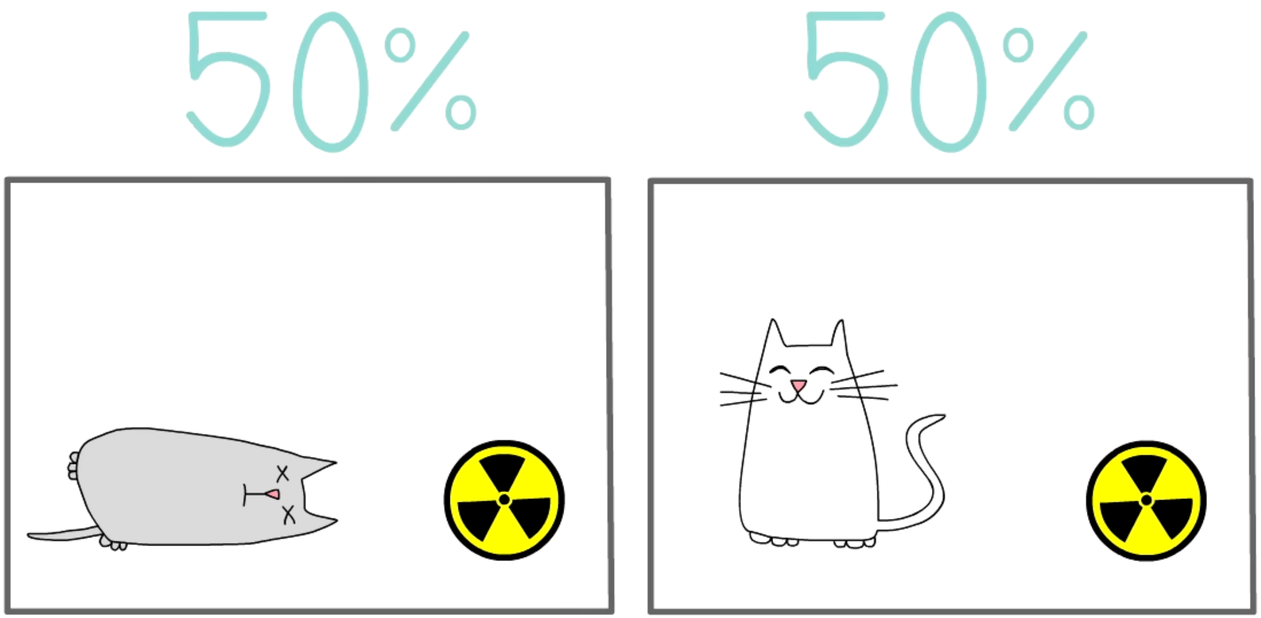
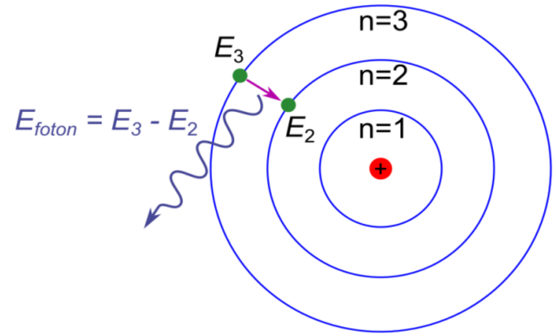
Schrödinger's Cat

- Can we have a macroscopic object in a superposition of states?

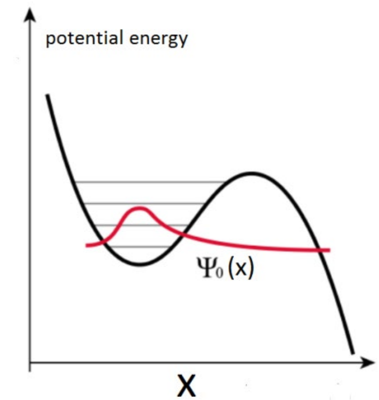


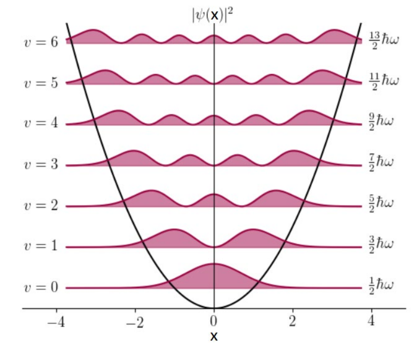
TED-Ex: Schrödinger's cat: A thought experiment in quantum mechanics - Chad Orzel

Quantum world

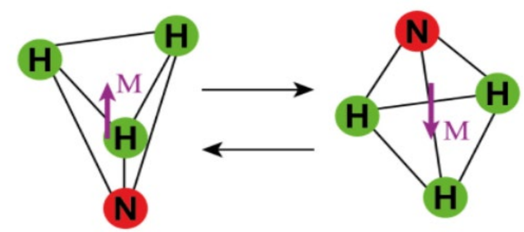


- Quantization of energy



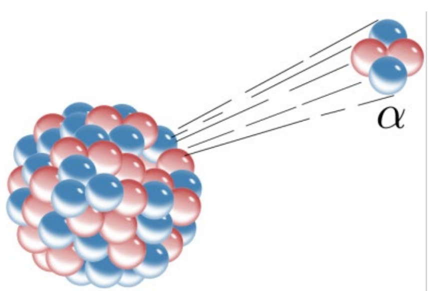


- Particles/atoms:
- are delocalized due to wave nature
- can tunnel



- can be in a superposition of states

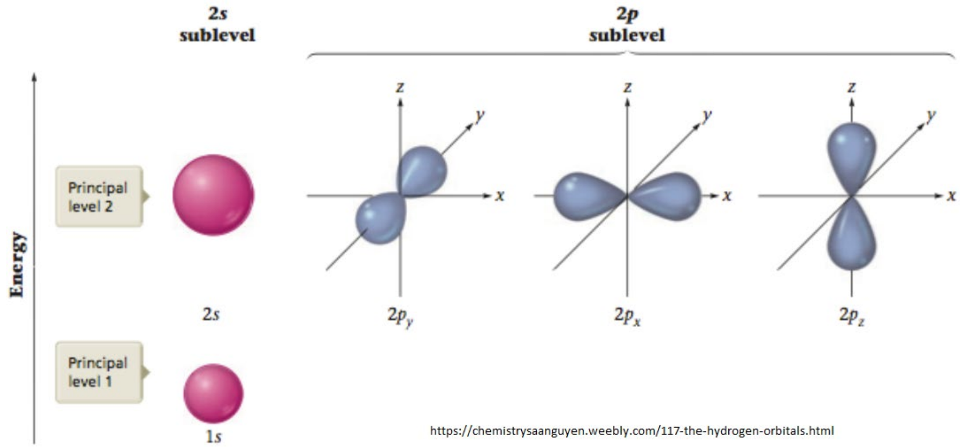
- Atoms can decay

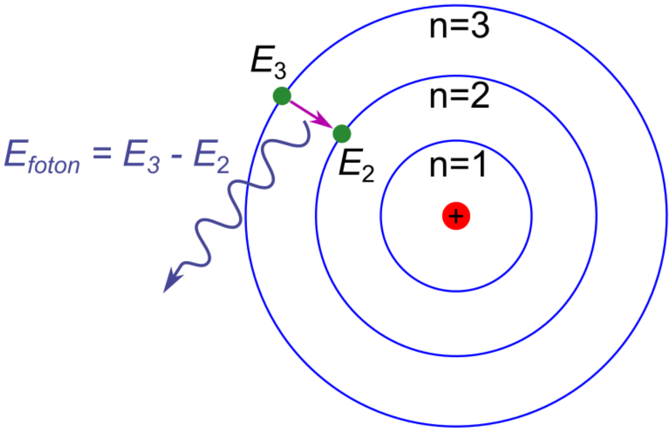


Interference/diffraction

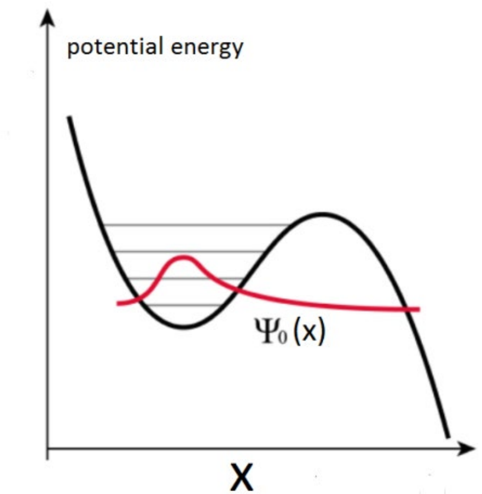
Quantized energy orbitals

- Well-defined discrete energy levels – hydrogen molecule
- Excitation/Emission



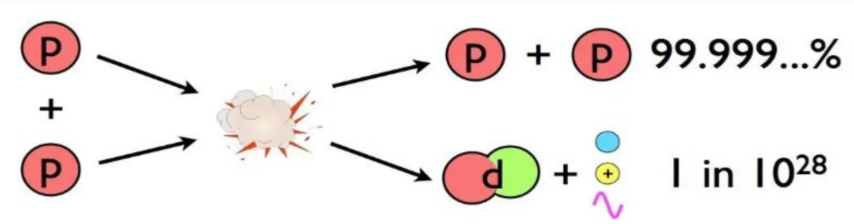


Electric charge tunneling

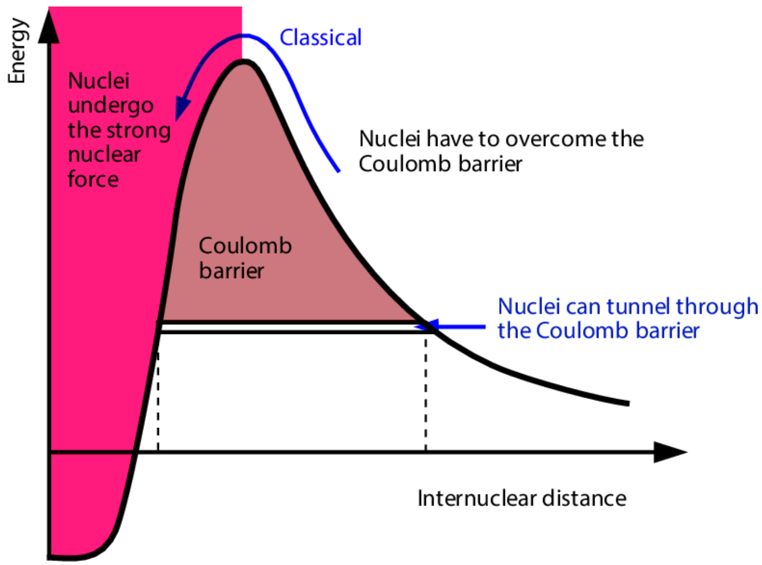


Nuclear Fusion in the Sun

$$p + p + e^- \rightarrow {}^2_1D + \nu_e + 1.442\text{MeV}$$

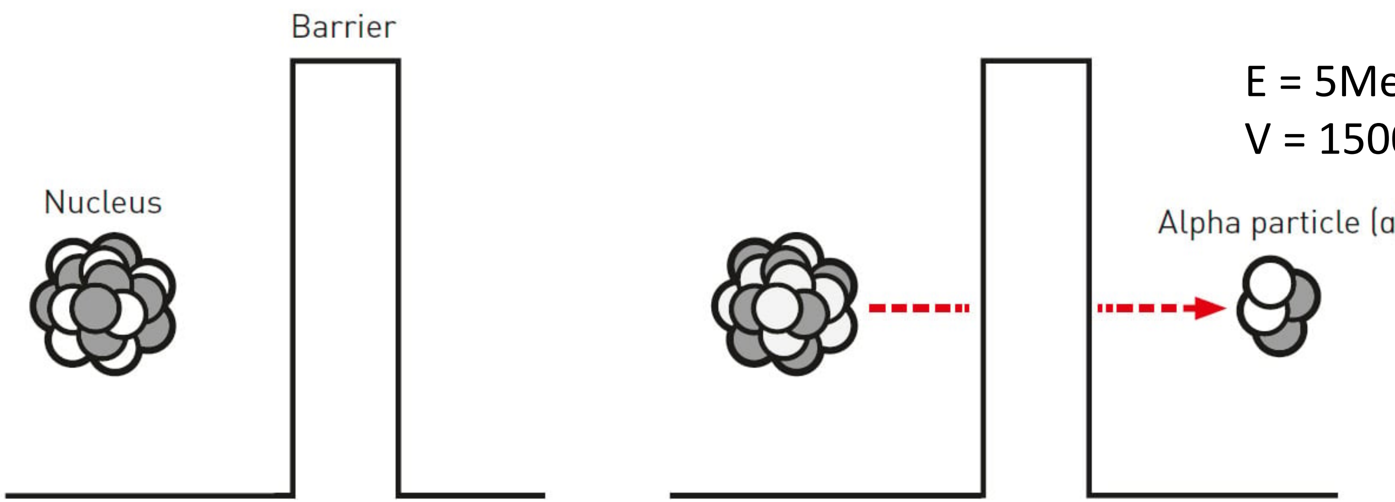


E.Siegel, *Beyond The Galaxy*



Decay of a radioactive nuclei

$$^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He^{2+}$$



E = 5MeV,
V = 15000 km/s

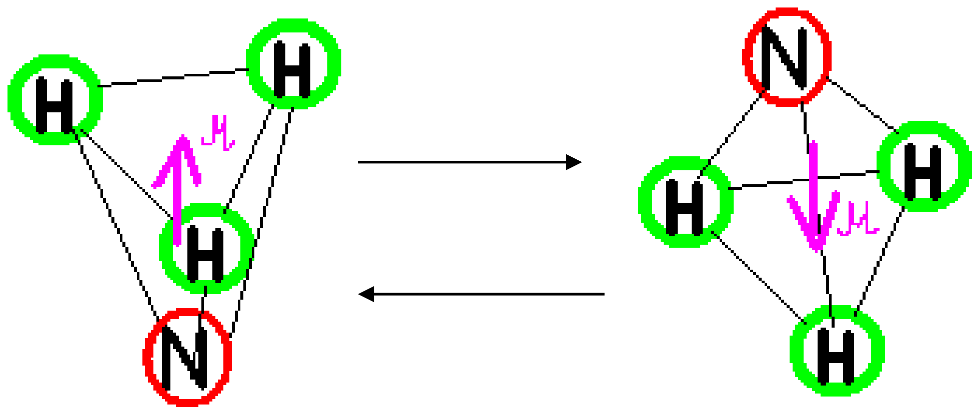
$T_{1/2}$= 4.5 billion years, half lifetime

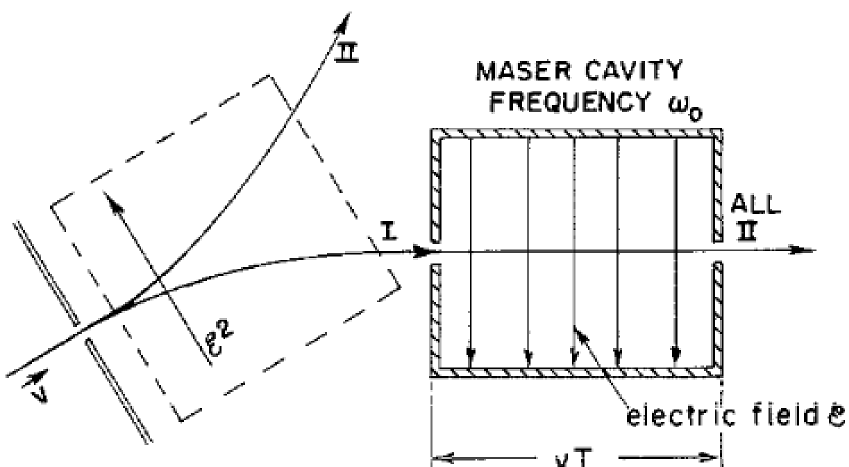
George Gamov, 1928

Quantum superposition

MASER

(microwave amplification by stimulated emission of radiation)





Feynman

(vol. III, Ch. 9)

Can macroscopic objects (ensembles of particles) behave in a quantum way, i.e. as they were a single particle?

Engineered *artificial atoms* consisting of many particles?

Can we find a macroscopic system, which would be described with a single wave-function?

A collective state needed... Superconductivity and superfuidity seem to be a good candidates.

Macroscopic wavefunction

$$\Psi(r) = \sqrt{\rho}e^{i\theta(r)}$$

$$j_s = \frac{-\hbar}{2m_e}(\nabla\theta - \frac{2e}{\hbar}A)\rho(2e)$$

j_s-supercurrent density

ρ – density of Cooper pairs

ϴ - macroscopic superconducting phase
The same for large number of Cooper pairs!!!

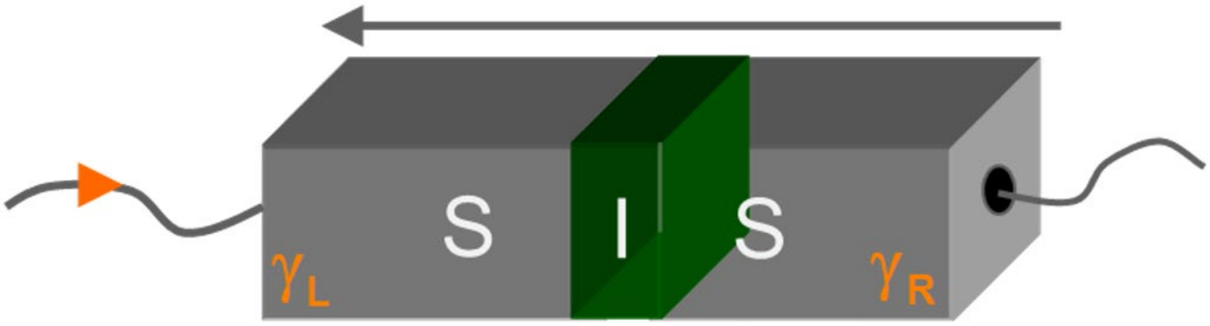
Can a system described with such a wavefunction behave like an atom?

Can we see quantum phenomena for a macroscopic object?

- Superconducting wave function describes collective behavior of many Cooper pairs
- Spontanous symmetry breaking => phase becomes a macroscopic variable
- The sc condensation creates an energy gap in excitation spectrum => Cooper pairs do not want to interact with environment
- It vastly reduces effects of environmental decoherence and should promote the presence of quantum effects.

Tunnel Josephson junction

$\gamma = \gamma_R - \gamma_L$



1st Josephson relation: $I_J = I_0 \sin\gamma$

2nd Josephson relation: $\frac{d\gamma}{dt} = \frac{2\pi}{\Phi_0} V$

Nobel, Brian Josephson, 1973

Nobel prize for showing that a Josephson junction is an artificial atom

VOLUME 55, NUMBER 18 PHYSICAL REVIEW LETTERS 28 OCTOBER 1985

Measurements of Macroscopic Quantum Tunneling out of the Zero-Voltage State of a Current-Biased Josephson Junction

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(Received 26 July 1985)

The escape rate of an underdamped ($Q \approx 30$), current-biased Josephson junction from the zero-voltage state has been measured. The relevant parameters of the junction were determined *in situ* in the thermal regime from the dependence of the escape rate on bias current and from resonant activation in the presence of microwaves. At low temperatures, the escape rate became independent of temperature with a value that, with no adjustable parameters, was in excellent agreement with the zero-temperature prediction for macroscopic quantum tunneling.

VOLUME 55, NUMBER 15 PHYSICAL REVIEW LETTERS 7 OCTOBER 1985

Energy-Level Quantization in the Zero-Voltage State of a Current-Biased Josephson Junction

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Department of Physics, University of California, Berkeley, California 94720, and Materials and Molecular Research Division, Lawrence Berkeley Laboratory, Berkeley, California 94720

(Received 14 June 1985)

We report the first observation of quantized energy levels for a macroscopic variable, namely the phase difference across a current-biased Josephson junction in its zero-voltage state. The position of these energy levels is in quantitative agreement with a quantum mechanical calculation based on parameters of the junction that are measured in the classical regime.

E-beam lithography

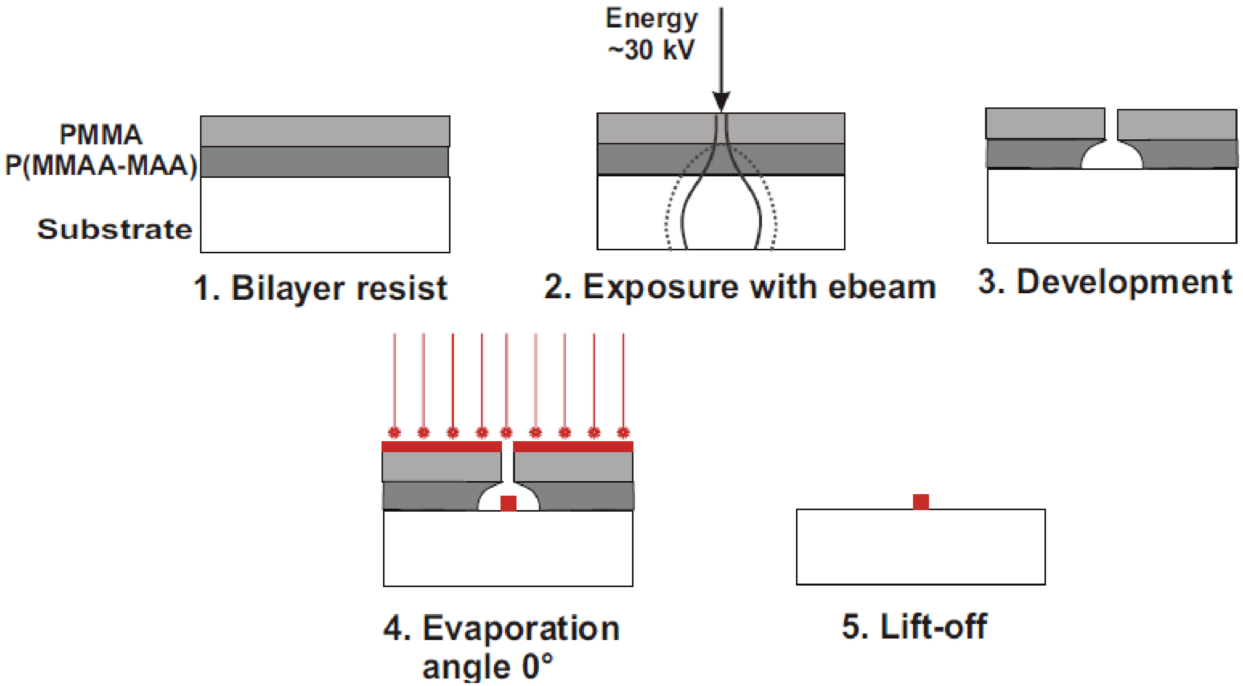
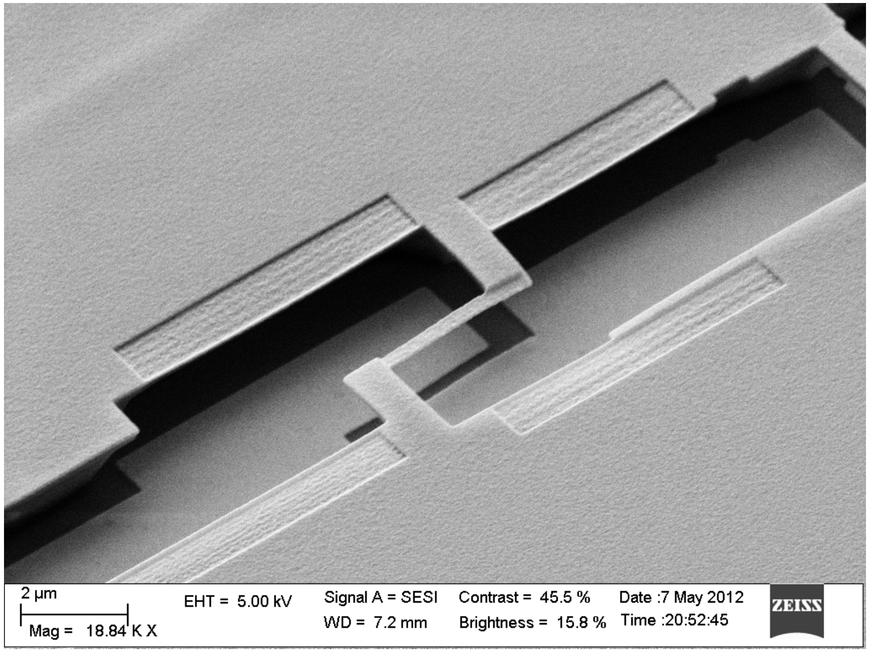
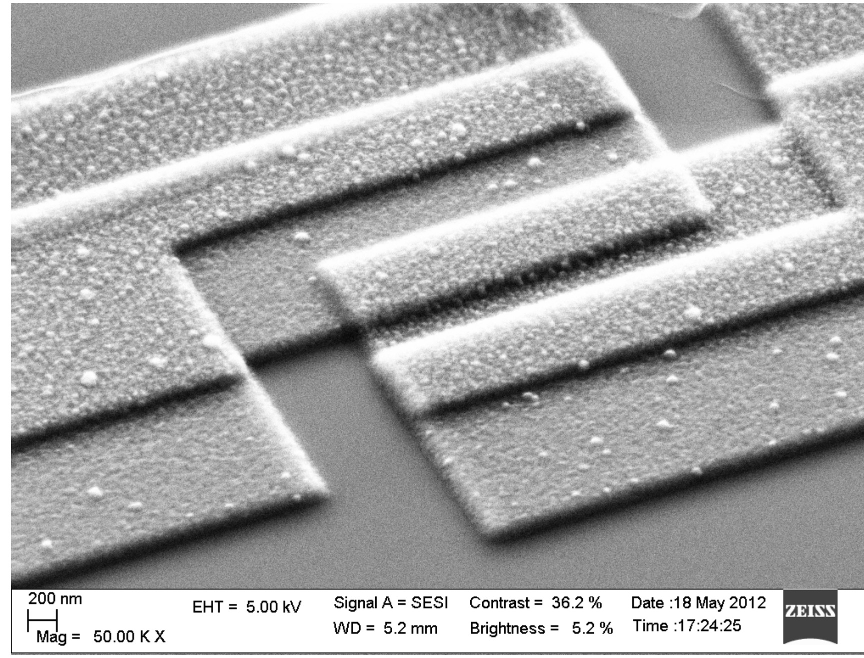


Figure 2.1 Fabrication of metallic structures by using positive electron beam lithography and evaporation techniques. Bilayer resist is used to achieve the undercut structure shown in step 3.





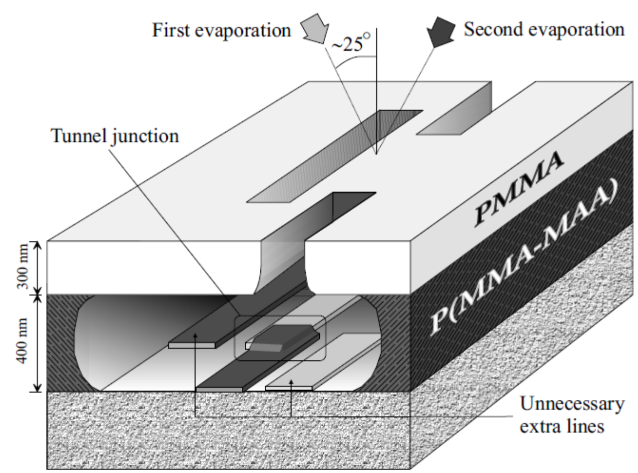
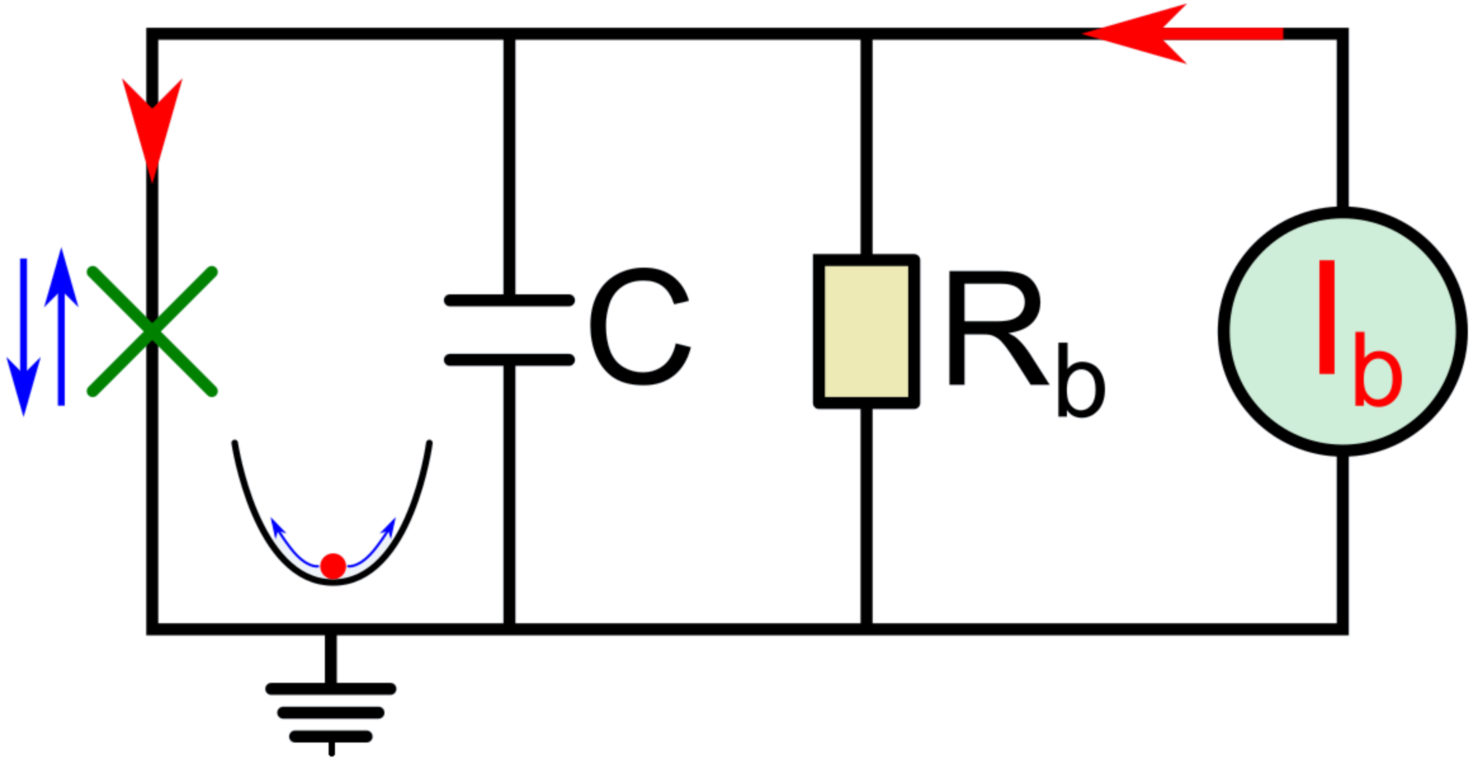


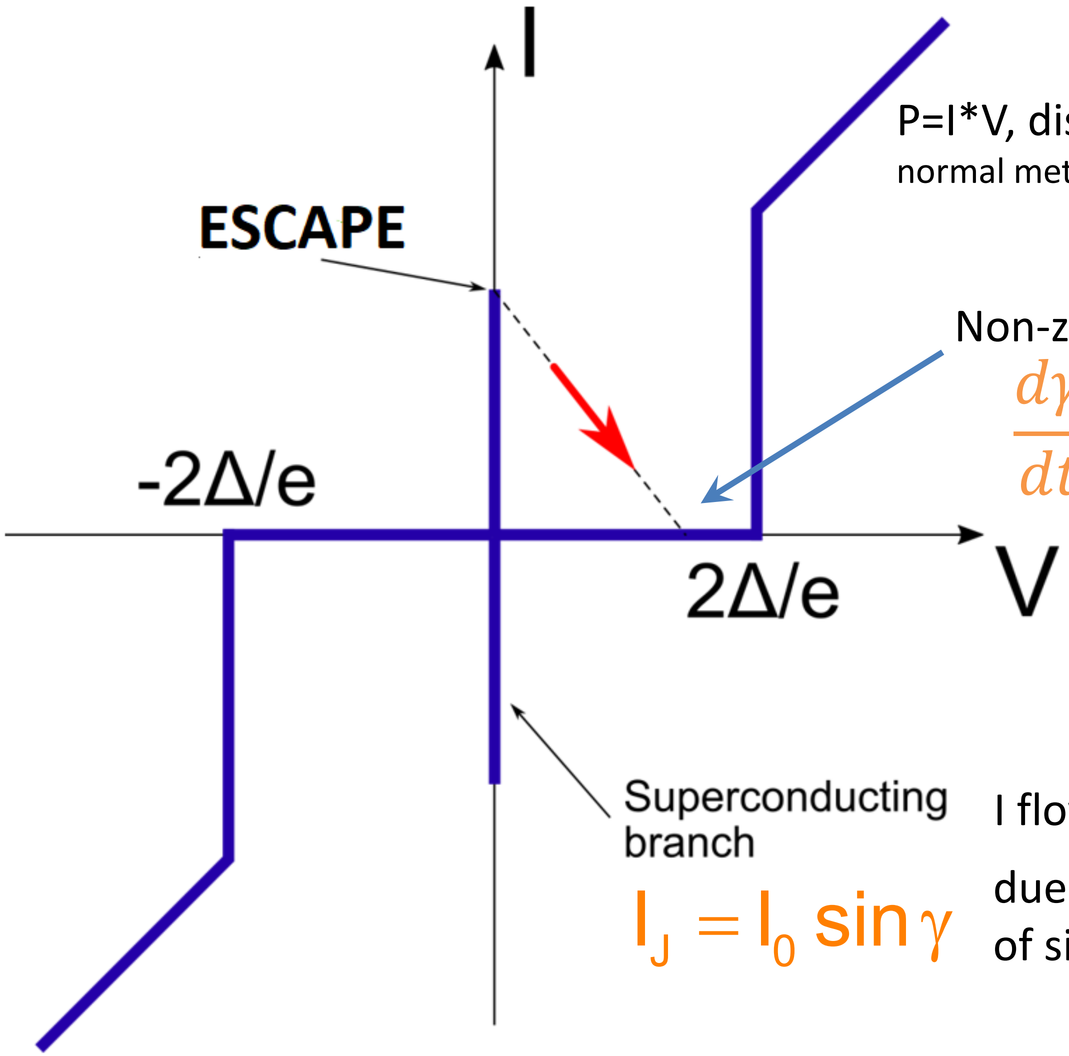
FIGURE 2.1 Principle of the self-aligning shadow evaporation technique. The tunnel junction is formed between the two metallic layers (usually Al) evaporated at different angles so that they overlap slightly. The tunnelling barrier is formed by oxidising the first layer before evaporating the second one. As a side effect of the technique there will be some extra lines, which are not used in measurements.

Junction embedded in electric circuit



The junction behaves as a nonlinear inductance.
The circuit is very similar to that of a harmonic oscillator.

Metastable state of the junction



P=I*V, dissipated power,
normal metal behavior above the gap

Non-zero voltage state

$$\frac{d\gamma}{dt} = \frac{2\pi}{\Phi_0} V$$

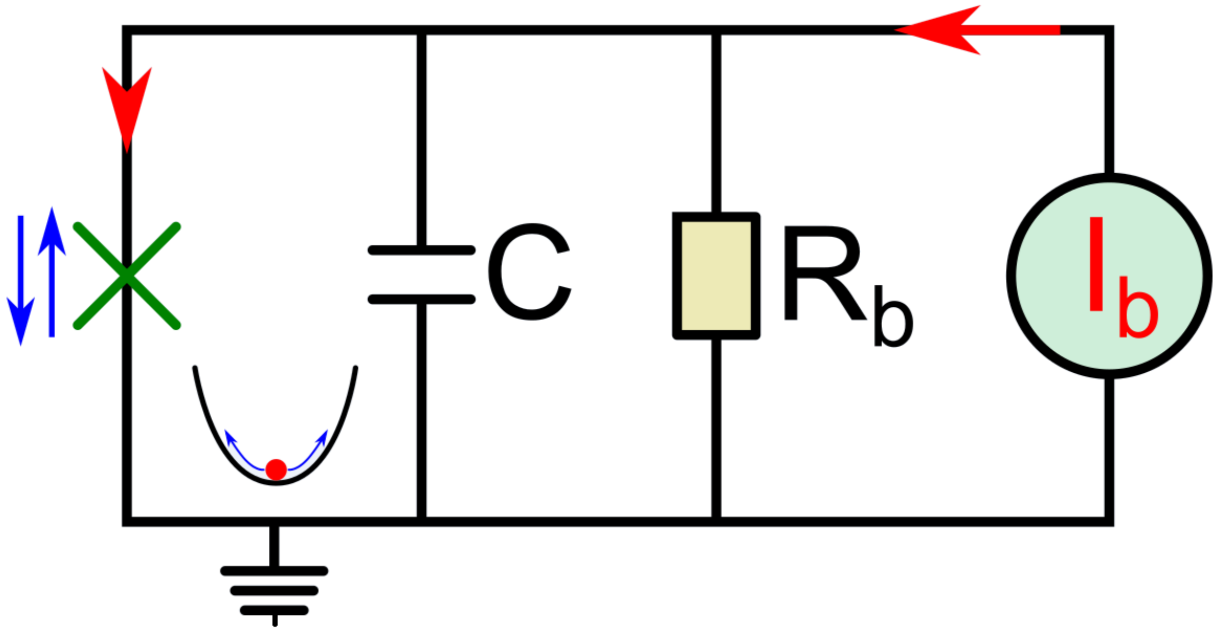
$$I_J = I_0 \sin\gamma$$

I flows, but V=0 =>P=0

due to tunnelling
of single Cooper pairs

RCSJ model

(Resistively and Capacitively Shunted Junction)



$$I_b = I_R + I_C + I_{JJ} = \frac{V}{R} + C\frac{dV}{dt} + I_0 \cdot \sin\gamma$$

RCSJ model

$$I_b = I_R + I_C + I_{JJ} = \frac{V}{R} + C\frac{dV}{dt} + I_0 \cdot \sin\gamma$$

2nd Josephson formula $\dot{\gamma} = \frac{1}{\varphi_0} \cdot V$

$$I_b = \frac{\varphi_0}{R}\dot{\gamma} + C\varphi_0\ddot{\gamma} + I_0 \sin\gamma$$

$$m = C\varphi_0{}^2 \quad , \quad \omega_0 = \left(\frac{I_0}{C \cdot \varphi_0}\right)^{1/2}, b = \frac{\varphi_0{}^2}{R}, Q_0 = \frac{\omega_0}{b/m} = RC\omega_0, k = \varphi_0 I_0 = E_J$$

$$\ddot{\gamma} + \frac{\omega_0}{Q_0}\dot{\gamma} + \omega_0{}^2\left(\sin\gamma - \frac{I_b}{I_0}\right) = 0$$

It looks like harmonic oscillator,

but now restoring force is not - kγ (as in the Hook's law), but:

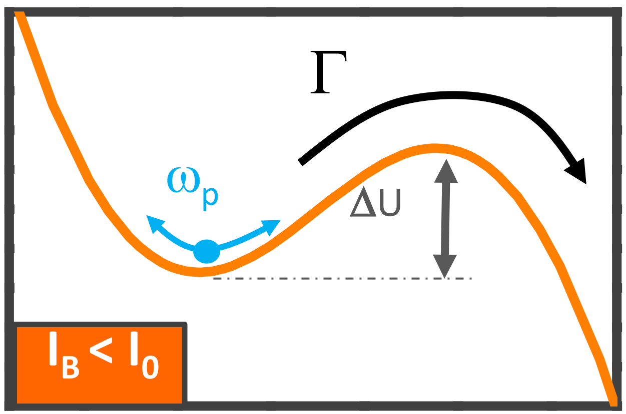
$$F = -k\left(\sin\gamma - \frac{I_b}{I_0}\right) = -\nabla E_p$$

$$E_p = +k\int\left(\sin\gamma - \frac{I_b}{I_0}\right)d\gamma = -E_J\left(\cos\gamma + \frac{I_b}{I_0}\gamma\right) \quad \textit{tilted washboard potential}$$

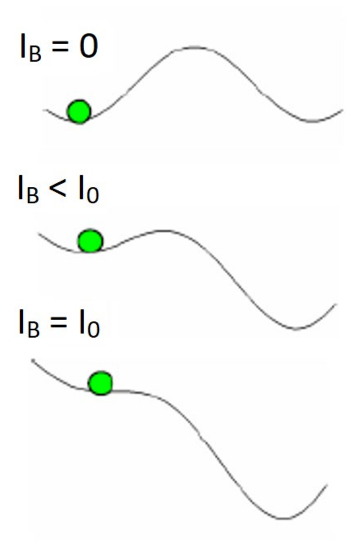
Dynamics of a fictious particle in a tilted washboard potential

$$E_p = -E_J(\cos\gamma + \frac{I_b}{I_0}\gamma)$$

The potential is tilted with the bias current.



γ- superconducting phase

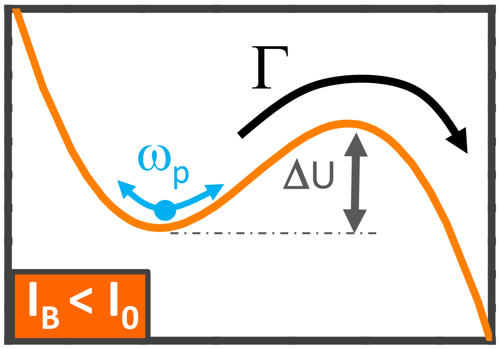


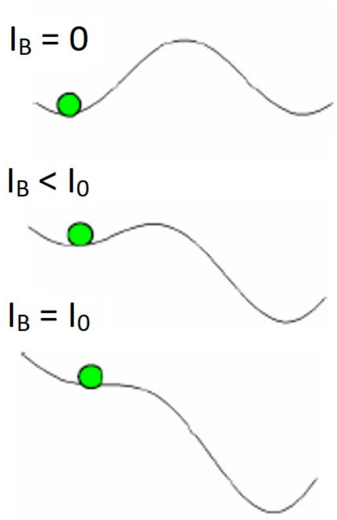
The oscillations of phase correspond to oscillating current across the junction.

$$I_J = I_0 \sin\gamma$$

M. Foltyn, M. Zgirski,
Gambling with Superconducting Fluctuations,
Phys. Rev. Applied 4, (2015)

Switching: Escape out of metastable state



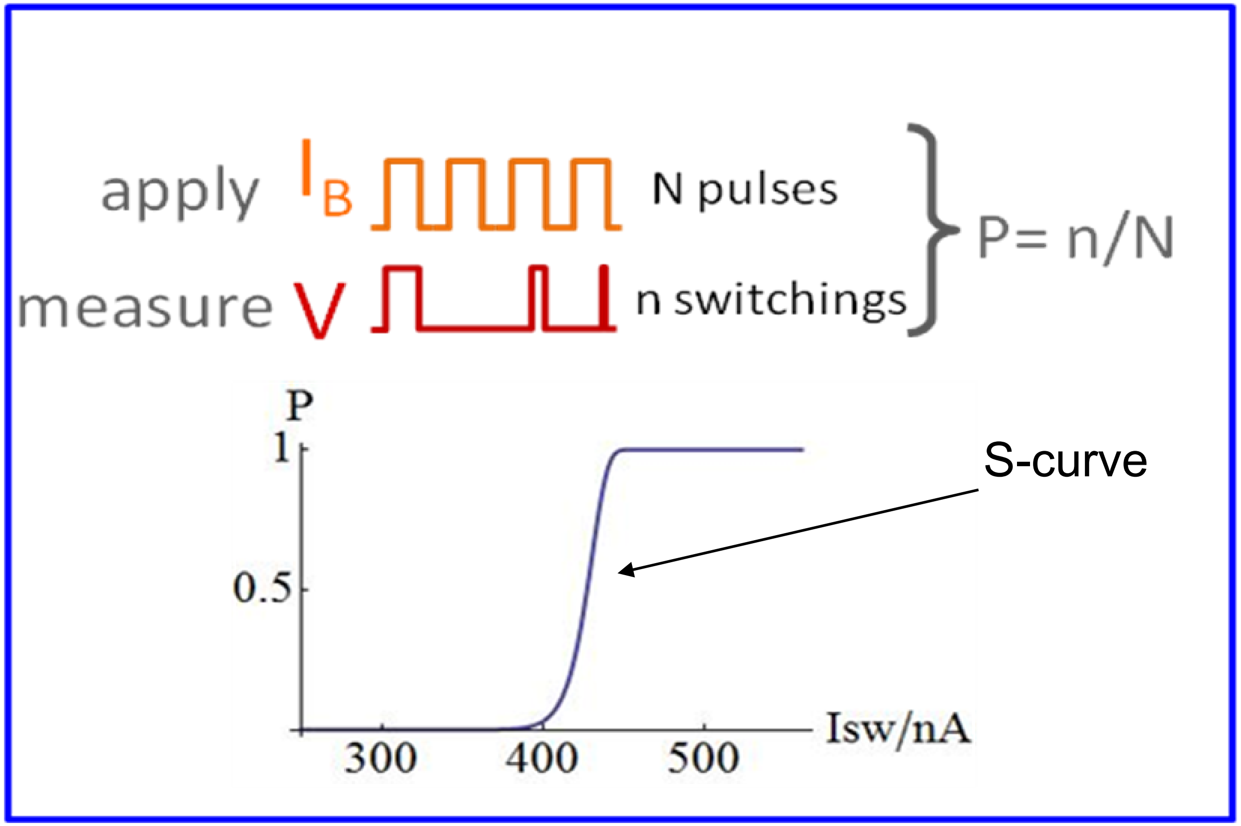


Escape rate Γ / lifetime in a metastable state τ:

$$\Gamma = \frac{\omega_p}{2 \cdot \pi} \exp(-\frac{\Delta U}{k_B T}), \quad Arrhenius \;\; law \qquad \frac{1}{\tau} = \Gamma$$

$$\Delta U(s) = \frac{4 \cdot \sqrt{2}}{3} \cdot E_J (1 - s)^{3/2}, \; s = \frac{I_b}{I_0}, \; \omega_p = \omega_{p0} (1 - s^2)^{1/4}$$

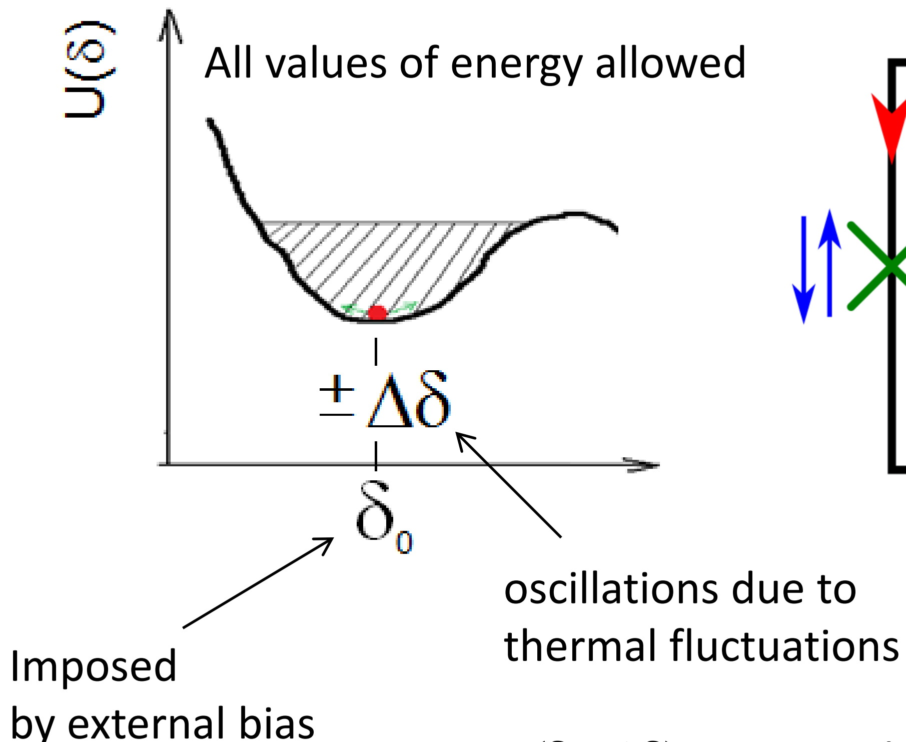
Extraction of the JJ lifetime τ from the switching probability P

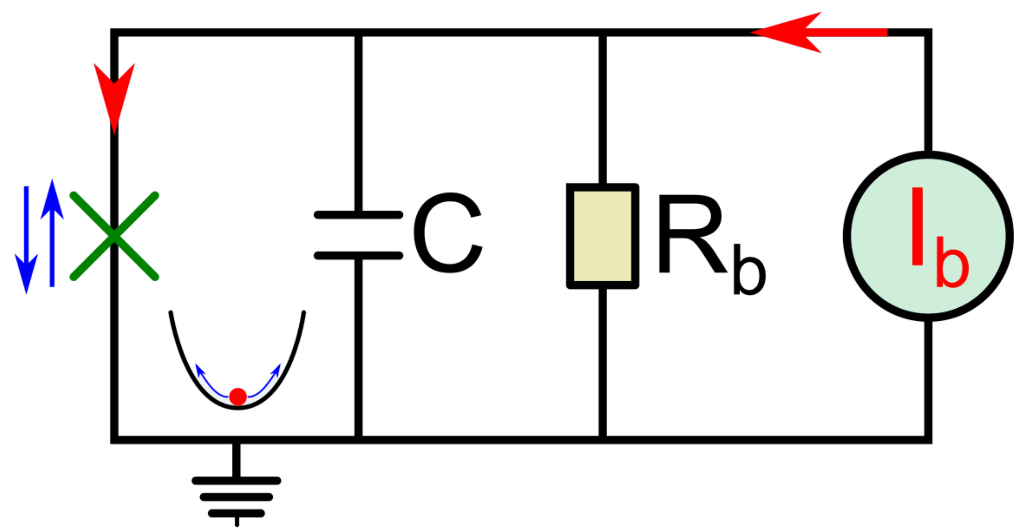


$$P = 1 - \exp(-\frac{T}{\tau}), \quad \frac{1}{\tau} = \Gamma$$

M. Zgirski, M. Foltyn, A. Savin, K. Norowski, *Flipping-Coin Experiment to Study Switching in Josephson Junctions and Superconducting Wires*, **Phys. Rev. Applied 11**, 054070 (2019)
M. Foltyn, M. Zgirski,
Gambling with Superconducting Fluctuations, **Phys. Rev. Applied 4**, 024002 (2015)

Junction as a classical oscillator

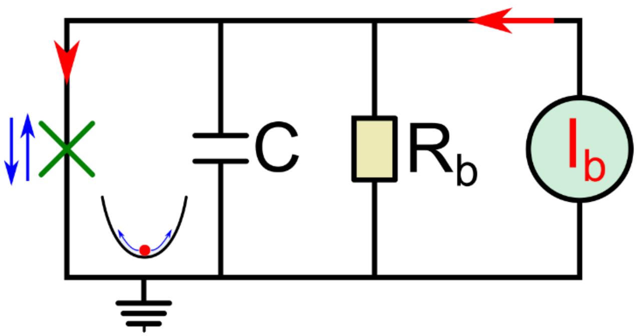




$I = I_0 \sin(\delta_0 + \Delta\delta)$ 1st Josephson relation

For small $\Delta\delta$: $I = I_0 \sin(\delta_0) + I_0 \cos(\delta_0)\Delta\delta$ => $I = I_b + (I_0^2 - I_b^2)^{0.5}\Delta\delta$

Escape in a resonance



$\tau(0)$ – JJ lifetime in a metastable state

$\tau(P)$ – JJ lifetime in a metastable state in the presence of microwaves

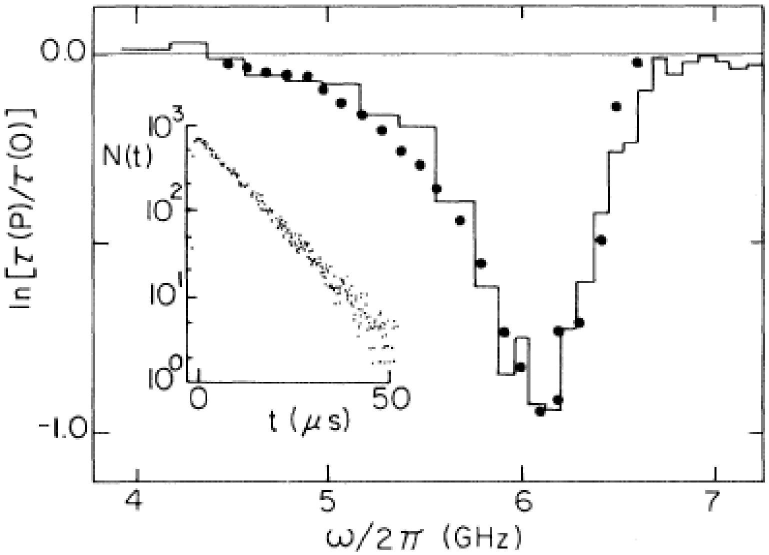
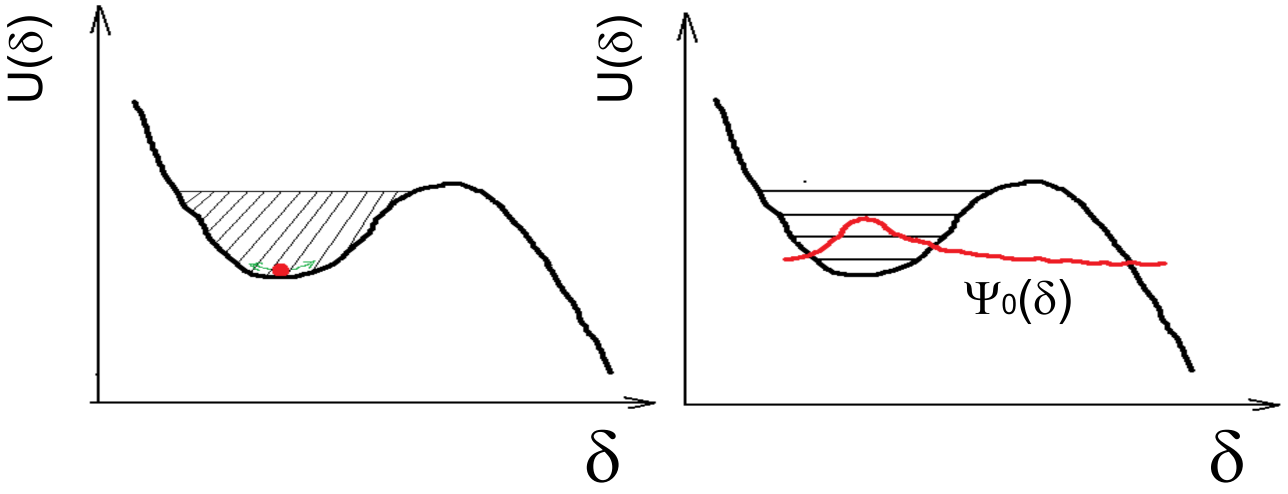


FIG. 3. Resonance in escape time vs microwave frequency. Dots represent measured values of $\ln[\tau(P)/\tau(0)]$ for a junction at 4.2 K with $I_0 = 4.64\ \mu$A, $I = 3.07\ \mu$A, and $\tau(0) = 8.4\ \mu$s. Solid line represents results of numerical simulation. Inset shows exponential distribution of switching events for the same junction in the absence of microwaves.

M.H.Devoret, J.M.Martinis, D.Esteve and J.Clarke, *Resonant Activation from the Zero-Voltage State of a Current-Biased Josephson Junction,* Phys. Rev. Lett. **53**, 1260 (1984)

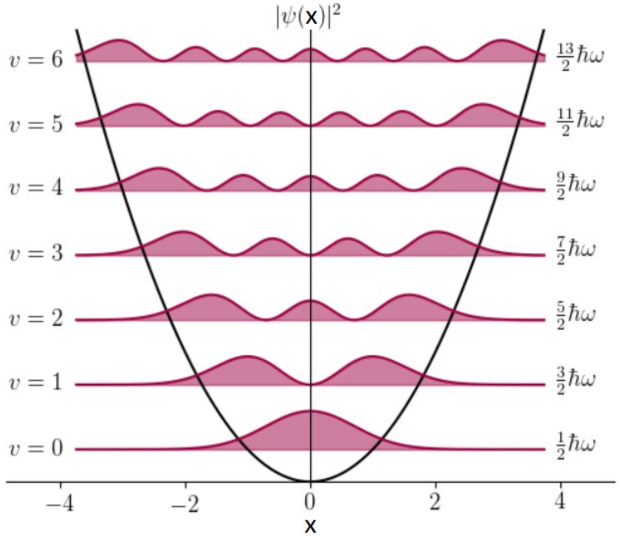
JJ – artificial atom?



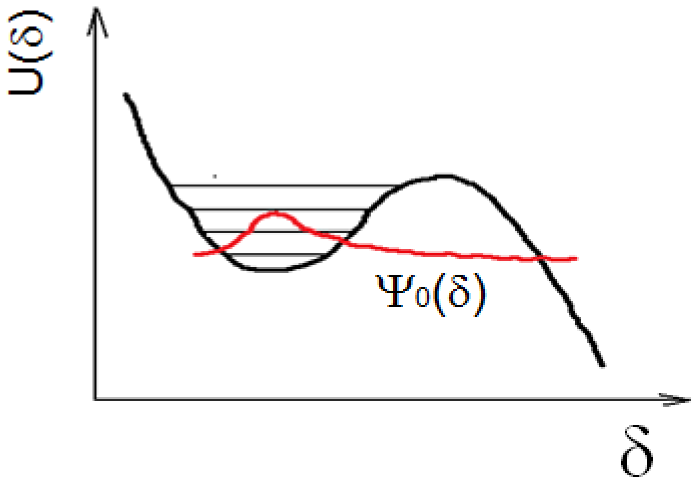
- Can the wavefunction escape from a local minimum by tunneling?
- Is energy of charge oscillations on a JJ quantized in analogy to a quantum harmonic oscillator?

Delocalization of the electrical current value

Quantum harmonic oscillations in a real space:



Quantum oscillations of phase:

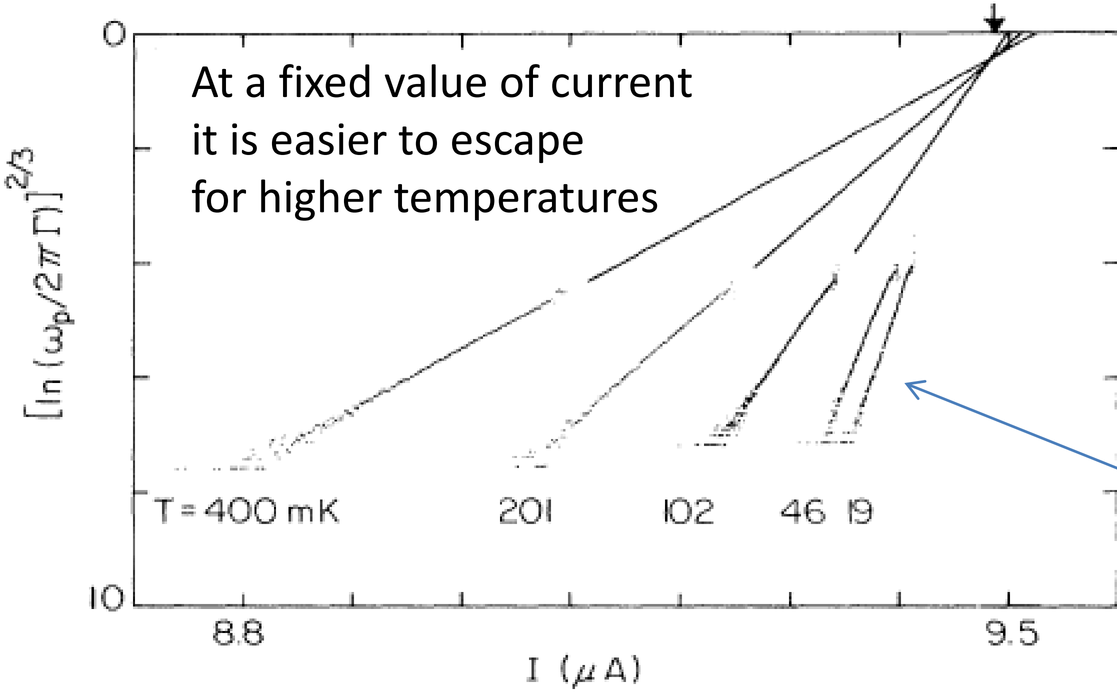


Delocalization of phase means blurring of the macroscopic current flowing through the junction!!!

=> Quantum Electrical Circuit !!!

Escape rate vs. temperature

I_0-critical current

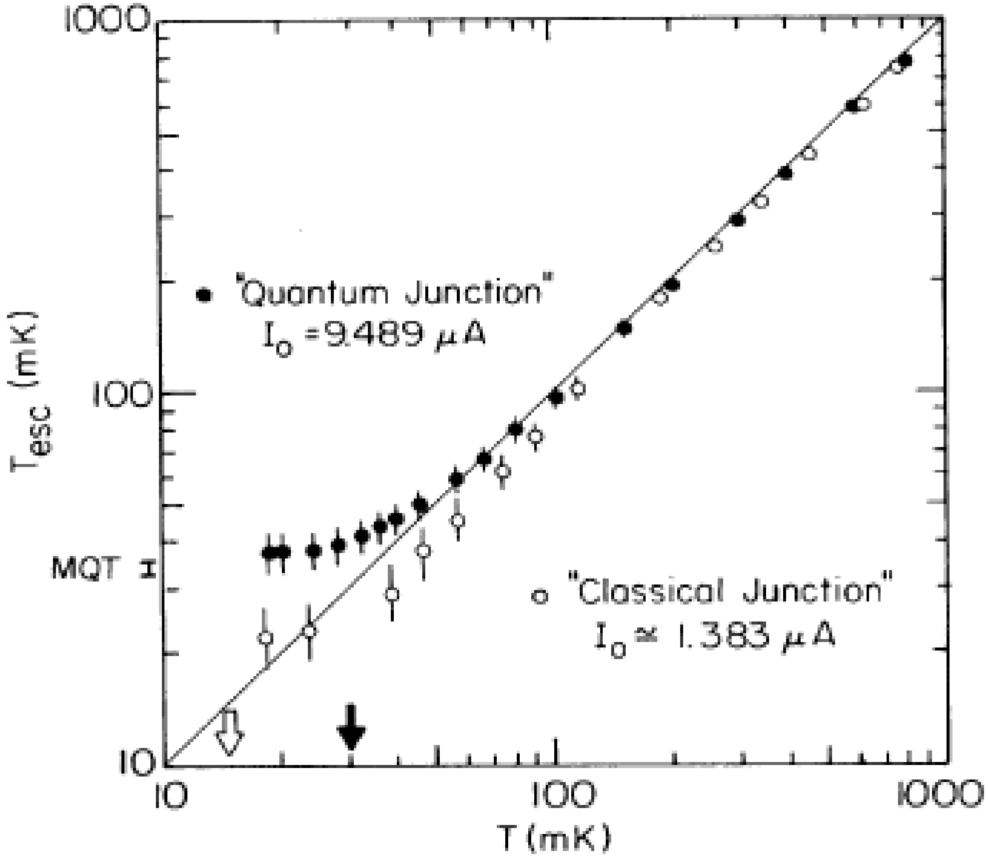


Temperature of electromagnetic environment can be extracted from the slopes

not temperature dependent

FIG. 1. $[\ln(\omega_p/2\pi\Gamma)]^{2/3}$ vs I for five values of temperature. Lines that intersect the current axis have been drawn through the data in the thermal regime, at the three highest temperatures. The arrow indicates the value of I_0 obtained after corrections for the prefactor were made.

M.H.Devoret, J.M.Martinis, J.Clarke*, Measurements of Macroscopic Quantum Tunnelling of the Zero-Voltage State of a Current-Biased Josephson Junction,* Phys.Rev.Lett. **55**, 1908 (1985)



$$\Gamma = \frac{\omega_p}{2\cdot\pi}\exp\left(-\frac{\Delta U}{k_B T_{esc}}\right)$$

$T_{esc} > T$

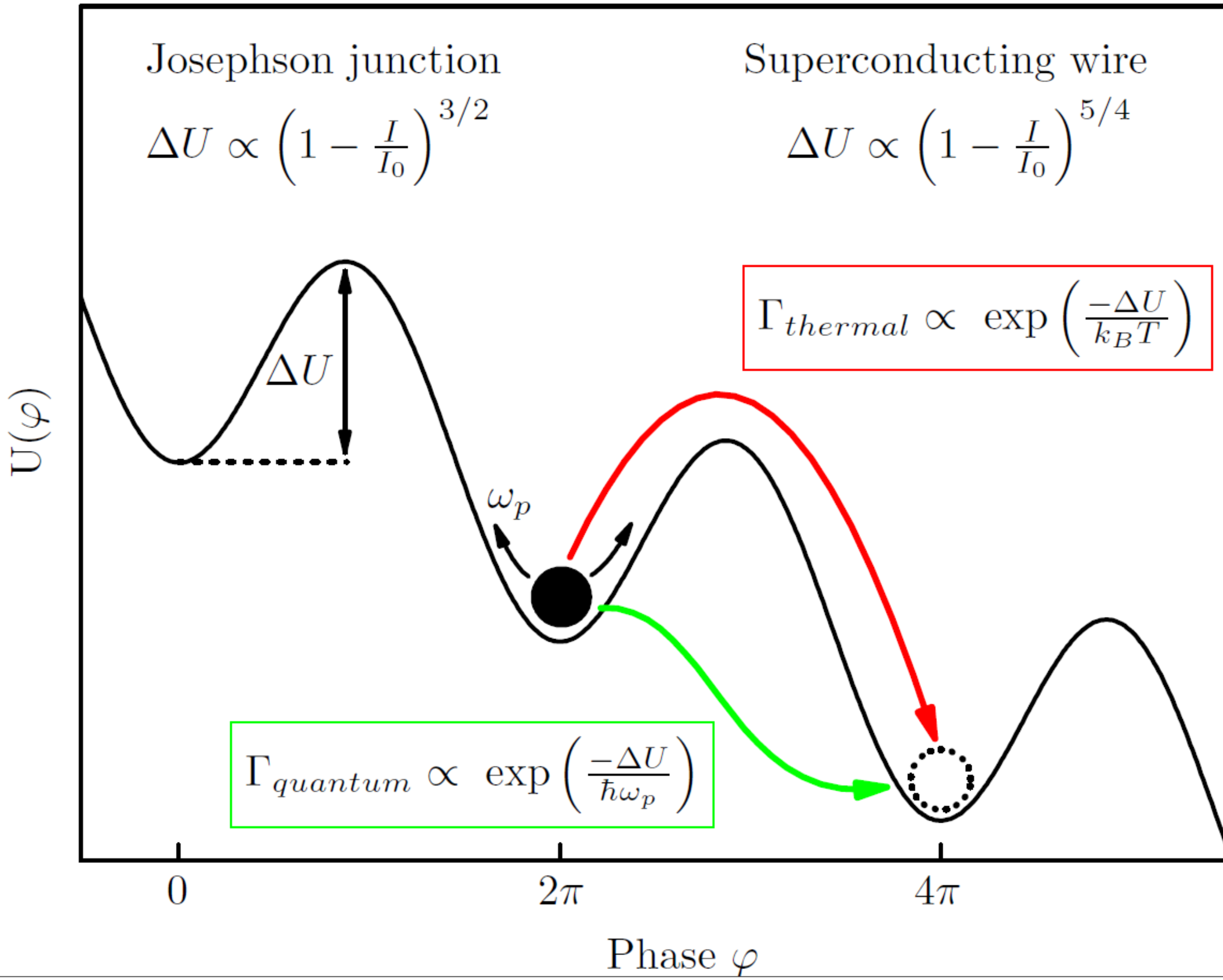
The temperature of the JJ decay is larger than temperature of the electromagnetic environment.

A new escape mechanism !!!

FIG. 2. T_{esc} vs T for two values of critical current for $\ln(\omega_p/2\pi\Gamma)=11$. The solid and open arrows indicate the predicted crossover temperatures for the higher and lower critical currents, respectively. The prediction of Eq. (5) for the higher critical current is indicated at the left.

M.H.Devoret, J.M.Martinis, J.Clarke, *Measurements of Macroscopic Quantum Tunnelling of the Zero-Voltage State of a Current-Biased Josephson Junction,* Phys.Rev.Lett. **55**, 1908 (1985)

Escape from the tilted washbord potential

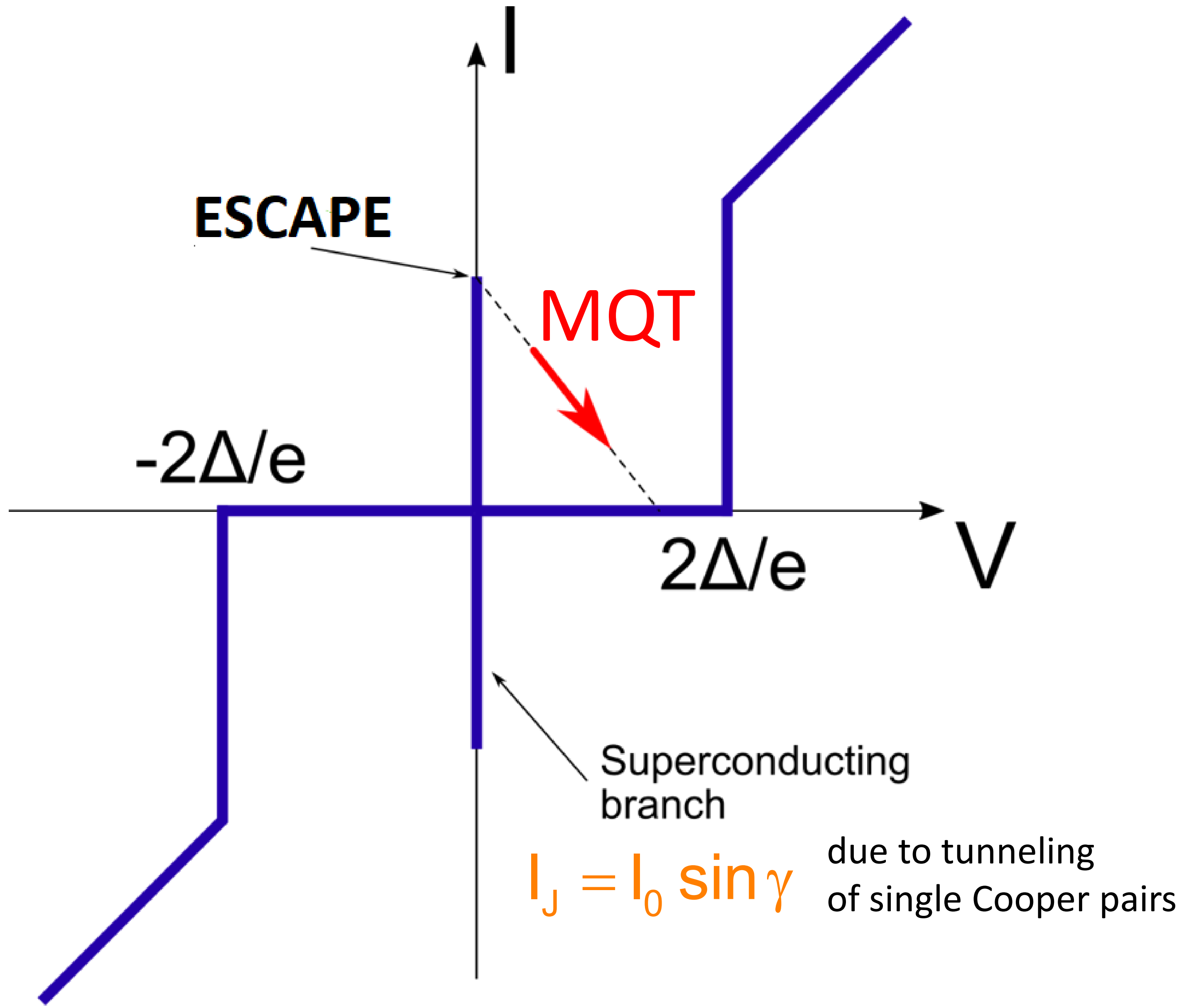


M. Foltyn, M. Zgirski, *Gambling with Superconducting Fluctuations*, **Phys. Rev. Applied 4**, (2015)

Important distinction

- MQT is NOT a tunneling of many Cooper pairs simultanously across a tunneling barrier defined in a real space.

- MQT is a „switching" of the collective state of many Cooper pairs between two macroscopic wavefunctions, although the two configurations are separated by an energy barrier which forbids the classical transition.



I
ESCAPE
MQT
-2Δ/e
2Δ/e
V
Superconducting branch
$I_J = I_0 \sin\gamma$
due to tunneling
of single Cooper pairs

Quantized energies in the trapping potential ?

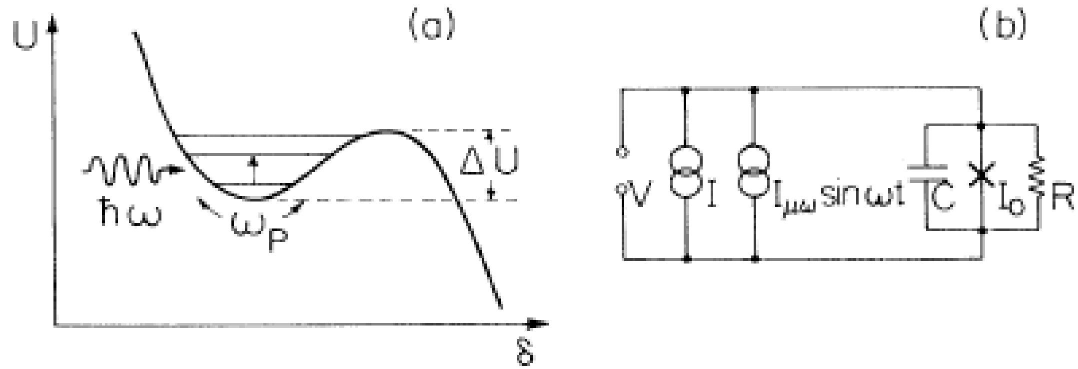
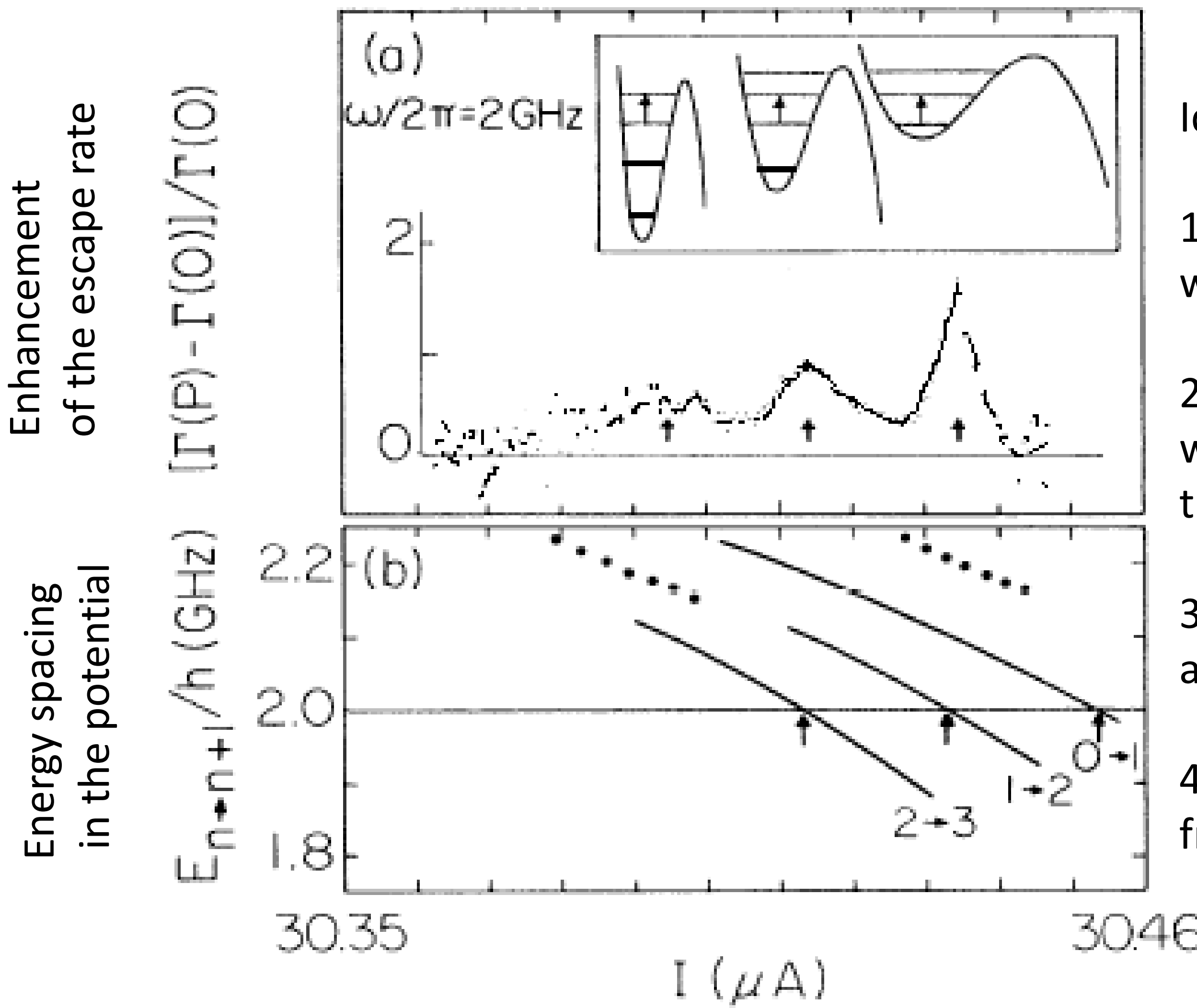


FIG. 1. (a) Cubic potential U vs phase difference δ showing three energy levels. Transition from the ground state to the first excited state induced by a photon of frequency $\omega/2\pi$ is shown. (b) Model of current-biased Josephson junction loaded with a resistor and irradiated with an external microwave current source.

J.M.Martinis, M.H.Devoret, J.Clarke*, Energy-Level Quantization in the Zero-Voltage State of a Current-Biased Josephson Junction,* Phys.Rev.Lett. **55**, 1543 (1985)

Quantized energies in the trapping potential – resonant activation



Idea:

1. Irradiate junction with a microwave photon

2. Adjust the level spacing with the bias current to match the energy of the photon

3. Force the transition to a higher energy level

4. Observe easier escape from the higher level

Experimental evidence – modification of the 0->1 energy spacing

Idea:

1. Tilt the potential with current

2. For a larger tilt potential is more „open”
$\Rightarrow$ energy spacing is smaller

3. Excite the circuit with a resonant microwave photon

4. Observe enhancement of the escape due to excitation of the current oscillations to the higher level

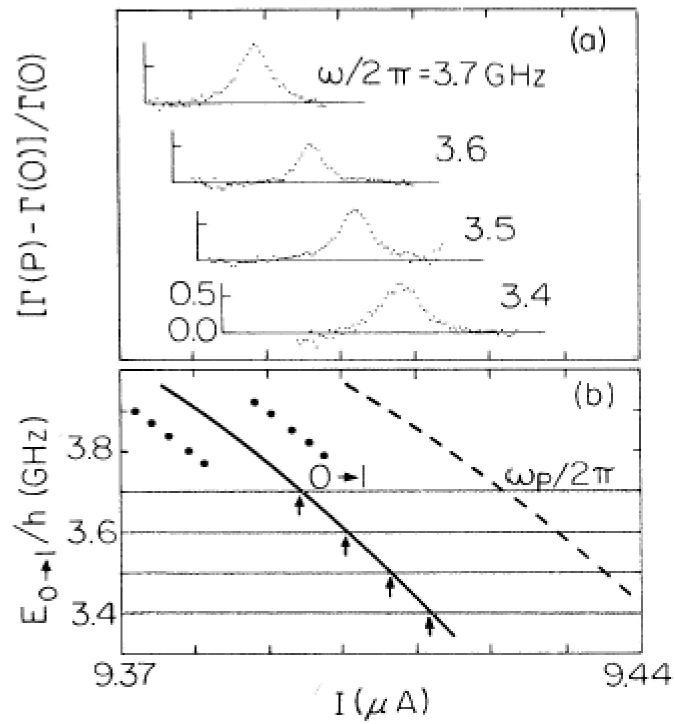


FIG. 3. (a) $[\Gamma(P)-\Gamma(0)]/\Gamma(0)$ vs I for a 10×10-μm^2 junction at 18 mK for four microwave frequencies. (b) Calculated energy-level spacing $E_{0\rightarrow 1}$ vs I for $I_0 = 9.489 \pm 0.007$ μA and $C = 6.35 \pm 0.4$ pF. Dotted lines indicate uncertainties due to errors in I_0 and C. Arrows indicate values of bias current at which resonances are predicted. Dashed line indicates plasma frequency.

Phase Qubit – How to tell the ground state from the excited state?

John Martinis, *Superconducting Phase Qubits*, Quantum Inf Process (2009) 8:81–103

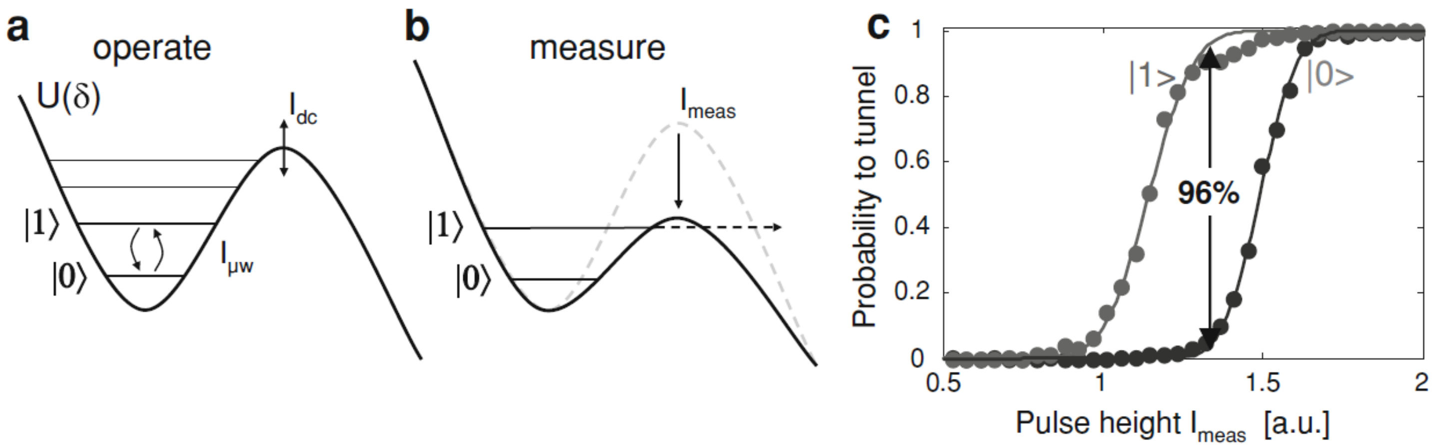
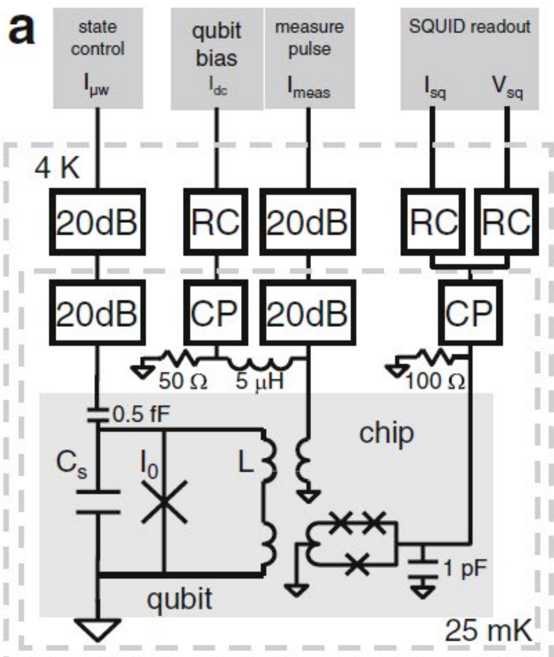
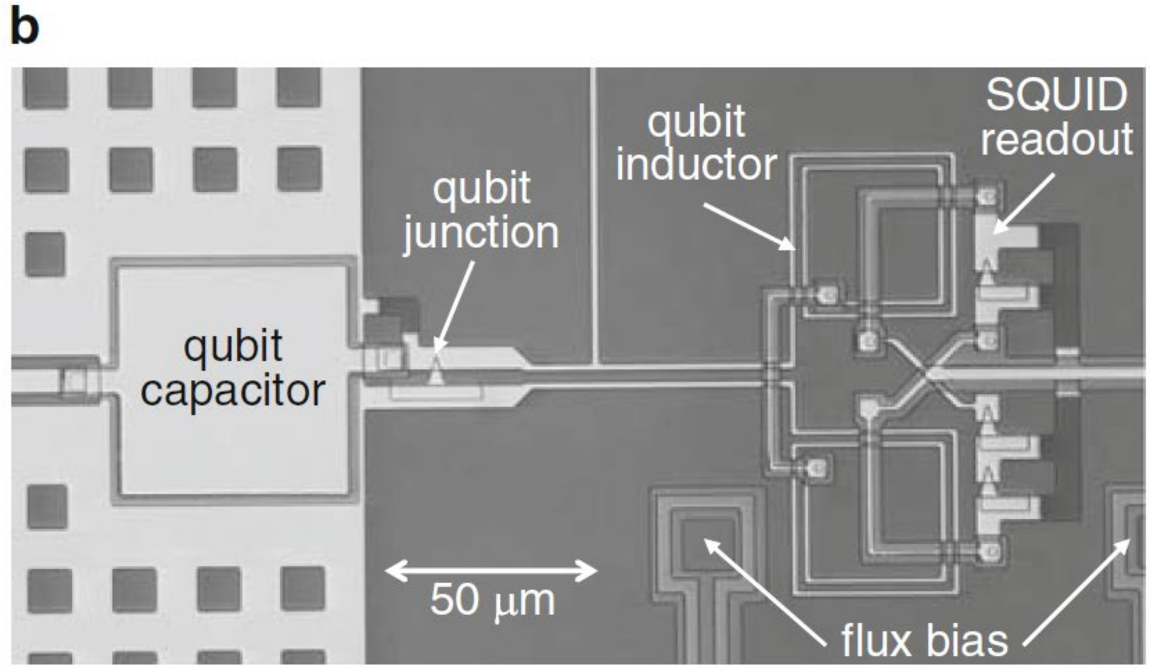


Fig. 1 **a** Plot of non-linear potential $U(\delta)$ for the Josephson phase qubit. The qubit states $|0\rangle$ and $|1\rangle$ are the two lowest eigenstates in the well. The junction bias I_{dc} is typically chosen to give 3–7 states in the well. Microwave current $I_{\mu w}$ produces transitions between the qubit states. **b** Plot of potential during state measurement. The well barrier is lowered with a bias pulse I_{meas} so that the $|1\rangle$ state can rapidly tunnel. **c** Plot of tunneling probability versus I_{meas} for the states $|0\rangle$ and $|1\rangle$. The arrow indicates the optimal height of I_{meas}, which gives a fidelity of measurement close to the maximum theoretical value 96%

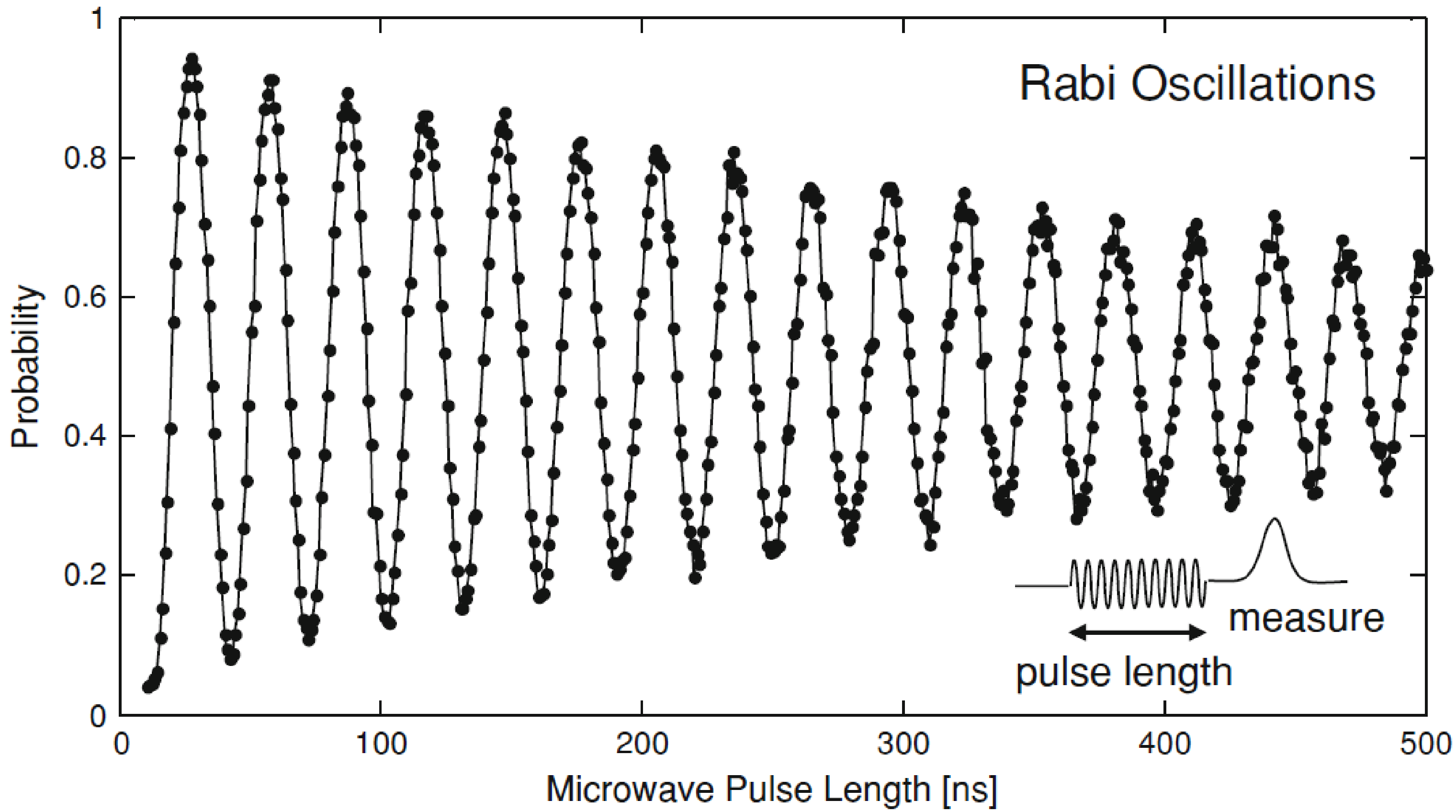
Phase Qubit (2)

John Martinis, *Superconducting Phase Qubi*ts, Quantum Inf Process (2009) 8:81–103



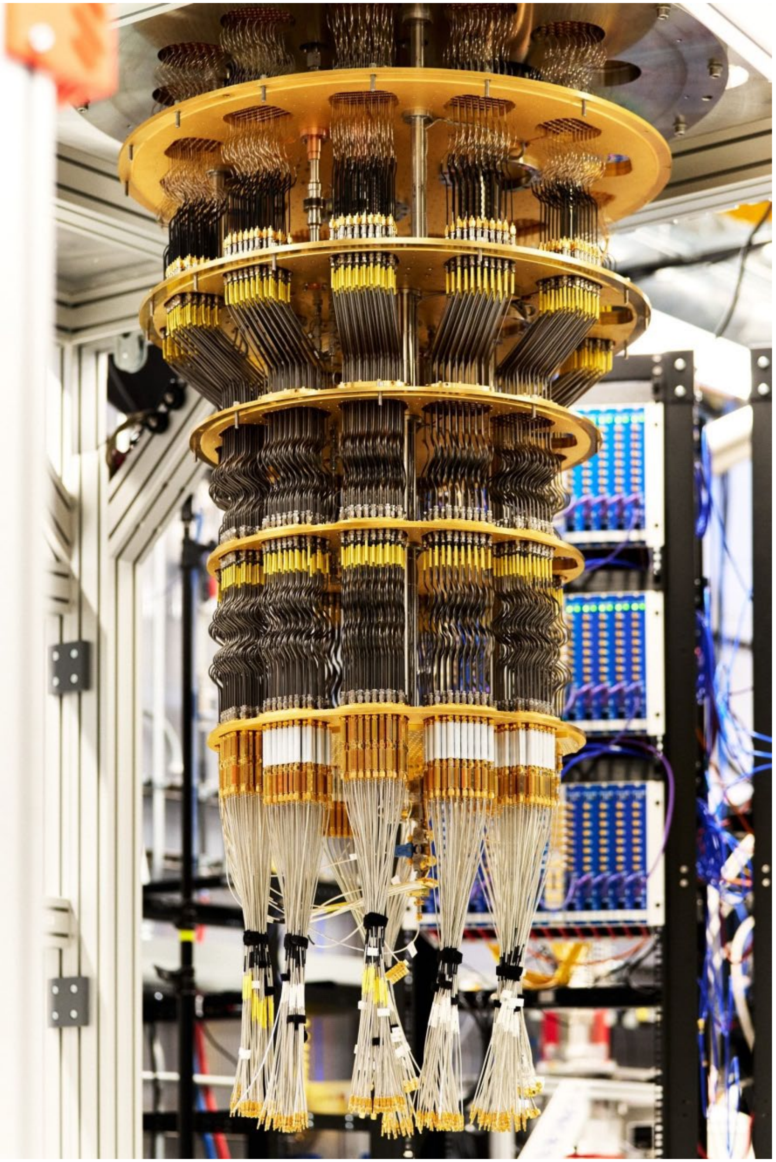


Phase Qubit (3)

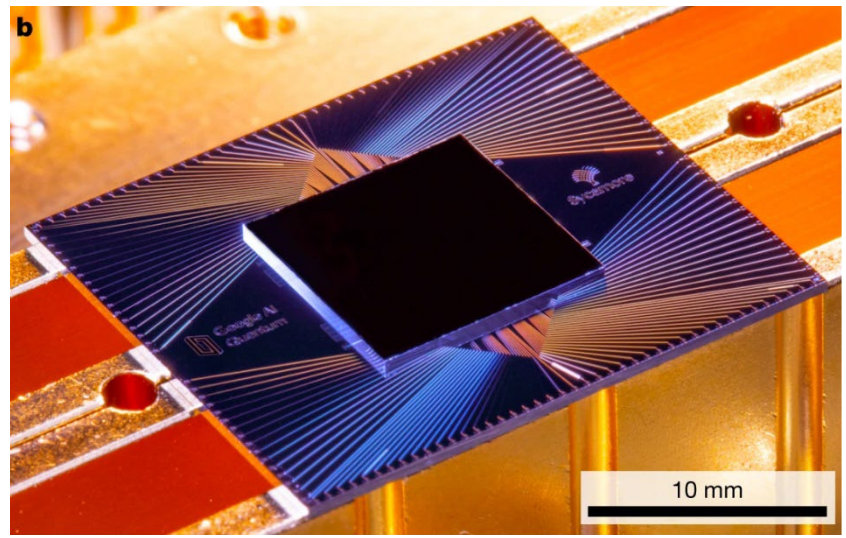


John Martinis, *Superconducting Phase Qubits*, Quantum Inf Process (2009) 8:81–103

A look ahead

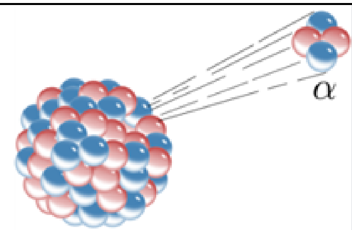
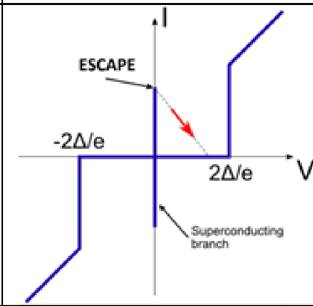
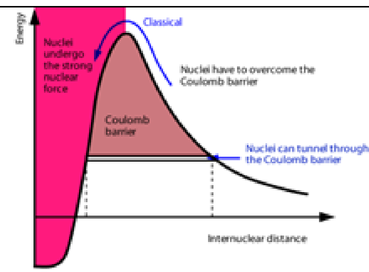
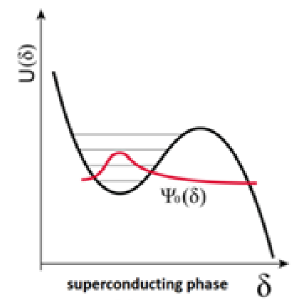
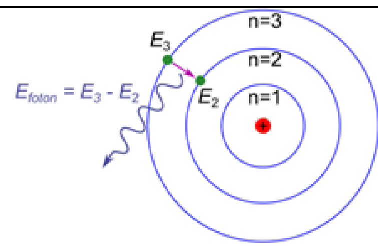
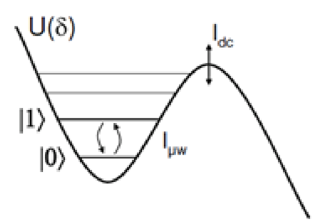
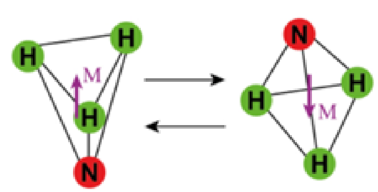
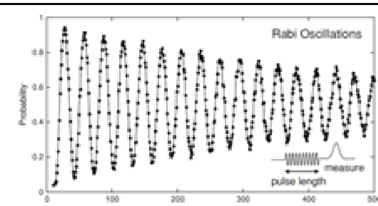


Google's Quantum Computer,
October 2025

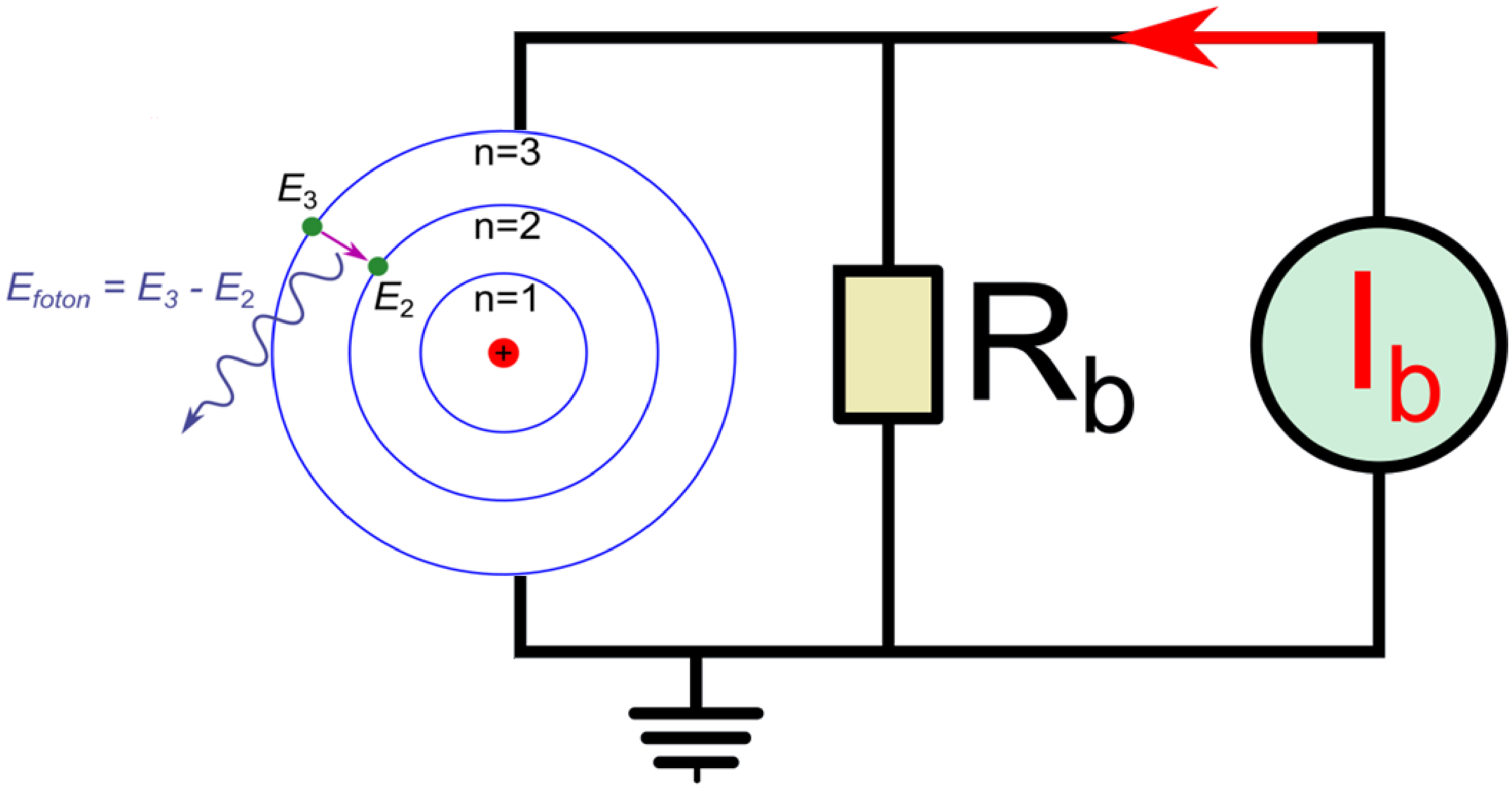


Arute, F., Arya, K., Babbush, R.,… **John Martinis**
Quantum supremacy using a programmable superconducting processor,
Nature 574, 505–510 (2019)

Recap

	Atom/Elementary particle	JJ – artificial atom	Notes
Decay			For JJ - Escape tunable with applied current, temperature, magnetic field !!!
Tunneling			For JJ tunneling involves simultaneous change in a superconducting phase for many Cooper pairs
Energy quantization			Energy spacing tunable with applied current, magnetic field and circuitry in which a JJ is embedded
Superposition			Coherent evolution between JJ energy levels is a basis for a superconducting qubit

Artificial Atoms for Quantum Electronics



http://coolphongroup.ifpan.edu.pl

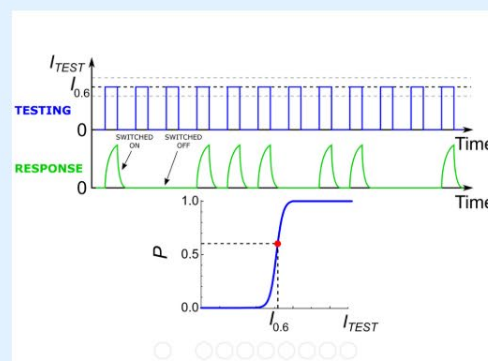
http://coolphongroup.ifpan.edu.pl

Presentations Team Lab Teaching Accomplishments

Welcome to CoolPhon Group

We are building and testing pioneering low temperature functional nanostructures with a special emphasis on superconducting devices. Our expertise involves fast time-resolved thermometry of nanostructures based on the probing of Josephson junctions with short current pulses, which we call the switching thermometry. It allows us to trace the thermal transients with a temporal resolution of a single nanosecond, which appear in the response to a local dissipation. We can directly investigate the dynamics of various energy relaxation channels at the nanoscale. Our second field of interest is vortex electronics. It involves building nanodevices for controlling and manipulating of superconducting vortices to present new functionalities e.g. memory cells and logical elements for application in quantum computing.

See our publications

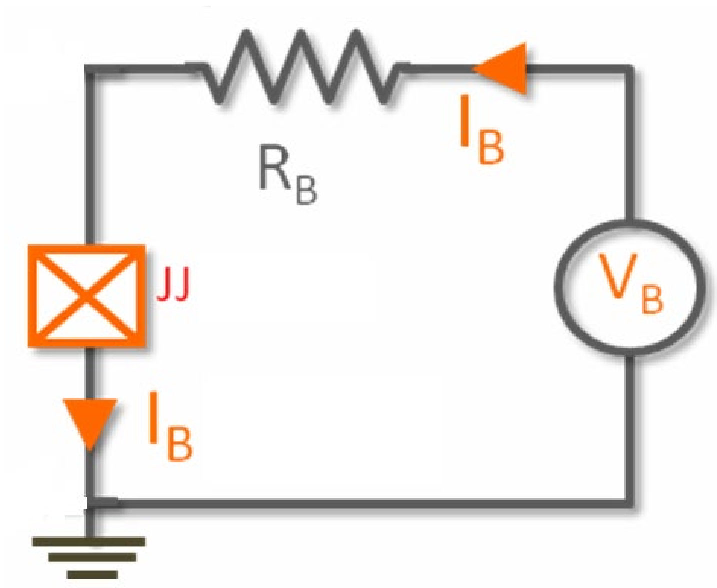


RCSJ model

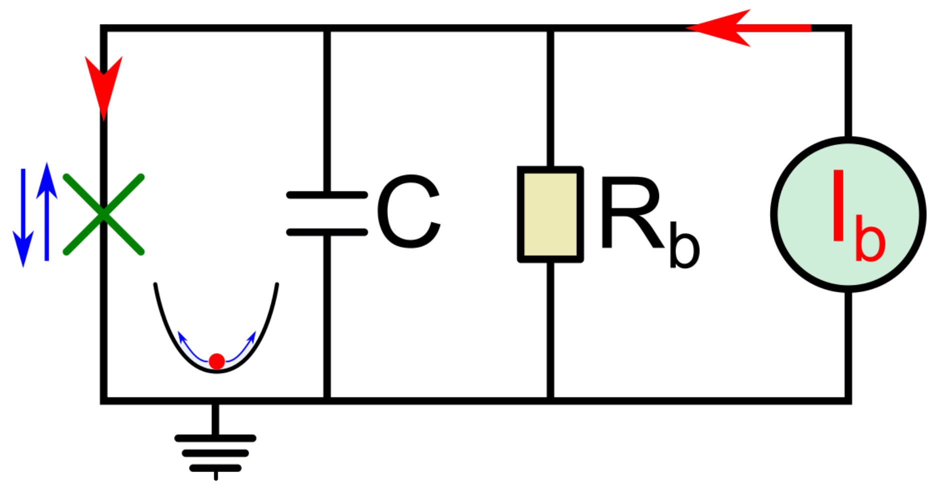
(Resistively and Capacitively Shunted Junction)

Thevenin equivalent

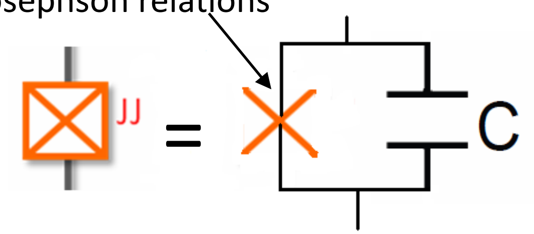
Norton equivalent



⇔

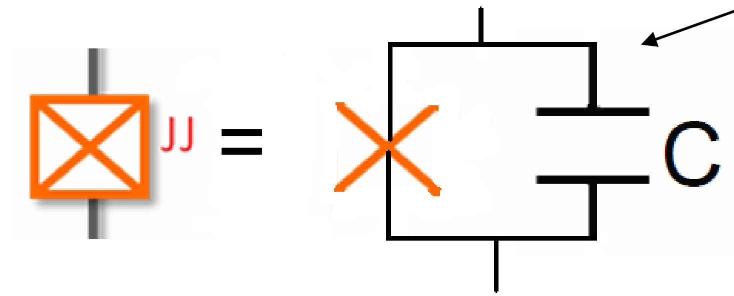


Pure Josephson element obeying Josephson relations



$$I_b = I_R + I_C + I_{JJ} = \frac{V}{R} + C\frac{dV}{dt} + I_0 \cdot \sin\gamma$$

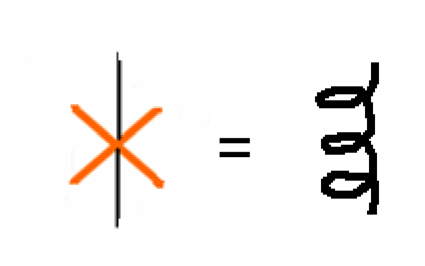
JJ is a nonlinear inductance



Anharmonic quantum oscillator,

=>

Not equal distance between energy levels



L nonlinear

$$V = L\frac{dI}{dt}$$ **linear inductance**

$$V = L(I)\frac{dI}{dt}$$ **nonlinear inductance**

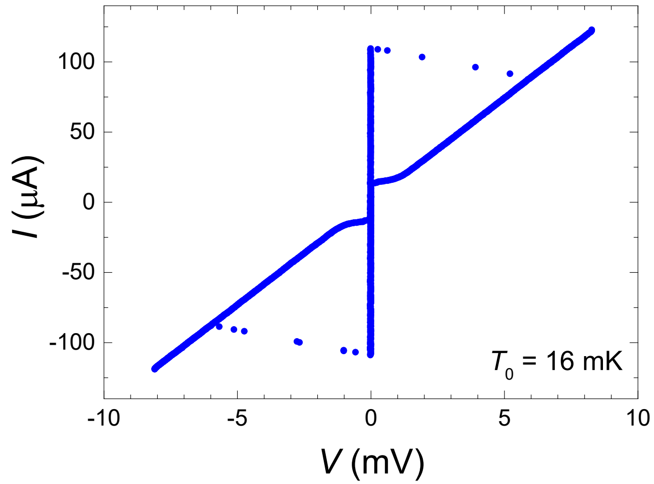
What does

a nonlinear inductance mean?

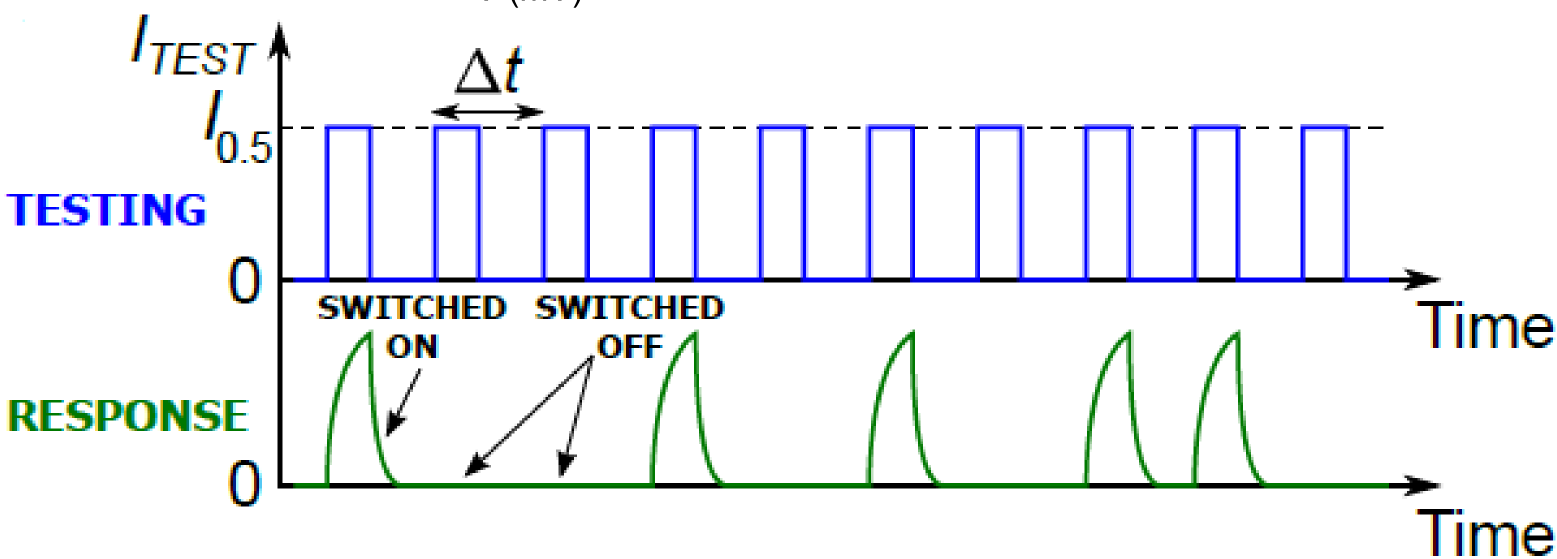
$$L(\delta) = \frac{\varphi_0}{I_0 \cos(\delta)}$$

Reflects kinetic inertia of Cooper pairs accelerated through the junction

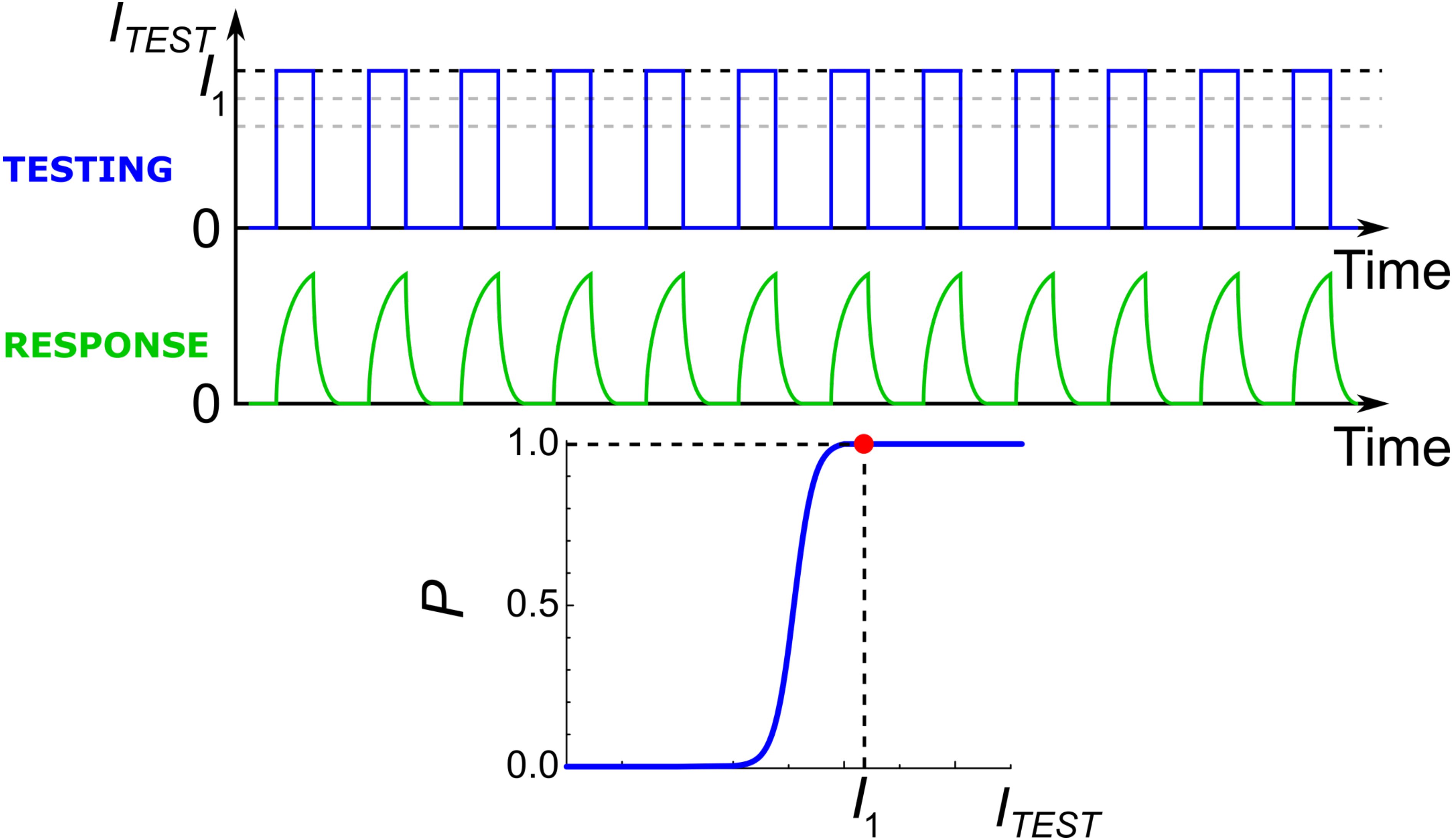
Switching measurements of the junction



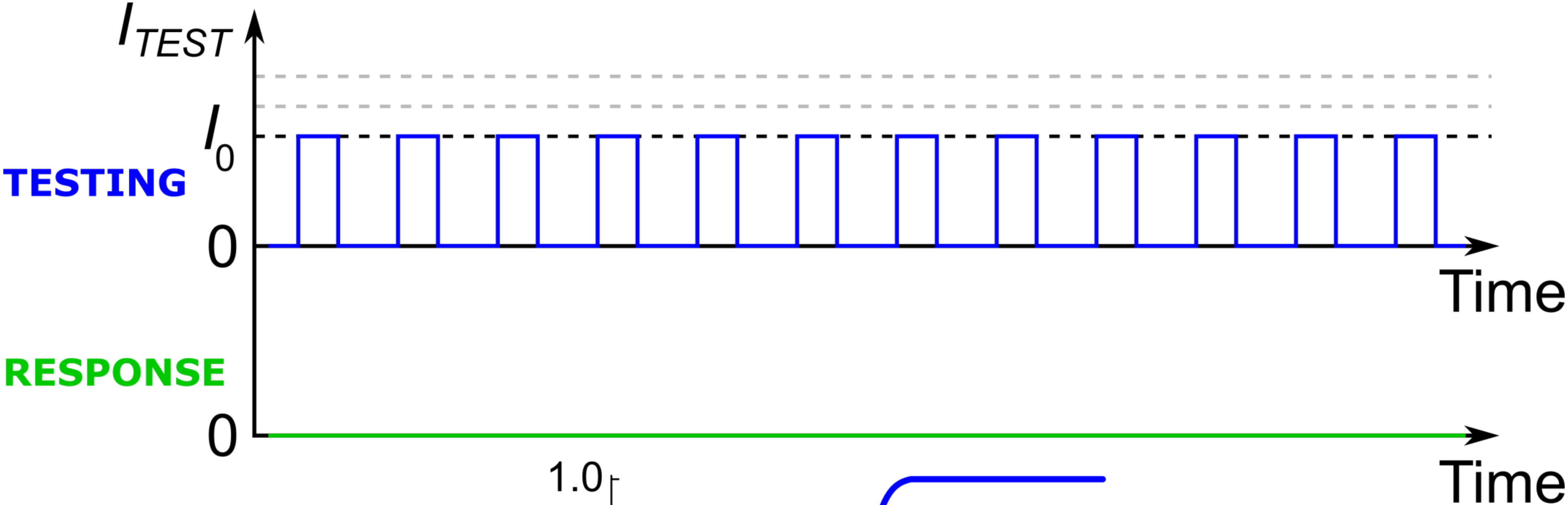
P=n/N

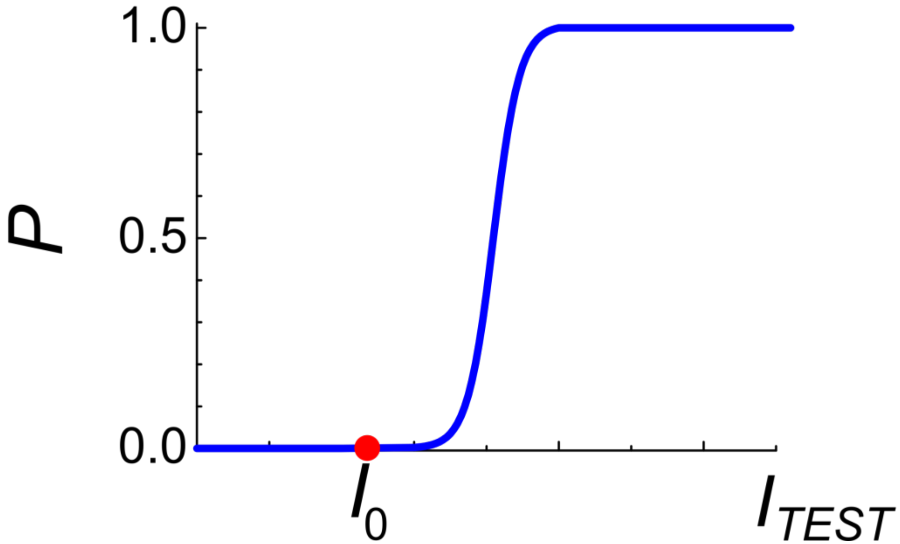


Testing JJ with current pulses

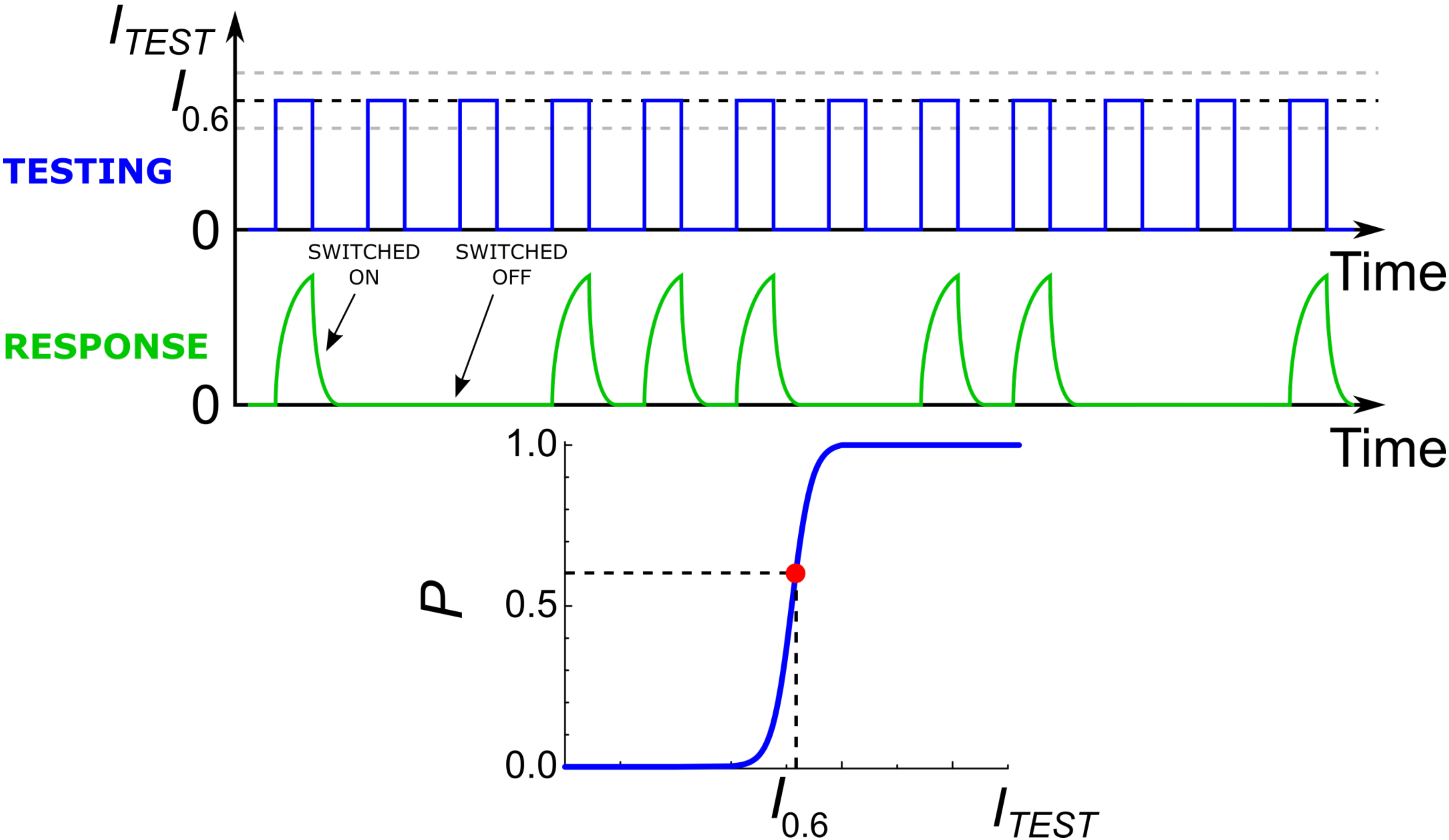


Testing JJ with current pulses





Testing JJ with current pulses



RCSJ model

$$I_b = I_R + I_C + I_{JJ} = \frac{V}{R} + C\frac{dV}{dt} + I_0 \cdot \sin\gamma$$

$$\dot{\gamma} = \frac{1}{\varphi_0} \cdot V$$

$$I_b = \frac{\varphi_0}{R}\dot{\gamma} + C\varphi_0\ddot{\gamma} + I_0 \sin\gamma$$

Harmonic oscillator:

$$m\ddot{x} + b\dot{x} + kx = 0$$

$$E(t) = E_0 e^{-\gamma t} (energy\ of\ damped\ harmonic\ oscillator)$$

$$\gamma = \frac{b}{m}, \qquad Q = \frac{\omega_0}{\gamma}, \qquad -kx = -\nabla E_p$$

$$\ddot{x} + \frac{\omega_0}{Q}\dot{x} + \omega_0^{\ 2} x = 0$$

Q - quality factor, amplitude of harmonic oscillator falls by a factor of e in Q/π cycles of free oscillations

First, consider γ->0, I_B=0 and map it into harmonic oscillator:

$$C\varphi_0^{\ 2}\ddot{\gamma} + \frac{\varphi_0^{\ 2}}{R}\dot{\gamma} + \varphi_0 I_0 \gamma = 0$$

$$m = C\varphi_0^{\ 2}\ , \ \omega_0 = \left(\frac{I_0}{C \cdot \varphi_0}\right)^{1/2}, b = \frac{\varphi_0^{\ 2}}{R}, Q_0 = \frac{\omega_0}{b/m} = RC\omega_0, k = \varphi_0 I_0 = E_J$$

Back to full equation:

$$\ddot{\gamma} + \frac{\omega_0}{Q_0}\dot{\gamma} + \omega_0^{\ 2}\left(\sin\gamma - \frac{I_b}{I_0}\right) = 0$$

$$\ddot{\gamma} + \frac{\omega_0}{Q_0}\dot{\gamma} + \omega_0{}^2(\sin\gamma - \frac{I_b}{I_0}) = 0$$

It looks like harmonic oscillator, but now restoring force is not - kγ (as in the Hook's law), but:

$$F = -k(\sin\gamma - \frac{I_b}{I_0}) = -\nabla E_p$$

$$E_p = +k\int(\sin\gamma - \frac{I_b}{I_0})d\gamma = -E_J(\cos\gamma + \frac{I_b}{I_0}\gamma) \quad \textit{tilted washboard potential}$$

Classical limit

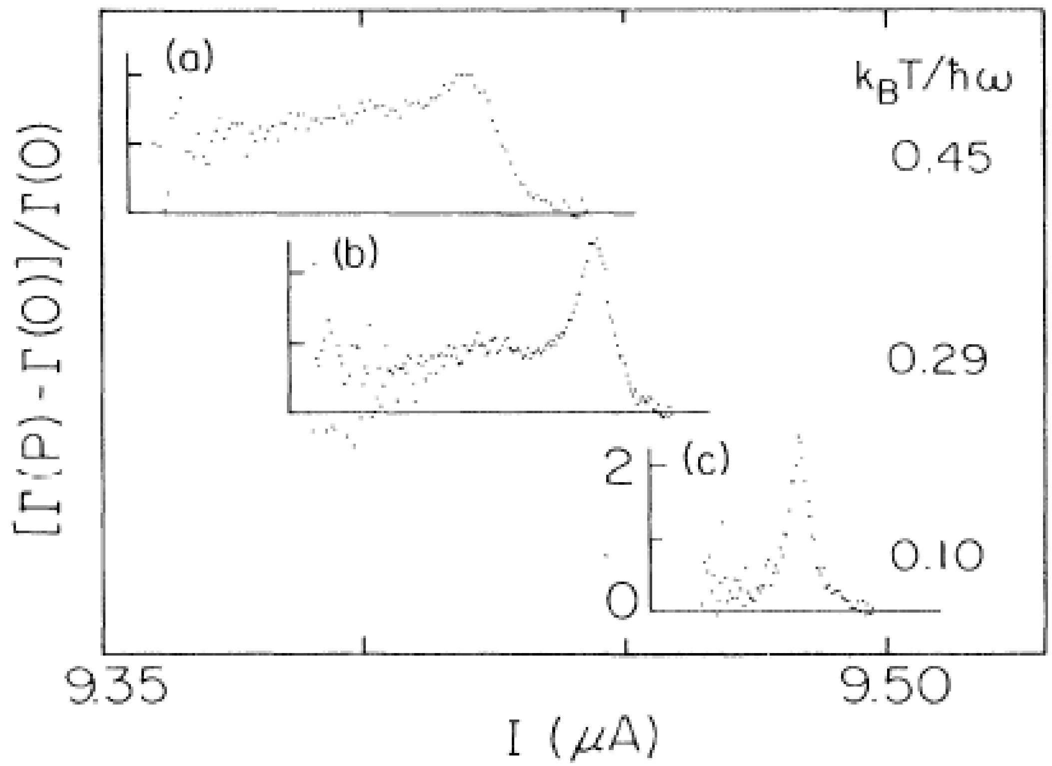


FIG. 4. $[\Gamma(P)-\Gamma(0)]/\Gamma(0)$ vs I for the junction of Fig. 3 with $I_0=9.57$ μA and $C\approx 6.35$ pF at three values of $k_BT/\hbar\omega$. The microwave frequencies are (a) 4.5 GHz, (b) 4.1 GHz, and (c) 3.7 GHz.

Escape rate vs. temperature

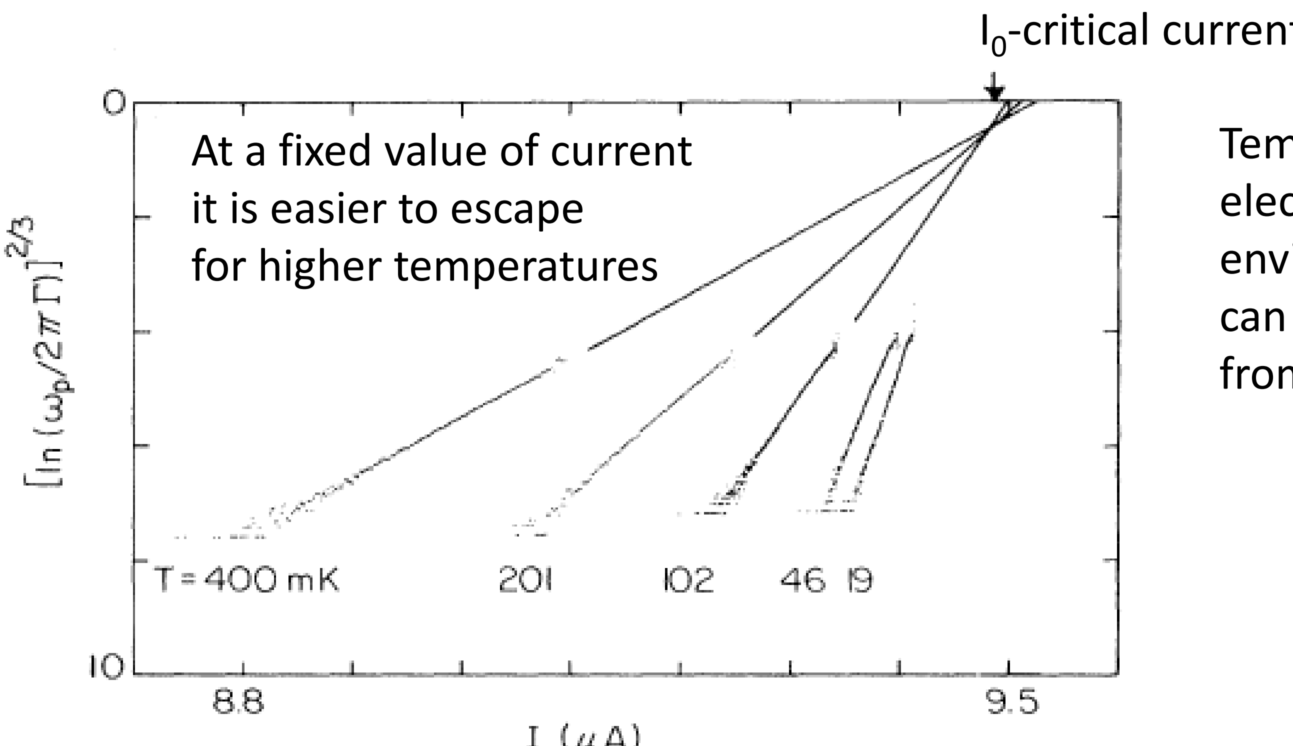
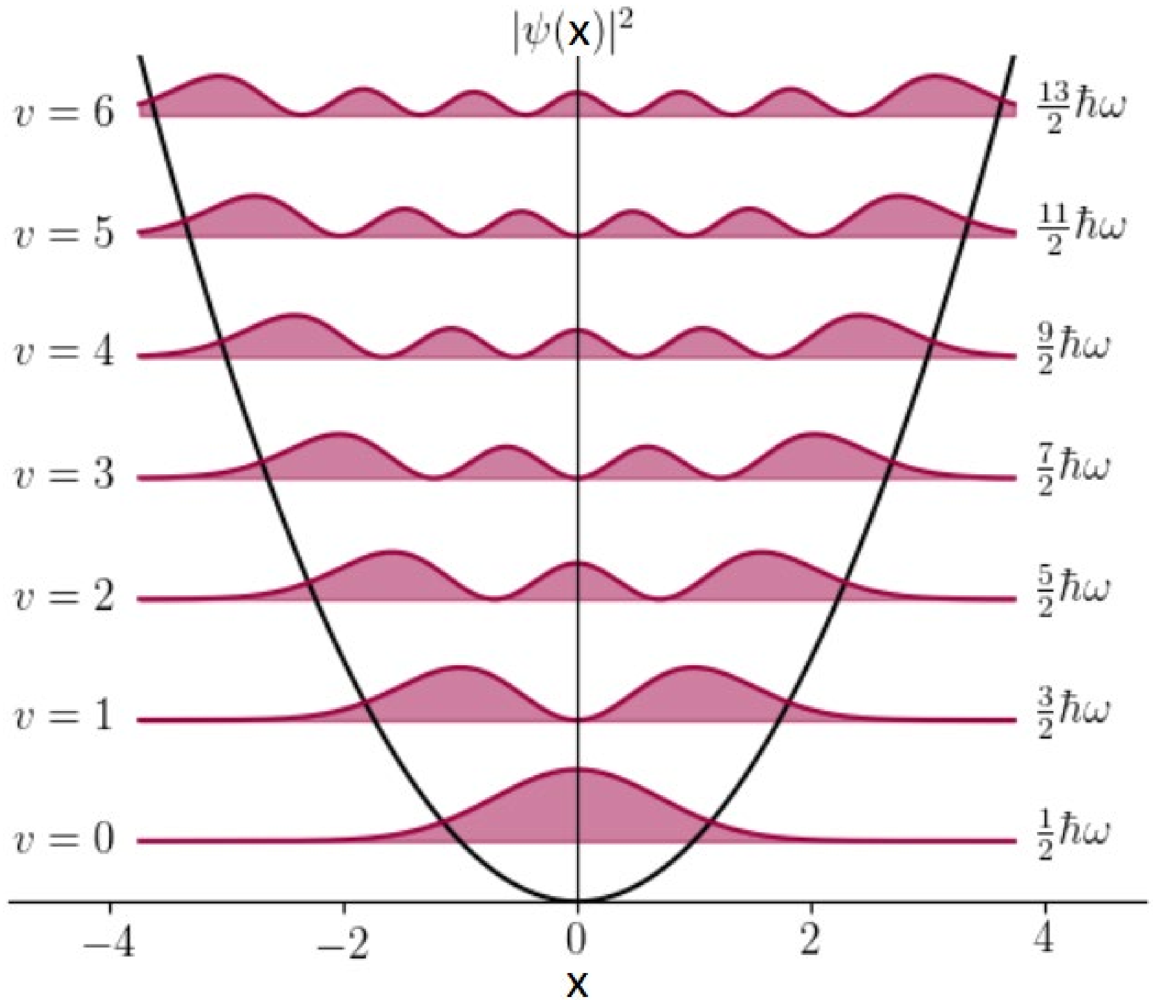


FIG. 1. $[\ln(\omega_p/2\pi\Gamma)]^{2/3}$ vs I for five values of temperature. Lines that intersect the current axis have been drawn through the data in the thermal regime, at the three highest temperatures. The arrow indicates the value of I_0 obtained after corrections for the prefactor were made.

Temperature of electromagnetic environment can be extracted from the slopes

M.H.Devoret, J.M.Martinis, J.Clarke, *Measurements of Macroscopic Quantum Tunnelling of the Zero-Voltage State of a Current-Biased Josephson Junction,* Phys.Rev.Lett. **55**, 1908 (1985)

Delocalization (wave nature)



X – degree of freedom

Maser (Laser), NMR etc.

Deterministyczne otrzymywanie stanu wzbudzonego i podstawowego i wzbudzonego itd:

$$|0\rangle \to (|0\rangle + |1\rangle)/\sqrt{2} \to |1\rangle \to (|0\rangle - |1\rangle)/\sqrt{2} \to |0\rangle$$

Impuls π obraca układ ze stanu podstawowego do wzbudzonego lub odwrotnie:

$$|\langle g\,|\,\psi(t)\rangle|^2 = \cos^2\left(\frac{\mu B_x}{2\hbar}t\right)$$

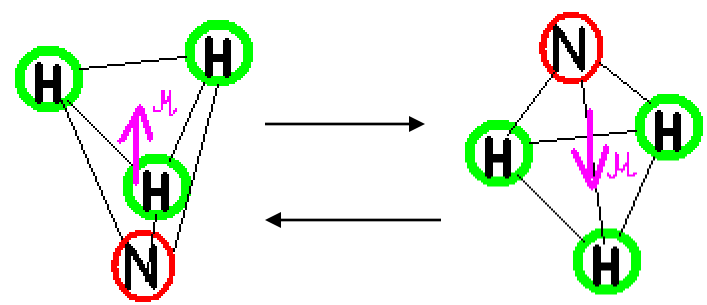
$$\frac{\mu B_x}{2\hbar}t_\pi = \frac{\pi}{2}; \quad t_\pi = \frac{\hbar\pi}{\mu B_x}$$

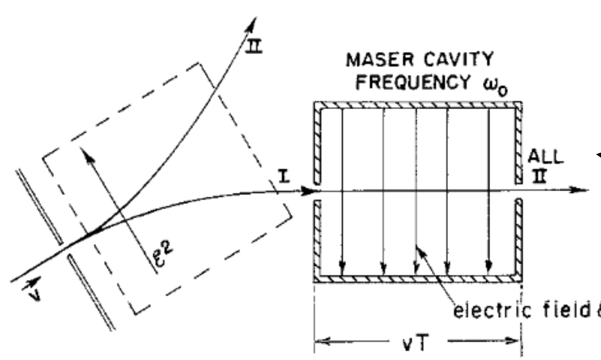
NMR – nuclear magnetic resonance

Badanie relaksacji poprzecznej i podłużnej magnetyzacji (równanie Blocha)

-> dr inż. Tykarski, Politechnika Warszawska

MASER (microwave amplification by stimulated emission of radiation)





from

Feynman

(vol. III, Ch. 9)